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Research of filtration through sorted river gravel

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Abstract – Experimental results of fluid flow through homogeneous permeable media – sorted river gravel of diameter 22.5, 12.2, 5.4 and 3.9 mm – the paper presents.Experiments for hydraulic gradients corresponding to Darcy's law up to 7,5 (post Darcy's movement) has been undertaken.Using a capillary tube model of filtration, parameters of the movement has been determined: porosity, pores diameter, tortuosity, Forchheimer type quadratic relationship of hydraulic gradient, the dynamic and static specific area and friction factor law in function of pore diameter Reynolds number.

Keywords – Newtonian fluid, post Darcy's filtration, uniform geometry of solid phase.

1. INTRODUCTION

At the beginning of the last century, Forchheimer [1], Slichter and others the incongruity of movement of water within filters with respect Darcy's linear law observed, the obtained discharge of filters being lower than that computed by relationship

$$Q = A \cdot k \cdot i \tag{1}$$

The head loss of filters has been greater than that obtained by (1). Closed to viscous friction contribution, the head loss has to contain part of kinetic energy losses too. The new assumption considered, and based on them three new empirical relationships for the hydraulic gradient have been proposed:

a) power type:

$$i = \alpha \cdot V_0^{\beta} \tag{2}$$

b) quadratic polynomial:

$$i = a \cdot V_0 + b \cdot V_0^2 \tag{3}$$

c) cubic polynomial

$$i = a \cdot V_0 + b \cdot V_0^2 + c \cdot V_0^3 \tag{4}$$

Within pores of permeable materials liquids flow in a curve path lines, their curvature being of the same order like grains size, so the way of liquid is longer than that the bed height, the elementary liquid currents having tortuosities and variable geometry on their length, so local loss of head occurs due to velocity modification. The head loss developing two terms have contribution: the first proportional the superficial velocity by viscous resistance at the pores wall (computed after Poiseuille), and the second proportional the kinetic head, due to inertial resistance [2].

Post Darcy's filtration has many technical applications for: fluid raw material extraction [13], flows within permeable hydraulic structures [11], movement over permeable walls [12], atmospheric pollution dispersion on dense built up areas [14], oxygen-carbon dioxide changing in vegetation zones, fire propagation in forests [14], in permeable bed channels hydraulic [9], rapid filters, in medicine [10] etc.

2. DESCRIPTION OF POST-DARCY'S FILTRATION

Description of fluid movement through permeable media different types of models, from simple to complex, are used:

- *Capillary tube model of the permeable media* consider that bed void is formed in a set of cylindrical small diameter equivalent curve tubes within that the loss of head occurs like in circular pipes, having equivalent roughness equal the fictive tubes diameter (Fig. 1.) [1, 2, 3, 4, 5].

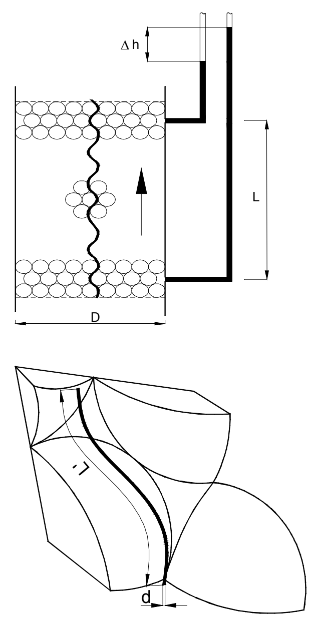

Fig. 1. Scheme of the capillary tub model of permeable media the paper uses this model presented in [1].

Presume the permeable granular material, firmly fixed by upstream and downstream permeable walls, is formed in a set of *m* tubes of diameter *d*, tortuous

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length $L^{'}$, total surface S. The total surface of the set of tubes is considered the solid granular material surface exposed to fluid current.

The total volume of the column of diameter D and length L is W. Tortuosity of the permeable media is:

$$\tau = \frac{L^{'}}{L} \quad (5)$$

Static specific surface A_s is considered the ratio:

$$A_s = \frac{\text{mean surface area of the particles}}{\text{mean volume of solid}}$$

and dynamic specific surface A_d:

$$A_d = \frac{\text{surface area presented by the particles to the flow}}{\text{volume of solid}}$$

resulting:

$$X = A_d / A_s \le 1 \quad (6)$$

Maximum for $X=1$ happen for punctually reciprocal contact of solid particles (ex. spheres).

Fictive tubes diameter for granular permeable material, with porosity n is:

$$d = \frac{4 \cdot n}{A_d \cdot (1-n)} \quad (7)$$

The mean velocity in tubes as a function of superficial velocity is:

$$V = V_0 \cdot \frac{\tau}{n} \quad (8)$$

The pressure drop for a range of pores Reynolds number over passing the limit of creeping flow, including week and strong inertia and transition zones, where:

$$\mathrm{Re}_d = \frac{V \cdot d \cdot \rho}{\mu} \quad (9)$$

could be considered as sum of two terms:

- the first, proportional to the flow velocity, is due to viscous resistance at the walls of the pores, expressed by Poiseuille equation:

$$\left(\frac{\Delta p}{L}\right)_{viscous} = \frac{2 \cdot \mu \cdot \tau^2 \cdot (1-n)^2 \cdot A_d^2}{n^3} \cdot V_0 \quad (10)$$

- and the second, due to inertial resistance, loss of energy caused by direction changing and roughness. Pores with very rough pipes are assimilate, the equivalent roughness k_e having the same range of magnitude as the diameter d of pores. Accepting, by analogy from circular pipes, for Darcy-Weisbach coefficient, Nikuradze relationship, the inertial pressure drop:

$$\left(\frac{\Delta p}{L}\right)_{inertial} = 0{,}0968 \cdot \frac{\rho \cdot \tau^3 \cdot (1-n) \cdot A_d}{n^3} \cdot V_0^2 \quad (11)$$

will be obtained.

The total relative pressure drop becomes:

$$\left(\frac{\Delta p}{L}\right)_{total} = \left(\frac{\Delta p}{L}\right)_{viscous} + \left(\frac{\Delta p}{L}\right)_{inertial} \quad (12)$$

In form of general major linear head loss in pipes:

$$h_r = \lambda \cdot \frac{L}{D} \cdot \frac{V_0^2}{2 \cdot g} \quad (13)$$

results:

$$\lambda = \frac{64}{\mathrm{Re}} + 0{,}7743 \quad (14)$$

or the friction factor:

$$f = \frac{\lambda}{4} = \frac{16}{\mathrm{Re}} + 0{,}194 \quad (15)$$

Instead of relationship (12) customary is the form:

$$\frac{\Delta p}{L \cdot V_0} = M \cdot V_0 + N \quad (16)$$

with

$$M = 0{,}0968 \cdot \tau^3 \cdot \rho \cdot A_d \cdot \frac{1-n}{n^3} \quad (17)$$

and

$$N = 2 \cdot \tau^2 \cdot \mu \cdot A_d^2 \cdot \frac{(1-n)^2}{n^3} \quad (18)$$

Tortuosity τ and dynamic specific surface A_d being micro scale amounts of the permeable media, their determination by direct measurements present some difficulties.

Experimental and statistical drawing of (16) by macroscopic averaged pressure drop measurements Δp on length L, as the superficial velocity function V_0, obtained by flow Q measurement on the whole cross section A and porosity n practical measurements permit computation of tortuosity τ and dynamic specific surface A_d. During experimentation liquid temperature has to be measured, it influences viscosity and density of the liquid. Using method of least squares M and N parameters of (16) results, then:

$$A_d = \left[\frac{N^3 \cdot (0{,}0968 \cdot \rho)^2}{M^2 \cdot (2 \cdot \mu)^3} \cdot \frac{n^3}{(1-n)^4}\right]^{\frac{1}{4}} \quad (19)$$

and:

$$\tau = \left[\frac{M^2 \cdot 2 \cdot \mu \cdot n^3}{N \cdot (0{,}0968 \cdot \rho)^2}\right]^{\frac{1}{4}} \quad (20)$$

are computed. Relationship (16), experimentally calibrated, permits computation of the friction factor:

$$f = \frac{2 \cdot \Delta p \cdot n^3}{L \cdot \tau^3 \cdot \rho \cdot A_d \cdot (1-n) \cdot V_0^2} \quad (21)$$

3. EXPERIMENTAL EQUIPMENT

The experimental equipment contains a vertical cylindrical infiltrometer, with upstream and downstream fixing permeable walls, with upwards water circulation for continuous air evacuation (Fig. 2., Photo 1., 2.) [15].

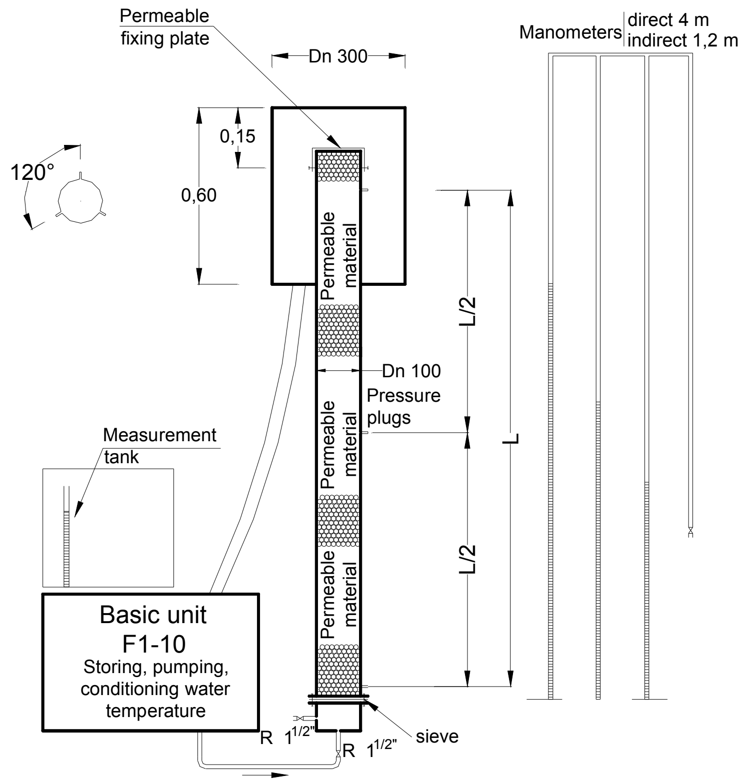

Fig 2. Experimental model and investigated parameters

Photo 1. General view of the equipment with direct manometers

Photo 2. General view of the equipment with indirect manometers

Infiltrometer diameter is D=100 mm, its total length 2.50 m, length between extreme pressure intake port (PIR) L=2.00 m. On its upstream part there is a demountable inlet section, bordered by the upstream permeable wall. On its downstream end the experimented porous material is fixed by other permeable wall that compress it. Loading the experimental granular material is done under vibrations, the compact lying being assured. The infiltrometer by a flexible 1 1/2” hose is supplied and the flow regulation by two 1 1/2” check valves assured. Water storage and pumping is realized by F1-10 ARMFIELD bench, with characteristics H_{pump}=18 m water column and flow Q=1.5 l/s, modified for experimental water temperature automatic regulation commended by a contact thermometer.

The infiltrometer with three pressure intake port at equidistant $\Delta L=1$ m is provided. Each PIR consist in three $d_{PIR}=0.8$ mm orifices at 120^o. Flow measurement is volumetrically, maximum relative flow errors being $\delta Q \leq 0{,}001$. Pressure drop by inclined tube micro manometers (for $\Delta h \leq 0{,}1$ m water column), direct differential manometers ($0{,}1 \leq \Delta h \leq 4$ m water column) and indirect differential manometers with mercury ($4 \leq \Delta h \leq 15$ m water column) are measured. Pressure pulsations at pump by air chamber on pumping pipe are damped. On pressure transmission to manometers capillary sections are inserted for the same purpose. The demountable section of the infiltrometer is provided by an inlet/outlet tap (1/2") for porosity measurements.

4. RESULTS

Experiences with uniform diameter sorted river gravel, as permeable solid material, and drinking water, as liquid phase, on the described installation has been undertaken [15]. Four diameter gravel has been used, (table 1).

Table 1. Geometric characteristics of the used gravel

Nr.	Dn (mm)	Sieves used	
		Shape	d, a (mm)
1	3.9	square square	2.5 4
2	5.4	square round	4 6
3	12.2	round square	10 12
4	22.5	round round	20 25

Diameters of gravel passed over square shape sieves the average of hypotenuse and cathetus has been considered.

During experiments the stable lay of gravel has been assured. Aspect of the river gravel used is shown in photo 3.

Dn 3,9 mm Dn 5,4 mm Dn 12,2 mm Dn 22,5 mm

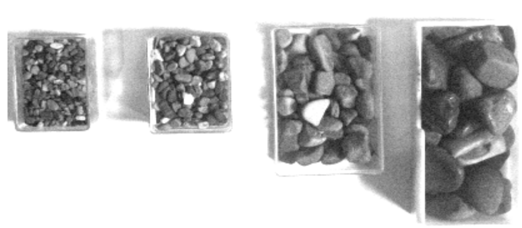

Photo 3. River gravel used in experiments

Loss of head measurements in a function the superficial velocity, expressed in relative hydraulic gradient - $i/V_0 = f(V_0)$ are presented in Fig. 3-6.

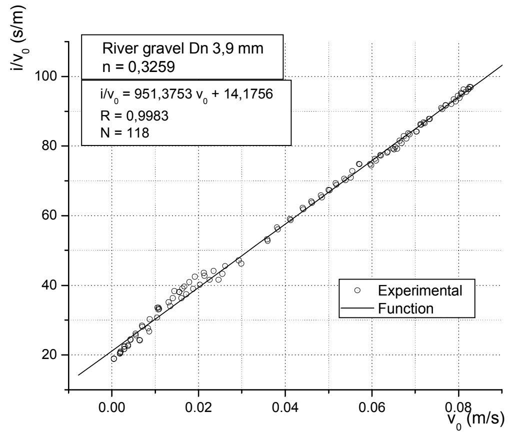

Fig. 3. Experimental relative hydraulic gradient for Dn=3.9 mm gravel

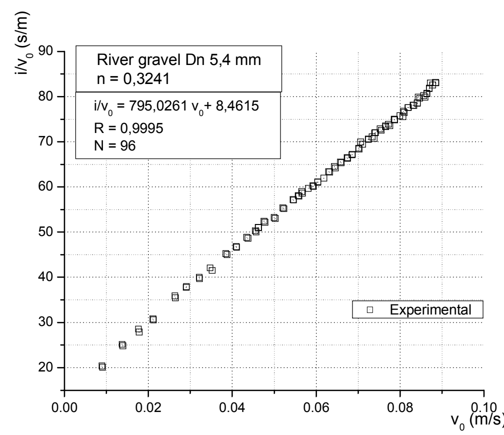

Fig. 4. Experimental relative hydraulic gradient for Dn=5.4 mm gravel

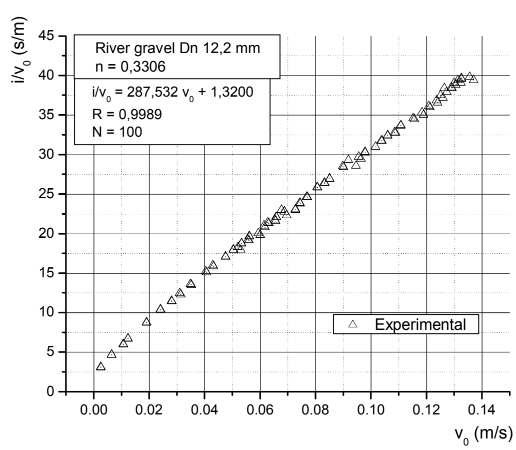

Fig. 5. Experimental relative hydraulic gradient for Dn=12.2 mm gravel

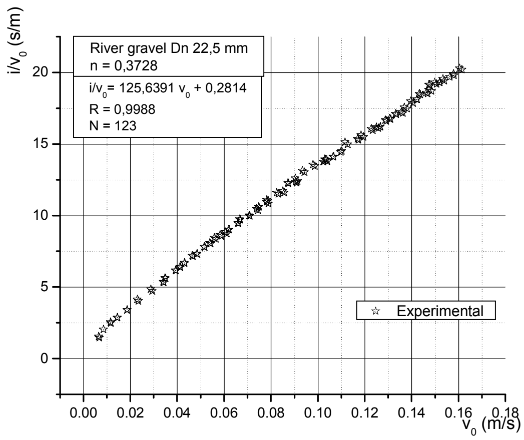

Fig. 6. Experimental relative hydraulic gradient for Dn=22.5 mm gravel

The hydraulic gradient i for the experimented permeable material with respect superficial velocity V_0 corresponds to Fig. 7.

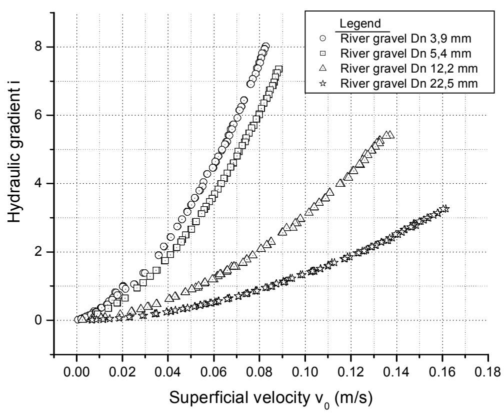

Fig. 7. Hydraulic gradient in the superficial velocity function for river gravel

Other computed hydraulic characteristics refer: M and N parameters of (16), computed for $\Delta h=\Delta p/\gamma$ with determinating coefficient R, specific static and dynamic surfaces A_s, A_d, tortuosity τ, (20), diameter of fictive tubes d, (7), mean velocity in the pore v, (8), pores Reynolds number Re_d, (9), and friction factor f, (21). Experimental conditions and part of determined hydraulic characteristics are presented in Table 2.

Friction factor $f=\lambda/4$, (21), in a pore Reynolds number function, Re_d, for all experimental diameters is draw in Fig. 8.

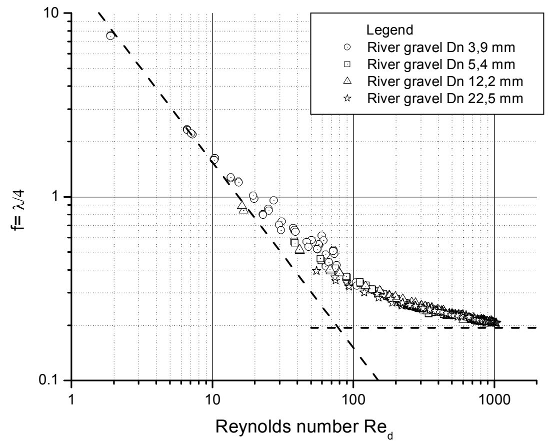

Fig. 8. Pore friction versus pore Reynolds number

5. CONCLUSIONS

Relationship (15) exact enough describes friction factor $f=\lambda/4$ in the wide field of studied pore Reynolds number (Re_d=2 - 1000) for river gravel. The studied Reynolds number field joins week and strong inertia and transition to turbulence of the movement.

In engineering practice Darcy's law could be accepted for $Re \leq 4{,}3$, errors due to kinetic losses being up to 5%. More pretentious works claim higher precision, for errors up to 1% the limit pore Reynold's number is $Re_d \leq 0{,}8$.

Figures 3 - 6 show an easy curvature of experimental data, and the quadratic relationship (2), (16) are approximations of the phenomena. Corrections of Darcy's law for inertial and transition zones have to be polynomial [1, 7, 8], at least cubic.

Table 2. Filtration parameters through homogenous river gravel

Dn (mm)	M (s^2m^{-2})	N (s^1m^{-1})	n	θ (°C)	NME	τ	A_d (m^{-1})	A_s (m^{-1})	X	10^3d (m)
3,9	951,3753	14,1756	0,3251	18,0	118	1,486	1496	1538	0,973	1,288
5,4	795,0261	8,4615	0,3241	18,0	96	1,542	1107	1111	0,996	1,733
12,2	287,532	1,3200	0,3306	19,5	100	1,484	481	492	0,978	4,103
22,5	125,0391	0,2814	0,3728	19,2	123	1,579	266	267	0,996	8,938

APPENDIX

a – coefficient, $m^{-1}\cdot s^{1}$;
NME – number of experiments in a set, dimensionless;
b – coefficient, $m^{-2}\cdot s^{2}$;
Q – flow, $m^{3}\cdot s^{-1}$;
c – coefficient, $m^{-3}\cdot s^{3}$;
Re_d – pore diameter Reynold's number, dimensionless;
d – pore tub equivalent diameter, m;
S – total surface of solid particles, m^{2};
d_b – gravel diameter, m;
V – mean pore velocity, $m\cdot s^{-1}$;
dn_b – gravel nominal diameter, m;
V_0 – superficial velocity, $m\cdot s^{-1}$;
f – friction factor, dimensionless;
W – total volume of the infiltrometer, m^{3};
g – gravity acceleration, $m\cdot s^{-2}$;
X – coefficient, dimensionless;
Δh – loss of head, m water column;
i – hydraulic gradient, dimensionless;
k – filtration coefficient, $m\cdot s^{-1}$;
k_e – equivalent roughness, m;
m – number of fictive pores, dimensionless;
n – porosity, dimensionless;
Δp – pressure drop, Pa;
A – cross section area, m^{2};
A_d, A_s –dynamic and static specific area, m^{-1};
D – infiltrometer diameter, m;
M – coefficient, $Pa\cdot s^{2}\cdot m^{-3}$;
N – coefficient, $Pa\cdot s\cdot m^{-2}$;

Greek Letters

α – coefficient, $m^{-\beta}\cdot s^{\beta}$;
β – coefficient, dimensionless;
γ – unit weight, $N\cdot m^{-3}$;
λ – Darcy-Weisbach coefficient, dimensionless;
ν – kinematic viscosity, $m^{2}\cdot s^{-1}$;
μ – dynamic viscosity, $Pa\cdot s$;
ρ – density, $kg\cdot m^{-3}$;
τ – tortuosity, dimensionless;

6. ACKNOWLEDGMENTS

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