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A SHORT OVERVIEW OF SOME RESULTS RELATING THE MODELING OF FLOW- AND TRANSPORT PROCESSES IN POROUS MEDIA

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Abstract: A short overview of some own results relating the modelling of flow- and transport processes in porous media obtained in the last decades is discussed. There will be referred to the following themes: general aspects of the mathematical modelling and the reliability of the basic equations of the groundwater flow with results embodied in several theorems related to the groundwater flow properties and to the validity of the classical linearization of the basic equations for unsteady groundwater flow; mathematical modelling and calculus methods for drainage and groundwater supply/recover/recharge systems including classical analytical solutions for subsurface drainage related to the drain spacing and to the reversible facilities in sub irrigation and the possibilities to apply elevated methods such the Analytical Element Method, Boundary Element method and Multi Objects Method (AEM_BEM_MOM) and as well as their coupling for modelling complex groundwater Supply Recovery/ Recharge (GWRS/RC);

1. INTRODUCTION, BASIC ECUATIONS

It is known that the *flow equation* and the *balance equation (continuity equation)* constitute the fundamental equations for modelling groundwater flow and different transport or transfer processes in porous media. Currently will be considered the saturated groundwater flow when the **flow equation** is the Darcy Equation and its generalized differential forms for 1D, 2D and 3D case. It describes the movement of the fluid e.g. water in the saturated aquifer as continuum, and represent the mathematical relationship between the specific discharge (flow velocity) V of the fluid in the aquifer and the hydraulic gradient (piezometric head gradient). The introduction of the continuum models for the aquifer–groundwater system enable the mathematical representation of the system's characteristics in spatial functions like as *h(x, y)* and *v(x, y)* in the case of 2D steady flow and *h(x, y,t)* and *v(x, y,t)* in the case of 2D unsteady flow. The **balance equation** or continuity equation for groundwater flow will be derived through the balance of fluid masses taking into account the inflow and outflow in a controlled volume of the porous media, including the effects of source, leakage or groundwater recharge on the elementary volume.

From these equations for groundwater flow currently a *basic equation* will be derived in term of piezometric head.

The basic equations for 2D groundwater flow obtained combining both Darcy and balance equations lead to the well known Boussinesq's nonlinear partial differential equations of second order in term of piezometric head as solution function h(x,y,t) ((Bear, 1972):

$$n\frac{\partial h}{\partial t}-\nabla.(kh\nabla h)=\begin{cases}0 & \text{without leakage}\\ q_{L/km} & \text{with leakage}\end{cases} \quad (1)$$

In the case of the steady flow the basic equation becomes:

$$\nabla.(kh\nabla h)=\begin{cases}0 & \text{without leakage}\\ q_{L/km} & \text{with leakage}\end{cases} \quad (2)$$

In the same way we obtain the basic equations for other processes which superimposed on the groundwater flow like: spreading or recovery of LNAPL (light non-aqueous phase fluid) above of the groundwater table, transport of in ground water dissolved pollutant, heat transfer and other processes. In these cases the basic equation of the considered process and the basic equation of the groundwater flow form a coupled equation system.

There are to mention also some other forms of the basic equations which currently are used in the technical applications:

-The governing equation for free LNAPL transport in homogeneous (i.e. k_l, k_w are constant) unconfined aquifer in term of LNAPL thickness (h_l) over of the ambient groundwater has the form (Carapcioglu at all 1996), (Liao and Aral 2000), (David 2004):

$$n_l\frac{\partial h_l}{\partial t}+\vec{V}_w\nabla(\frac{k_l}{k_w}h_l)-k_l\nabla.(k_lh_l\nabla h_l)=\pm Q_l \quad (3)$$

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where n_l denotes the porosity, V_W is the velocity of an existing ground water flow and k_l, k_w is the hydraulic conductivities for LNAPL respectively water . If V_W =0 eq. (4) coincide with the equation (1) modeling unsteady groundwater flow in unconfined aquifer in term of the entire groundwater depth (h).

-The governing equation of pollutant transport or heat transfer by conduction in aquifer is derived on the same way and it known as:

$$\frac{\partial \phi}{\partial t} + \frac{\vec{V}_{pw}}{R} - D\nabla \cdot (\nabla \phi) = 0 \qquad (4)$$

In this equation Φ is the polutant concentration ($\Phi=C$) or the temperature field ($\Phi=\theta$) and R is the retardation factor. V_{pw} is the pore velocity of an existing ground water flow. D denote the diffusion/dispersion or the thermal diffusivity accordance to the considered process.

These basic equations allow the solving numerous important technical applications of the groundwater engineering. To obtain unique solutions for practical problems, boundary conditions and initial conditions are necessary. So will be formulated boundary value problems (BVP) or initial- and boundary value problems (IBVP).

In the well known monographs are discussed several results for general theoretically solution sensibilities as well as for practical applications Muskat (1937), Polubarinova-Kochina, P. Ya. (1962), Bear, J.(1972), Gheorghita, St., (1966) and other famous authors. I myself have some general properties of the groundwater flow was derived as theorems, the in the Communications of the French Academy were published, David (1969 and 1971). In addition it is to mention also the monograph published in 1998 in Germany (David, I., Groundwater hydraulics, Couvillier Publisher).

The main task of the engineer is to find solutions of such BVP and IBVP problems, which enable the planning of different systems like drains, groundwater supply and recharge systems (GWRS/RC), estimation of light non-aqueous phase liquids (LNAPL) spreading on the groundwater and her recovery, in situ treatment of iron-polluted groundwater and solving heat transfer problems by geothermic energy plants etc.

Only a small number of exact solutions of this nonlinear partially differential equation are known to date but they are very effective to a rapid analyze the influence of different parameter and to prove the reliability of the approximated solutions and of the numerical methods.

These kinds of solutions are developed especially several decades ago while numerical solutions are grown in importance in the last decades. I appreciate that approximations and the analytically approaches remain always actually if it is possible it to obtain.

In the present paper only some own results concerning both aspects analytical and numerical solution will be presented.

2. ABOUT THE RELIABILITY OF THE LINEARIZED BOUSSINESQ'S PARTIAL DIFFERENTIAL EQUATION

The first aspect which we discuss refers to the reliability of the linearized form of the general nonlinear Boussinesq's partial differential equation (1) or (3).

It is known that a very effective approach to obtain solutions of the Boussinesqu equation is the use of the linearized form of this equation**.** There are several techniques for linearization such nonlinear PDE presented in (Bear, 1972). The most widely used technique is to introduce a decomposition of the thickness h_l in the form

$$h_l(x,y,t) = h_a(t) + \delta h(x,y,t) \qquad (5)$$

$h_a(t)$ is the average thickness at time t and δh is the deviation of the h_l from h_a which usually is calculated as a simple average:

$$h_a(t) = \frac{\iint_\Omega h_l(x,y,t)d\Omega}{\iint_\Omega d\Omega} = \frac{1}{\Omega}\iint_\Omega h_l(x,y,t)d\Omega \qquad (6)$$

If the thickness $h_l(x,y,t)$ varies only slightly relating to the average thickness $h_a(t)$, i.e.

$$\delta h << h_a \qquad (7)$$

the nonlinear equation (1) becomes a linear PDE one:

$$\frac{\partial h_l}{\partial t} - D(t)\nabla.\nabla(h_l) = 0 \quad \text{where} \quad D(t) = \frac{k_l}{n_l}h_a(t) \qquad (8)$$

This linear PDE, which approximate very closed the original nonlinear equation (1), can be interpreted as a two-dimensional diffusion equation with time dependent diffusion coefficient $D(t)$.

For this linear PDE vast number of solutions are obtained for various boundary conditions dealing with groundwater flow (e.g. source solutions, error function solutions, successive steady state solutions and so an); (Polubarinova-Kochina 1962; Bear 1972) which are unanimous admitted.

Liao and Aral (2000) used the same average technique for modelling LNAPL spreading. We have shown (David 2005) that this approach can lead to significantly large errors. For that purpose the solutions of the linearized equation obtained with the simple averaging method (6) will be compared with the exact solution of the nonlinear equation for a radial symmetrically spreading of an LNAPL mound of constant volume on the horizontal groundwater table. Some results we can see in Figure 1.

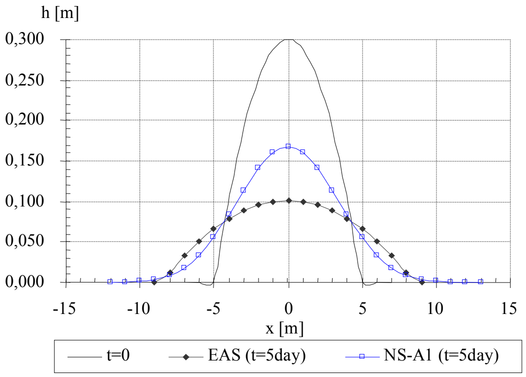

Figure 1 LNAPL thickness profiles by a radial symmetrical spreading as function of time t and x=0 at y=0 calculate as: EAS-exact solution and NS-A1- using simple average thickness (6)

We can see that by LNAPL mound thickness appear large errors especially at x=0 where the error achieved about 50%.

To reduce the deviations of the LNAPL thickness obtained as solutions of the linearized equation using the simple average technique (13), in comparison with the exact solution, a new average technique for the average free product thickness will be suggested and tested:

$$h_a(t) = \frac{\iint\limits_{\Omega} h_l^2(x,y,t)\,d\Omega}{\iint\limits_{\Omega} h_l(x,y,t)\,d\Omega} \qquad (9)$$

In Figure 2 we can see the LNAPL spreading after 5 days exact solution (EAS) and numerically (NS-A2) using the proposed second order average thickness (9)

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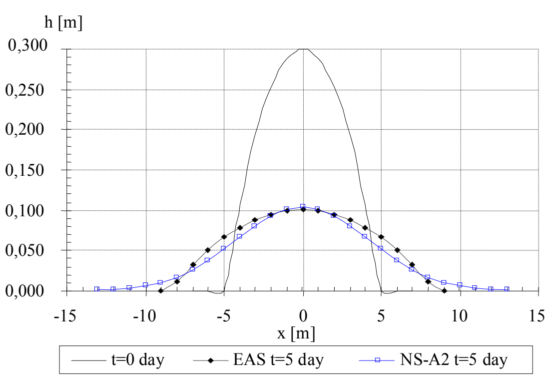

Figure 2 LNAPL thickness profiles by a radial symmetrical spreading as function of x=0 at y=0 after t=5days calculated as: EAS-exact solution and NS-A2- using the proposed second order average thickness (7)

The results shown that the new Weighted Domain Averaging Technique (9) which has been proposed, reduce substantially the calculus errors and consequently allows the improve of the existing numerical and semi-analytical calculation methods for LNAPL mound spreading and migration.

3. MATHEMATICAL MODELING OF DRAINAGE SYSTEM

The first own research regarding the Drainage systems I have developed in the 70th years with colleagues Wehry and Man which is summarised in a book entitled "Actual problems in drainage technique" published in 1982. These results were further developed in recent years especially through developing of computer programs using the elaborated theoretical basis. In this direction it is to mention the paper entitled "Subsurface drainage and its facilities in reversible sub irrigation" from Teusdea, David and Mancia, (2008) published by DAAAM International, Vienna, Austria in Annals of DAAAM for 2008 & Proceedings of the 19th International DAAAM Symposium. In this paper, the previously developed formulas from David (1983 and 1985) was used to calculate the drain- spacing under consideration of both functions of a drainage system, as drainage and as sub irrigation (Fig.3).

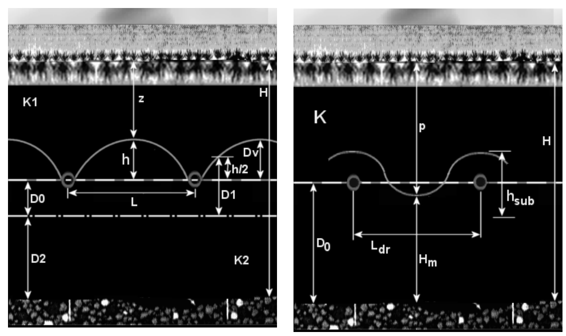

Figure3. a) Scheme of the subsurface drainage structure; b) Scheme of the sub irrigation structure.

4. MODELLING OF GROUNDWATER SUPPLY AND GROUNWATER RECHARGE SYSTEMS

Sustainable groundwater management demands the use of large-scale models which are 1st order models in spatial dimension includes large-scale elements and components such as rivers, lakes, natural groundwater recharge and also groundwater supply and recharge systems (GWRS/RS). Regarding this scale, the effect of a GWRS/RS can be expressed with only global discharge of these objects.

From the 1st order modelling which usually will be realised using FEM the domain of influence of the GWS/RS can be bounded and represents a model region of 2nd order (Fig. 2b) in which the components of the GWS/RS like wells, partially penetrated wells, well with laterals, drains etc. are of great importance. To capture the local characteristics of the flow in the vicinity of the objects of a GWS/RS, like partially penetrating wells or wells with laterals which usually act like singularities a local analysis of the flow features is necessary i.e.3nd order modelling

Whereas in the last decades large-scale flow- and transport models were further developed, mainly by FEM, objects of water supply and recharge are still dimensioned with formulas which have become obsolescent. These formulas but also the FEM-models are inadequate to take account of inner flow in the objects as well as of their mutual interactions in the near-region. A very powerful method for the treatment of singularities in domains with corners and in infinite domains was introduced by using a special mesh grading. To capture the essential characteristics of flow in the vicinity of the objects of a GWS/RS, like partially penetrating wells or wells with laterals, a suitable alternative is to use basis functions of the potential theory.

In the present paragraph basis functions of 2D- and 3D potential theory will be used to build analytical- and boundary elements (EM and BEM) and so called integrated objects for 2nd and 3th order modelling of GWS/RS. We will be discuss especially several own results developed in the last decade.

The analytical element method (AEM) for modelling ground-water flow in shallow regional aquifer which incorporate local 2D/3D flow features as well have been developed and put together by (Strack 1989) and (Haitjema 1995). Several results are obtained by (David 1977) for modelling radial collector well with laterals using semi-analytical methods (e.g. we are used the conformal mapping technique and line strength distributions) and by (David et *al.,* 1995, 1998, 2002) for modelling groundwater flow generated by groundwater supply and recharge systems (GWR/RS) in a bounded flow domain using coupled AEM and BEM. The partially penetrating well (pW) is one of the most important component of a GWR/RS. The modelling of this 3D flow has been analysed by (Muskat 1937), (Polubarinova-Kochina 1962), (Dagan 1978) and (Haitjema 1982), (Haitjema *et al.,* 1988, 2000). The solutions obtained for pW or horizontal well (e.g. radial collectors of a well with laterals) are achieved by assuming that the flow is generated by a distribution of sources or line sinks of unknown strength along the well axis, whose values can be determined from the boundary conditions such as the given head on the well screen. The pW is divided into N intervals (i.e. well elements) and the source strength distributed along the well axis is replaced by the specific discharge as a constant strength for each interval i.e. $\psi_i=q_i$ (i=1,2, ...N). So the discharge Q of the pW can be calculate as a simple sum $Q=\Sigma\psi_i\Delta L_i= \Sigma q_i\Delta L_i$.

In numerous practically cases of GWR/S, it is necessary to take into account the internal head loss such as in the relatively long laterals of the collector wells, and in recharge wells like boreholes (pW) filled with gravel which are used for artificial recharge of the aquifer especially in wooded areas (David *et al.,* 1988, 2002). Since the inner head losses influence the piezometric head distribution along the well or drain, and the changed piezometric head leads to change of the outflow distribution (e.g. Figure 4 shows this for a recharge borehole filled with gravel). The modelling of such a coupled flow system requires iterative algorithmic procedures which will be presented later.

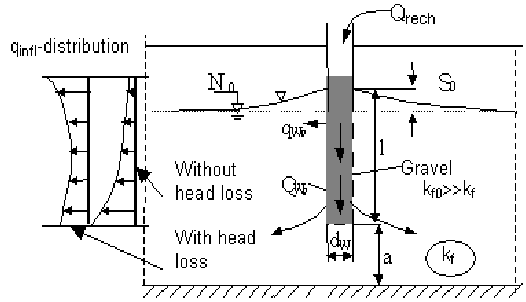

Figure 4 Scheme of the coupled internal and external groundwater flow of a partially penetrating recharge well or borehole filled with gravel.

To solve all these problems we developed two methods including computer programs also:

- Coupled Analytical- Elements and Boundary-Elements- Method (C-AEM/BEM) for GWRS/ RC-System comprising components whose radius is very much smaller her length which we designated as lines objects and

-Generalized Multi-Objects-Method (G-MOM) comprising complex 3D components including any general elements and sub regions with different permeability.

Some parts of these results are or will be presented and published by internationally Conferences (David 2004, 2005, 2010), (Zang & David, 2005).

The C-AEM/BEM -based modeling conside**r a** GWRS/RC-System comprising components such as partially penetrating Wells ($pW^{(j)}$), horizontal objects (hObj) (e.g. well with laterals (W_l), horizontal Drains (D_r) and partially penetrating Trenches (T_r)) in a bounded flow domain is shown in Figure 4. This local model obtains its boundary conditions through regional AEM or FEM modelling. The closed boundary at the local scale (Figure 5 a) is modelled with the indirect BEM, while the GWR/RS is modelled with the AEM through line strength distribution along the axis of its components (objects).

The potential function φ(M) for the local 3-D flow-system in a large extended plane domain D_o^+ with the boundary $C_o = C_{oH} \cup C_{oq} \cup C_{o\Sigma}$ (Figure 5 can be derived by superposition of all partial potentials which represent the contribution of the boundary conditions, recharge area and of the horizontal and vertical components of the GWR/RS.

$$\phi(M)=-\frac{1}{2\pi}\left[\int_{C_0}\psi(P)G(M,P)dl+\int_{D_\sigma}\sigma(P_\sigma)G(M,P_\sigma)d\Omega+\int_{L_{obj}}\psi_{hObj}(P_{hObj})\,G(M,P_{hObj})dl+\sum_{j=1}^{N_{pW}}\sum_{k=1}^{N_{W^{(j)}E}}\psi_k^{(j)}G_k^{(j)}(M,P_{W^{(j)}})\right]+c \quad ...(10)$$

$$M\in D_0^{+*}\cup D_\sigma\;;\;\; D_0^{+*}=D_0^{+}-\{W^{(j)}\cup hObj\},\; P\in C_0$$

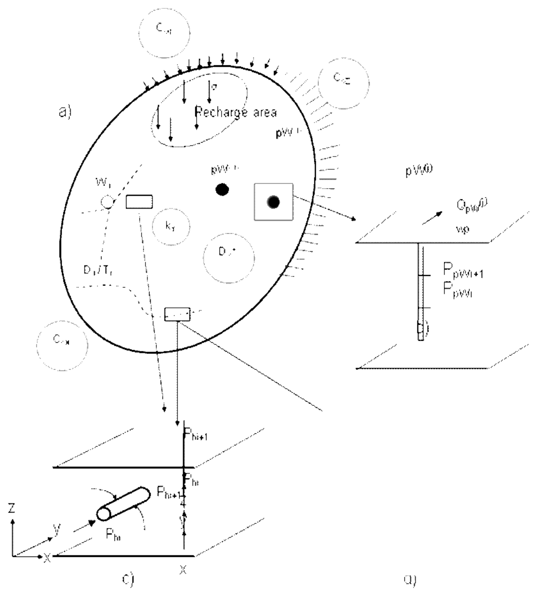

Figure 5 Scheme of the bounded flow domain (a) with several components of a GWRS/RC including partial penetrating Wells ($pW^{(j)}$)/(b) horizontal objects such as well with laterals (W_l), horizontal Drains (D_r)/(c) and partially penetrating Trenches (T_r)/ **(d)**

The specific discharge in the direction "n" $q_n(M)$ is given by Darcy's law:

$$q_n(M)=\frac{\partial \phi(M)}{\partial n_M}=-\frac{1}{2\pi}\Bigg[\int_{C_0}\psi(P)\,F(M,P)\,dl+\int_{D_\sigma}\sigma(P_\sigma)\,F(M,P_\sigma)\,d\Omega+$$

$$+\int_{L_{obj}}\psi_{hObj}(P_{hObj})\,F(M,P_{hObj})\,dl+\sum_{j=1}^{N_{pW}}\sum_{k=1}^{N_{W^{(j)}E}}\psi_k^{(j)}F_k^{(j)}(M,P_{W^{(j)}})\Bigg] \quad ..(11)$$

$$M\in D_0^+\cup D_\sigma\,,\ P\in C_0$$

The first term in both equations represent the effects of the boundary C_o with its given boundary conditions, given head as C_{oH}, inflow/outflow as C_{oq} and $C_{o\Sigma}$ as impervious boundary and are modelled by means of BEM. In the equations (1) and (2), ψ(P) is the strength of the singularities distributed along the boundary C_o, G(M,P) is the known logarithmic potential and F(M,P) its derivation in direction "n" located in M(x,y,z). For the numerical implementation, these terms are expressed with constant boundary elements i.e. unknown constant strength for each boundary element. The second term represents the effect of the recharge area D_σ, with σ(P) the recharge rate distribution. The third terms represent the effect of the horizontal objects, with the strength ψ_{hObj} distributed along the axes of these objects. These terms are implemented using a simplified procedure: 2D horizontal analytical elements as line sinks (LSE) with constant strengths and an additional head loss to model the 3D effects in the vertical plane (Figure 5 c, d). The additional head loss $\Delta h_{v(hObj)}$ in a point P of the hObj can be calculated (David 1977

The last terms of (1) and (2) represent the potential and specific discharges respectively, generated by partially penetrating recharge wells ($pW^{(j)}$) located in $W^{(j)}$, j = 1, 2, ... , N_{pW} with N_{pW} the number of the pWs.

For modelling pW the well length L_W is divided into well elements (pWE_k) with length L_{WEk} and the line sink (LS) distributions on the well axis has an unknown constant strength ψ_k for each well element (pWE_k). The unknown strength distributions ψ along the boundary, along GWRS/RC components (hObj, pW) and the integration constants c will be determined from the equation system which we obtain from the integral representations (10) and (11), taking into account the above described numerical approach and the given boundary conditions. For the hObj and $pW^{(j)}$ the boundary conditions are given heads on the drain/well wall (envelop/screen) respectively.

For the purpose of these calculations, an AEM/BEM-based Computer Programme has been developed and several results obtained (David *et al.,* 1995, 1998 2002) using the simplified assumption that the line sink strength distribution along the well and drain axis can be replaced with the specific discharge, e.g. for pW-elements

$$q_{pWE_{k,k+1}} \cong \psi_{pWEk,k+1} \quad (12)$$

So the total discharge is found by adding up the strengths:

$$Q_{pw}=\sum_{i=1}^{N_{WE}} q_{pWEk,k+1}L_{WEk}=\sum_{i=1}^{N_{WE}}\psi_{pWEk,k+1}L_{WEk} \quad (13)$$

A first example using the developed program is the specific discharge distribution along a collector pipe of a radial collector well (CWL) with three horizontal laterals (radial drain pipes) placed at the centre of a confined aquifer depicted in Figure 5.

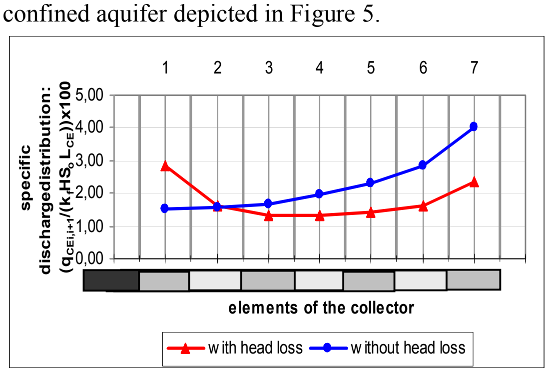

Figure 6 Influence of the inner head loss in the lateral on the discharge distribution per unit length for a radial collector well with three laterals

The parameters for this example (Figure 5) are: aquifer depth H=10m, distance from the vertical shaft centre to the end of the collector L_{CWL}=50m, length of an collector element L_{CE}=6,25m (7 inflow elements), influence radius of the well R=200m, diameter of the collector pipe d_{CWL}=0,25m, drawdown in the shaft S_0=10m, hydraulic conductivity of the aquifer k_f=0,003m/s and the roughness of the pipe wall of 3mm. We observe that the inner head loss can have a very important influence on the specific discharge distribution and on the total discharge as well ($Q_{CWwith\ hl}$= 0,78 $Q_{CWwithout\ hl}$) ; $Q_{CWwithouthl}/(k_f H S_0)$=3,02. On the basis of numerous examples, we come to the conclusion, that for collector wells with laterals the simplified assumption (12) can be accepted in all practical cases.

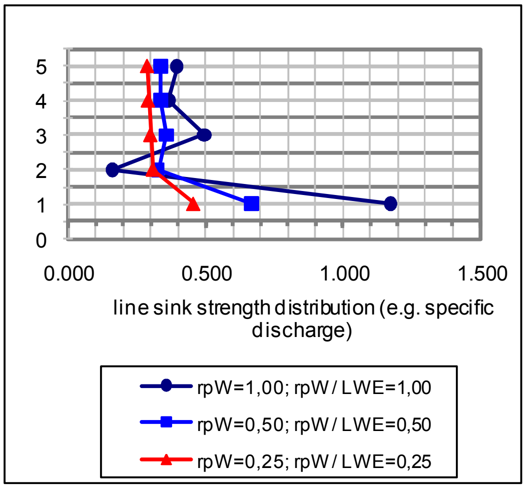

Figure 6 Dependence of the line sinks strength distribution along the well axis of a partially penetrating for different well radius (parameter rpW/L_{WE}) and for 5 LS

In Figure 6 the dependency of the strength distribution on the correlated quotient r_{pW}/L_{WE} (i.e. the well radius to length of the well element) can be demonstrated.

It can be shown that the line sink strength distributions obtained for r_{pW}=0,5m and r_{pW}=1.00m become oscillatory and so as discharge distribution is not realistic. Only the distribution for r_{pW}=0,25 come close to the realistic distribution. Furthermore, it is to be mentioned that

strength distribution also leads to the same results.

Results show that in the case of pW with a relatively large diameter, the discharge distribution along of the pW can be very different in comparison to the LS density (strength) distribution along the well axis, and therefore, invalidates the stated assumption (12). In this case, therefore, the line sink strength distribution has, among others, only a mathematical meaning as a singularity distribution along the pW axis and the assumption (12) does not apply.

So the assumption that equation (12) is only for well elements with a relatively small diameter in comparison to the length of the well element is valid, i.e.

$$d_W << L_{Wk,k+1} , \Psi_{LS\ Wk,k+1} \approx q_{Wk,k+1} \qquad (14)$$

For well elements with a large diameter L_{WE} , equation (12) is not more valid i.e.:

$$d_W > L_{Wk,k+1} , \Psi_{LS\ Wk,k+1} \neq q_{Wk,k+1} \qquad (15)$$

In this case an improvement of the existing method and its implementation was necessary which will be presented in the next.

The Generalized Multi-Objects-Method (G-MOM) comprising complex 3D components and sub regions with different permeability. One considers a bounded continuum domain Ω with the boundary Γ. The domain Ω includes several Non-Singularity-Objects (NSO) and Singularity-Objects (SO) as well as defined sub-domains:

$\Omega^{+}_{iNSO} \subset \Omega$, i=1,2,...,$n_{NSO}$ with the closed boundaries Γ_{iNSO} and
$\Omega^{+}_{jSO} \subset \Omega$, j=1,2,...,$n_{SO}$ with the closed boundaries Γ_{jSO}

In terms of groundwater flow in porous media the NSO represent inclusions with different hydraulic conductivities which can be also impervious or a hole filled with water. SO represent discharge or recharge objects like wells, drains or ditches located arbitrarily and shaped like a stretched tube.

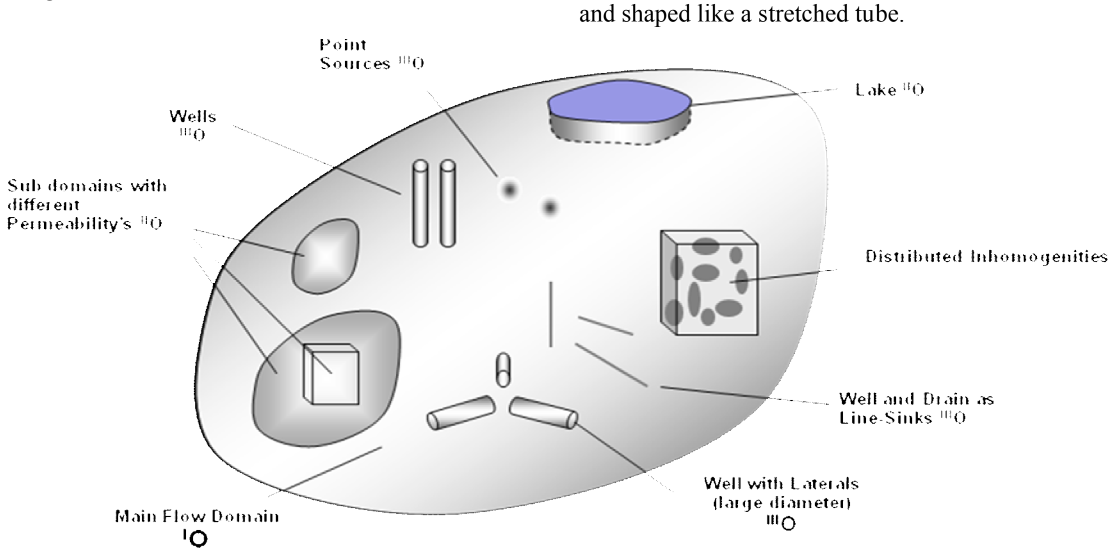

Figure 7 Scheme of Generalized Multi-Objects-Sytem comprising complex 3D components and sub regions with different permeability.

The flow domains are the interiors of all NSO (i.e. Ω^+_{NSO}) and their complementary region to Ω (i.e. Ω^+) are defined as follow:

$$\Omega^+_{NSO} = \bigcup_{i=1}^{n_{NSO}} \Omega^+_{iNSO} \quad \text{and}$$
$$\Omega^+ = \Omega - \Omega^+_{NSO} \cup \left(\bigcup_{j=1}^{n_{SO}} \Omega^+_{jSO}\right) \tag{16}$$

The searched solution is the potential function

$$\varphi(x) = \begin{cases} \varphi^+_{iNSO}(x),\ x \in \Omega^+_{iNSO},\ i=1,2,\ldots,n_{NSO} \\ \varphi^+(x),\ x \in \Omega^+ \end{cases} \tag{17}$$

which satisfied the following boundary conditions and contact conditions on the boundaries

$\varphi^+_{|\Gamma_\varphi} = \varphi_0,$ potential boundary Γ_φ

$\left.\dfrac{\partial \varphi^+}{\partial n}\right|_{\Gamma_q} = q_B$ flux boundary Γ_q

$\varphi^+_{|\Gamma_{jSO}} = \varphi_{0jSO}$ on the SO-boundaries

$\varphi^+_{|\Gamma_{iNSO}} = \varphi^+_{iNSO|\Gamma_{iNSO}}$ and

$\left.\dfrac{\partial \varphi^+}{\partial n}\right|_{\Gamma_{iNSO}} = \left.\dfrac{\partial \varphi^+_{iNSO}}{\partial n}\right|_{\Gamma_{iNSO}}$ on the NSO boundaries (18)

where

$\Gamma = \Gamma_\varphi \cup \Gamma_q;$

φ_0, φ_{0jSO} and $\left.\dfrac{\partial \varphi^+}{\partial n}\right|_{\Gamma_q}$ are gived functions

Mesh dependent methods, namely the Finite Differences Method (FDM) and the Finite Element Method (FEM) are able to obtain approximate descriptions of the domain geometry and the governing equations. Their major disadvantage is their tendency to generate extensively huge data sets and equation systems for three-dimensional problems. Another disadvantage appears when we put these mesh dependent methods into practice for NSO and SO. It is difficult if not completely impracticable to describe arbitrary distributed NSO and SO using FEM/FDM because it requires an update or costly regeneration of the 3D mesh after each modification of the external boundary or of the interior objects (i.e. the distribution and the shape of NSO and SO).

A more efficient approach to solve such complex potential problems is to find an adequate method which allows the determination of the required internal/external potential functions. It is possible by means of boundary integral representations using only boundary elements or distributed singularities to shape NSO and SO respectively.

The searched functions $\varphi^+_{iNSO}(x)$, $\varphi^+(x)$ can be represented as surface single layer potentials similar to the 2D problems analyzed by (DAVID 1995).

$$\varphi^+_{iNSO}(x) = \frac{1}{4\pi}\int_{\Gamma_{iNSO}} \frac{\psi_{\Gamma_{+iNSO}}}{r(\xi,x)} d\Gamma + c_i,\quad x \in \Omega^+_{iNSO} \quad \text{and} \tag{19}$$

$$\varphi^+(x) = \frac{1}{4\pi}\int_{\Gamma} \frac{\psi_\Gamma}{r(\xi,x)} d\Gamma + \frac{1}{4\pi}\sum_i \int_{\Gamma_{iNSO}} \frac{\psi_{\Gamma_{-iNSO}}}{r(\xi,x)} d\Gamma + \sum_j \int_{l_j^{(n)}} \frac{\psi_j}{r(\xi,x)} dl^{(n)} + c$$

where

$\psi^+_{+iNSO}(\xi)$, $\psi^+_{-iNSO}(\xi)$, $\psi^+_{\Gamma}(\xi)$ and $\psi^+_j(\xi)$ are unknown density distributions along the different boundaries. In the same representation $l_j^{(n)}$ are the spatial supports of the singularities to generate Singularity Objects (SO): point singularities (n=0), line singularities (n=1) or surface singularities (n=2).

In Figure 8 are presented some representative example calculated with the on MOM basis developed PC-program.

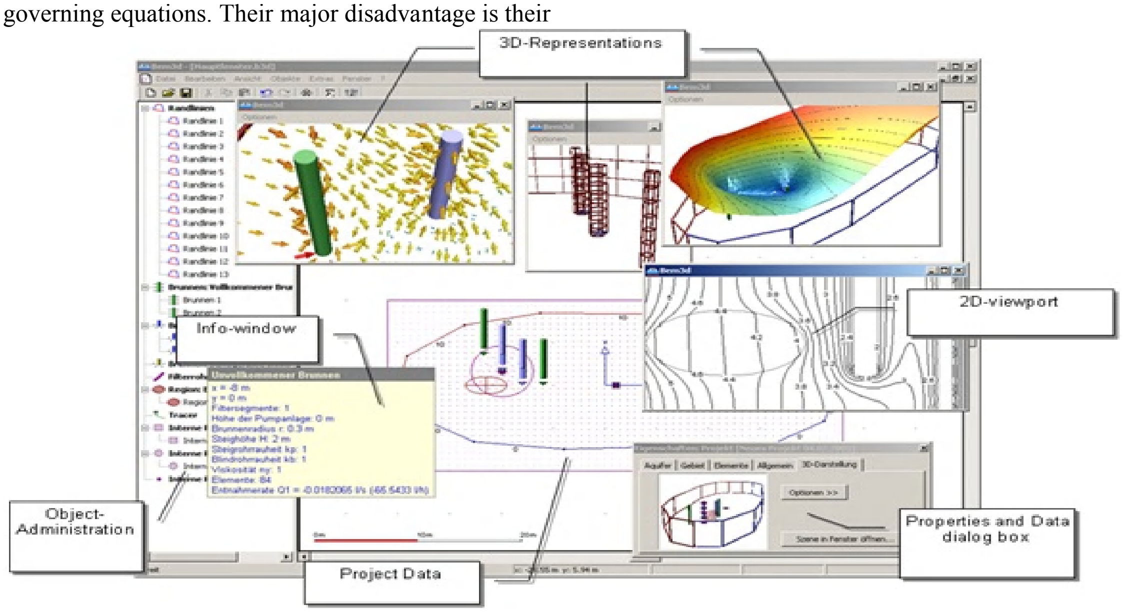

Figure 8 A summarized scheme of the developed Software-Implementation for GMOM

5. CONCLUSIONS

It is to note noted that although the area of the Groundwater modeling is a classic research field there are always new problems to investigate and to find better solution methods.

In this context the paper present some new results which supplemented the current methods. In this context the paper present some new results which supplemented the current methods. One of these is the proposal new average method for the linearization of the Boussinesq partially differential equation. The proposed linearization second order eliminates practically the approximation errors which are introduced by linearization. So it is possible the extension the application field of the linearized Boussinesq equation to solve new problems like a LNAPL spreading and recovery modeling.

The Generalized Multi-Object Method (GMOM) is able to solve groundwater flow problems of arbitrary geometrical configuration of 3D heterogeneities elegantly. We solve singular integral by distributing the density contours on additional boundaries, which lies outside or inside the real boundaries. As an extra advantage, this generates smooth potential distribution even close to and on the boundaries after discretisation and numerical integration. This mesh less method allows the reduction of the discretization.

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