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Simple and fast method to develop design formulas for the microwave three port unequal power divider

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Abstract – A simple and fast method to analyse and to obtain design formulas for the microwave three-port unequal power divider is proposed. The method is based on the equivalence of the isolation resistance with a two-port consisting of two transmission lines and a resistance parallel connected between them, such as a generalized ring coupler is obtained. Therefore, analysing this coupler, the all design formulas are obtained for the three-port unequal power divider.
Keywords: unequal power divider, ring coupler.

I. INTRODUCTION

The three-port power divider is a particular case of Wilkinson power divider [1], obtained for two outputs. Different new configurations and fabrication techniques for the three-port power dividers have been also reported recently (see for example [2], [3] and [4]).

The analysis and design of the three-port unequal power dividers have been subject for many papers. Closed design formulas for this power dividers, terminated on arbitrary impedances have been reported in [5], while more flexible formulas useful to design small-size dividers have been reported in [6]. The analysis methods used in [5] and [6] are based on the equivalent circuits for the even and odd operation modes and the transmission line equations, involving many computations.

In this paper, a simple and fast method useful to analyze and to design the three-port unequal power divider is proposed. The method is based on the equivalence of the isolation resistor with a two-port consisting of two transmission lines and a resistor parallel connected between them. It is obtained a generalized ring coupler. This coupler is analyzed and design formulas are developed. Because this coupler is equivalent with the three-port unequal power divider, these formulas may be also used to design the divider. This method leads to the same design formulas as in [6], but more straightforward.

II. EQUIVALENT CIRCUIT DESCRIPTION

The circuit diagram of the wellknown three-port unequal power divider is shown in Fig. 1a, consisting of four transmission lines, where Z_{L1} and Z_{L2} are the equivalent impedances from the nodes 2 and 3, towards the two outputs of the divider, where the load impedances Z_c are connected. This circuit may be redrawn as in Fig. 1b, this one being practically analysed in this paper.

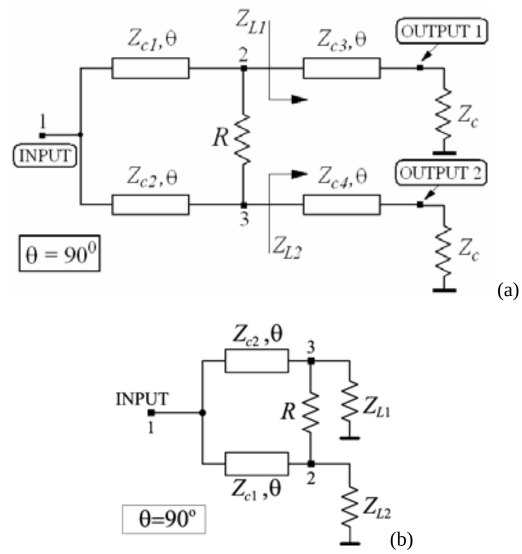

Fig. 1. The complete three-port unequal power divider (a) and the equivalent circuit analyzed by the method presented in this paper (b).

For $\theta = 90^0$, it is easily to show the circuit given in Fig. 2 is equivalent with a series connected resistance R if their $ABCD$ matrices are equal, i.e.:

$$\begin{bmatrix} 0 & jZ_{cy} \\ \dfrac{j}{Z_{cy}} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ R_A^{-1} & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -jZ_{cx} \\ -\dfrac{j}{Z_{cx}} & 0 \end{bmatrix} = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$

Therefore,

$$R = \frac{Z_{cx} Z_{cy}}{R_A} \tag{1a}$$

and

$$Z_{cx} = Z_{cy} \tag{1b}$$

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Therefore, for the central working frequency (when $\theta = 90^0$), the resistance R from Fig. 1b may be replaced by the circuit given in Fig. 2, obtaining the equivalent circuit given in Fig. 3.

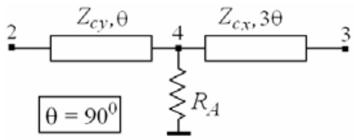


Fig. 2. The equivalent circuit used in this paper to replace the resistance *R* from Fig. 1b.

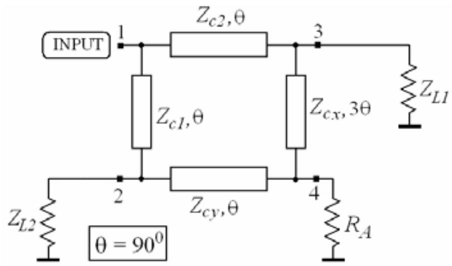


Fig. 3. The equivalent circuit used in this paper to analyze the three-port unequal power.

It is observed for $Z_{c2} = Z_{cy}$, $Z_{c1} = Z_{cx}$ and $Z_{L1} = Z_{L2} = Z_c$, this circuit is the wellknown ring coupler. Therefore, the unequal power divider shown in Fig. 1b may be seen as a generalized ring coupler (coupling different from 3dB, $Z_{L1} \neq Z_c$ and $Z_{L2} \neq Z_c$, where Z_c is the input impedance of the circuit, at the port 1).

The equivalent circuit given in Fig. 3 will be used in the next chapter to develop the design formulas for any coupling coefficient of the ring coupler. Because of the equivalence between this coupler and the three-port unequal power divider, these formulas will be also used as design formulas for the three-port unequal power divider, for any power ratio at the outputs.

III. DEVELOPING OF THE DESIGN FORMULAS

Considering the port 1 as the input port of the divider, the first design condition is developed imposing the voltage at the port 4 to be equal to zero, meaning that the port 4 is an isolated port ($S_{41} = 0$). This condition may be found starting from the voltage transfer function between the port 4 and the port 1. The equivalent two-port for this task is shown in Fig. 4 (see also Fig. 3).

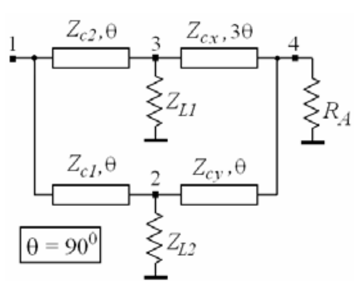


Fig. 4: The equivalent two-port used to develop the condition for which the port 4 is isolated.

It is easily to show that this condition is fulfilled for $1 + \frac{B_{1-2-4}}{B_{1-3-4}} = 0$, where B_{1-2-4} and B_{1-3-4} are the B elements of the $ABCD$ matrices for the two-ports which connect the nodes 1 and 4, through the node 2 and the node 3, respectively (see Fig. 4). Therefore, the port 4 is isolated with respect to the port 1 if:

$$\frac{Z_{c2}}{Z_{L1}} \cdot \frac{Z_{L2}}{Z_{c1}} = \frac{Z_{cy}}{Z_{cx}} \tag{2}$$

Another design condition for the ring coupler is for the ports 2 and 3 which must be isolated one from each other (the ports 2 and 3 are also the output ports for the power divider – see Fig. 1b and Fig. 3). The circuits used for the voltage transfer functions between the port 2 and the port 3 are given in Fig. 5a and Fig. 5b, considering the signal generator connected at the port 2 and at the port 3, respectively (for the first case, the load impedance connected at the port 3 is Z_{L1}, while for the second case, the load impedance connected at the port 2 is Z_{L2} – see. Fig. 3). Following the same procedure as for the condition (2), the ports 2 and 3 are isolated one from each other if:

$$\frac{Z_{c1} \cdot Z_{c2}}{Z_{cx} \cdot Z_{cy}} \cdot \frac{R_A}{Z_c} = 1 \tag{3}$$

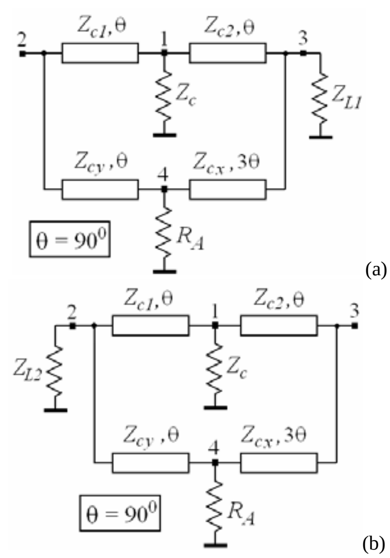


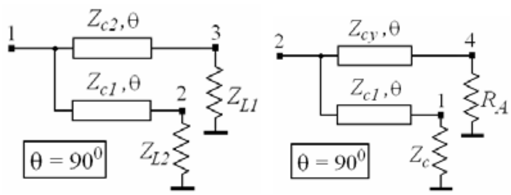
Fig. 5. The equivalent two-ports used to develop the condition for which the port 2 and the port 3 are isolated.

The last design conditions reffer to the impedance matching to the ports 1, 2 and 3. The equivalent circuits to find the input impedance at the ports 1, 2 and 3 are shown in Fig. 6a, b and c, respectively. These equivalent circuits have been obtained taking into account the port 4 is isolated and the ports 2 and 3 are isolated one from each other. Imposing the input impedance at the ports 1, 2 and 3 to be equal to Z_c, Z_{L2} and Z_{L1}, respectively, the following design conditions are obtained:

$$\frac{1}{Z_c} = \frac{Z_{L2}}{Z_{c1}^2} + \frac{Z_{L1}}{Z_{c2}^2} \tag{4a}$$

$$\frac{1}{Z_{L2}} = \frac{Z_c}{Z_{c1}^2} + \frac{R_A}{Z_{cy}^2} \tag{4b}$$

$$\frac{1}{Z_{L1}} = \frac{Z_c}{Z_{c2}^2} + \frac{R_A}{Z_{cx}^2} \tag{4c}$$



(a) (b)

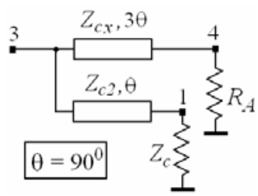


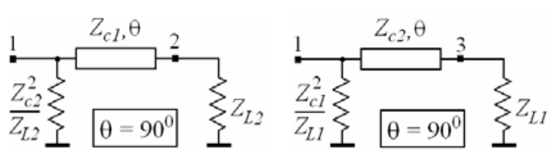
Fig. 6.

The equivalent circuits at the ports 1, 2 and 3, used to develop the impedance matching conditions.

(c)

The scattering parameters S_{21} and S_{31} are computed when the port 4 is isolated with respect to the input port 1 (the condition (2) is fulfilled). The equivalent two-ports for obtaining S_{21} and S_{31} are given in Fig. 7a,b (the load impedances at the ports 2 and 3 are equal to Z_{L1} and Z_{L2}, respectively – see Fig. 3).

It is easily to show that $S_{21} = Z_{L1} / B_{1-2}$ and $S_{31} = Z_{L2} / B_{1-3}$, where B_{1-2} and B_{1-3} are the B elements of the *ABCD* matrices for the two-port which connect the ports 1 and 2 and the two-port which connect the ports 1 and 3, respectively (see Fig. 7a,b without Z_{L1} and Z_{L2}).



(a) (b)

Fig. 7. The equivalent two-ports used to develop S_{21} (a) and S_{31} (b).

They are obtained:

$$S_{21} = -\mathrm{j}\frac{Z_{L2}}{Z_{c1}} \tag{5}$$

and

$$S_{31} = -\mathrm{j}\frac{Z_{L1}}{Z_{c2}} \tag{6}$$

If k is the ratio between the powers at the ports 2 and 1, because the load impedance at the port 2 is Z_{L2} and the input impedance at the port 1 is Z_c, by using (6):

$$k = \frac{Z_c Z_{L1}}{Z_{c2}^2} \tag{7}$$

In the same way, because the load impedance at the port 3 is Z_{L1}, by using (5) and taking into account the circuit is assumed without losses, then:

$$1 - k = \frac{Z_c Z_{L2}}{Z_{c1}^2} \tag{8}$$

Also, it is easily to show the condition (4a) is equivalent with (7) and (9a).

Taking into account (1b) and the conditions (2) and (4b,c), the formulas (7) and (8) lead to the following system of equations:

$$\frac{k}{1-k} = \frac{Z_{c1}}{Z_{c2}} = \frac{Z_{L2}}{Z_{L1}} \tag{9a}$$

$$\frac{1}{Z_{L2}} - \frac{1}{Z_{L1}} = \frac{Z_c}{Z_{c1}^2} - \frac{Z_c}{Z_{c2}^2} \tag{9b}$$

From equations (9a,b), imposing the k coefficient and also the load impedances at the ports 2 and 3 (Z_{L2} and Z_{L1}), the characteristic impedances Z_{c1} and Z_{c2} may be computed using the following formulas:

$$Z_{c1} = \sqrt{\frac{Z_c \cdot Z_{L2}}{1-k}} \tag{10a}$$

$$Z_{c2} = \sqrt{\frac{Z_c \cdot Z_{L1}}{k}} \tag{10b}$$

The resistance R from the power divider given in Fig. 1b may be computed taking into account (1a,b) and (3). It is obtained:

$$R = \frac{Z_{c1} Z_{c2}}{Z_c} \tag{11}$$

The formulas (10a,b) and (11) may be used to design the three-port unequal power divider shown in Fig. 1b. These design formulas obtained by using the analysis method presented in this paper are essentially the same as proposed in [6], but they were obtained more straightforward.

If Z_{L1} and Z_{L2} are known, then: $Z_{c3} = \sqrt{Z_{L1} Z_c}$ and $Z_{c4} = \sqrt{Z_{L2} Z_c}$ (see Fig. 1a), therefore the full circuit of the three-port unequal power divider may be designed.

IV. CONCLUSIONS

If the isolation resistor of the three-port unequal power divider is replaced by an equivalent two-port consisting of a resistance parallel connected between two transmission lines (of electrical lengths equal to θ and 3θ, where $\theta = 90^0$ for the central working frequency), it is obtained an equivalent circuit which may be analysed easily. Using this equivalent circuit, the same design formulas as reported by other authors (applying different other techniques) may be obtained very simple. Because the equivalent circuit used for the three-port unequal power divider is a generalized ring coupler (for coupling different from 3dB and load impedances different from Z_c - usually 50Ω), the design formulas obtained in this paper may be used for this kind of coupler, too.

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