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Considerations about the drawing of the open channels

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Abstract: Drawing the open channels flow of trapeze shape is not a just simple problem hence between the flow discharge and geometric parameters a nonlinear relationship there is. In the classic problems when the flow discharge and one of the geometrical parameters (like breadth and water depth) are given, the solution has one solution and that is even simple calculus to obtain it. An interesting problem appears when given data are represented by the flow discharge and velocity values and both breadth channel and water depth are requested. The problem has limitations and when the solution exists, it's possible to obtain two values for the geometric parameters. The paper presents a new approach of the last problem in the drawing of the open channels flow of trapeze shape with pertinent comment.

Keywords: open channels flow, drawing, solution limitation

1.INTRODUCTION

For a typical trapezoidal cross section by combining both the continuity and Chezy equations the governing equation is obtained that describing the relationship between discharge and geometric parameters:

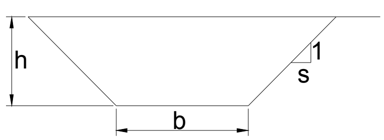

Figure 1. Definition sketch

$$Q = \frac{1(\beta + s)^{1.66}}{n(\beta + s')^{0.66}} h^{2.66} \sqrt{i} \qquad (1)$$

where:
Q-the open channel discharge [L^3T^{-1}];
n-Manning coefficient, dimensionless;
β-the channel bottom width b divided by the hydraulic depth h, dimensionless;
i- channel longitudinal slope, dimensionless;
s-bank slope of the channel, dimensionless;
s'- derived coefficient: $s' = 2\sqrt{1+s^2}$, dimensionless, and
h- the normal water depth in the channel, [L].

The drawing of such shape open channels consists in four different categories of problems, as follow:

i) Q, i, s, n and h parameters are given and the width b is requested;
ii) Q, i, s, n and b parameters are given and the hydraulic depth h is requested;
iii) Q, i, s, n and β (usually, but not always, the most efficient hydraulic section) parameters are given and the both width b and hydraulic depth h are requested;
iv) Q, i, s, n and the velocity v parameters are given and the both width b and hydraulic depth h are requested in so called permissible velocity method.

For the two categories (i) and ii) from the problems the design is based on the general relationship (1) and design curves. In the recent years the design can be performed by the trial-and-error method grace on the computers evolution.

In the third category, based on relationship (1), the normal depth can be directly obtained and by the most efficient hydraulic section propriety the channel width is obtained. All the above types of problems have an only one solution.

The last category of problem, called be some authors like permissible velocity method, will be discussed in the next section.

2. PERMISSIBLE VELOCITY METHOD

In this category of the problem the design must ensure that the flow velocity under uniform flow conditions is less than the permissible flow velocity one. The permissible flow velocity is defined as the mean velocity at which the channel bottom but also the sides are not eroded. This value depends upon many factors of which the most representative are the type of soil and the size of particles. Others factors are represented by the general hydraulic depth and the channel alignment that can induces secondary currents. From this reason in the case of

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the trapezoidal cross section which are erodible, the first step is to establish the side slope depending on the type of soil. The values wary from 1:1 in the case of clay to 3:1 in the case of sand (Figure 1).

The values of permissible flow velocity depending on the type of soil are presented in the Table 1, after [2].

Table 1. Maximum permissible flow velocity

No.	Material	Velocity (m/s)
1	Fine sand	0.6
2	Coarse sand	1.2
3	Earth	0.6-1.8
4	Grass-lined earth	1.8-2.4
5	Soft sand stone	2.4
6	Metamorphic stone	6.1

2.1 HYDRAULIC RADIUS ANALYTICAL SOLUTION [2]

In this method, the recommended steps to solve the problem are the following:

-from the specified material of the channel the Manning coefficient *n*, slide slope *s* and the maximum velocity *v* are selected;

-the required hydraulic radius *R* is computed from Manning formula:

$$\mathrm{R} = \left(\frac{\mathrm{nv}}{\sqrt{\mathrm{i}}}\right)^{1.5} \qquad (2)$$

-the required flow area is computed from continuity equation:

$$S = Q / v \qquad (3)$$

-wetted perimeter is obtained from:

$$P = S / R \qquad (4)$$

-by substitution of the bottom width in the perimeter and flow area expresions:

$$P = b + s' h$$
$$S = (b + h)h \qquad (5)$$

yelds the quadratic equation in h:

$$h^2 (s' - 1) - Ph + S = 0 \qquad (6)$$

By solving the quadratic equation the water depth is obtained ant than based on relationship (5) the bottom witdh b is calculated.

No discussions about the number of solutions are present in this section.

2.2 GRAPHIC- ANALYTICAL SOLUTION

In this method the main idea is to express the required flow area in terms of β parameter. By replacing the expresion of the normal water depth in the channel:

$$h = \left(\frac{Q}{v(\beta + m)}\right)^{0.5} \qquad (7)$$

in the governing equation (1) and by combining with relationship (3) the the required flow area in terms of β parameter is obtained:

$$S = \frac{\sqrt{i}}{n\, v} \frac{(\beta + s)^{0.33}}{(\beta + s')^{0.66}} \left(\frac{Q}{v}\right)^{4/3} \qquad (8)$$

In the equation (7) all the parameters are given, the remaining parameter is β which is used as variable. In this moment is simple to construct the graph $S = S(\beta)$. (Figure 2).

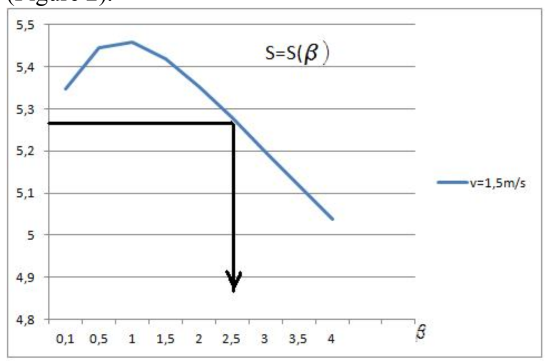

Figure 2. The graph $S = S(\beta)$ (one solution)

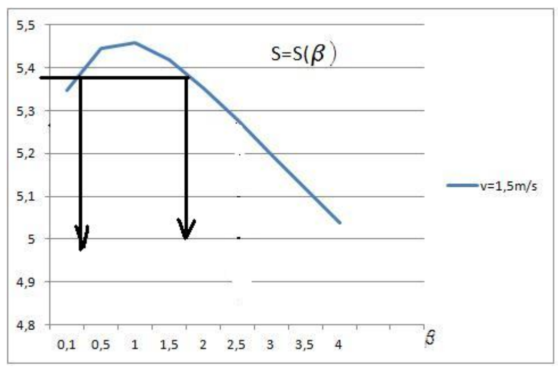

Figure 3. The graph $S = S(\beta)$ (two solutions)

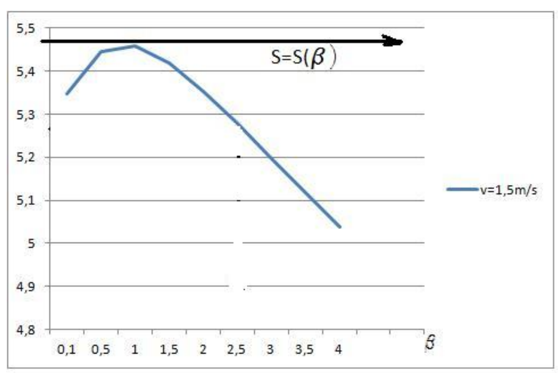

Figure 4. The graph $S = S(\beta)$ (no solution)

The graph was constructed with the numerical values: Q=5 m3/s, v=1.5 m/s (earth), m=1 (clay) the channel slope i=0.002 and the Manning coeficient n=0.014.

By entering in the graph with the value of required flow area (3) the parameter β results.

Once the β parameter is obtained the folowing relationship is used for normal water depth:

$$h = \left[\frac{nQ}{\sqrt{i}} \frac{(\beta + s')^{2/3}}{(\beta + s)^{5/3}} \right]^{3/8} \tag{9}$$

and the bottom width will result $b=\beta h$.

No discussions about the number of solutions are present even in this section. Aditional discussions will be made later.

2.2 ANALYTICAL SOLUTION [1]

In this method we use the notation:

$$A = \left(\frac{nQ}{\sqrt{i}}\right)^3 \left(\frac{v}{Q}\right)^4 \tag{10}$$

By using this parameter and the relationship (7) the quadratic equation in β is obtained:

$$A\beta^2 + \beta(2s'A' - 1) + s'^2 A - s = 0 \tag{11}$$

and, the solutions are:

$$\beta_{1,2} = \frac{1 - 2s'A \pm \sqrt{1 - 4A(s'-s)}}{2A} \tag{12}$$

For the begining the existence of one or two solutions must be checked the condition :

$$A \le \frac{1}{4(s'-s)} \tag{13}$$

or in other terms:

$$Q \ge 4v(s'-s)\left(\frac{nv}{\sqrt{i}}\right)^3 \tag{14}$$

If the condition is carry out than the quadratic equation (10) is solved and the value or values for in β are obtained.

After is solving the quadratic equation the problem is transformed in the problem iii) problem type in which, the normal water depth h is obtained with relationship (9), and the bottom width will result by the already known relationship $b=\beta h$.

3.DISCUSIONS

The hydraulic radius analytical solution seems to be a logic one by the presented steps and his simplity. There are not references about the number of solutions, or worse about the absence of this solution.

In the graphic- analytical solution the first comment is about the poorly accuracy because of the graph. The user has no indications about the limits of the horizontal scale used for β parameter and the grid refinement.

Also, like in the first solution, it is possible in some cases than the horizontal line having the necessary flow area value intersect the graph in two points, in other words to have two solutions. One single solution is possible to have only when the horizontal S line intersect the graph in the top, at the maximum value. In the case in which the horizontal line does not intersect the graph that means that are no solution of the problem, the user did not verify preliminary the existence of some solution.

Finally, the above comments should not have a great importance since the construction equipment used to accomplish a such open-chanel flow can not operate with an accuracy of centimeters units.

The third solution, so called analytical solution seems to be the properly one. This caracteristic is given by the following reasons: the condition of compatibility contained in equation (12) must be verified in order to find out the existence of one, two or none solution. This is usefull to ovoid the complicated and vain plans.

Other comments about the compatibility condition can be made below.

The condition of compatibility of the problem contained in equation (13) and (14) can be expressed by the function f_1 in terms of bank slope of the channel s as follows:

$$f_1 = \frac{1}{4(s'-s)} \tag{15}$$

or,

$$A \le f_1 \tag{16}$$

On the other hand, the quadratic equation (11) in β, $f(\beta)$, suggest that the mathematical condition to have at least one positive solution is expressed by another condition:

$$f(\beta = 0) \ge 0 \tag{17}$$

or,

$$s'^2 A - s \ge 0;\ A \ge \frac{s}{s'^2} \tag{18}$$

Let's note this new function in terms of bank slope of the channel s by f_2:

$$f_2 = \frac{s}{s'^2} \tag{19}$$

By combining the conditions (14) and (16) that produces:

$$f_1 < A \le f_2 \tag{20}$$

The last combination of conditions suggests than in order to have solutions for a such open channel flow design the parameter A defined by relationship (10) must be located between the function f_1 and f_2 as is presented in the Figure 5.

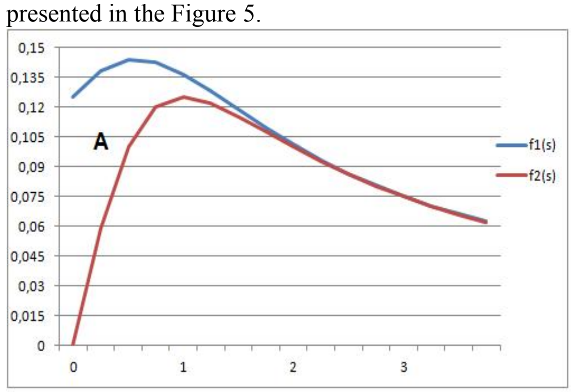

Figure 5. The range of solution for A parameter

That means than if the value of the parameter A is located outside of the range delimited by the curves $f_1(s)$ and $f_2(s)$ the design problem has no solution.

The interesting problem consists in fact that the A parameter which is expressed in terms of permissible velocity, Manning coefficient, channel slope and flow discharge depends only on single parameter consisting in bank slope. This suggest that in the case when is no solution and the bank slope remains constant, the parameters which can modified are in number of four.

This argument can be considered an advantage in the design of the open channel flow in the case of permissible flow velocity method.

4.CONCLUSIONS

Drawing the open channels flow of trapeze shape is a current practice in water engineering. There are four type of design problems consisting in establishing of the normal water depth or bottom width, or both.

The first three category of the design problems are no difficult and can be solved by design curves or by analytic method. All the categories of problems have solutions.

A more interesting problem occurs when given data consists in the flow discharge and velocity values and both breadth channel and water depth are requested.

This is so called permissible flow velocity case. The problem has limitations and when the solution exists, it's possible to obtain one, two values for the geometric parameters or none. There are three methods to solve this case.

The first one, named hydraulic radius analytical solution is a clear solution but there are not references about the number of solutions, or about the absence of this solution.

In the second solution named graphic-analytical solution the first problem appears about the poorly accuracy because of the used graph. The user has no indications about the limits of the horizontal scale used for bparameter and the grid refinement.

The third solution named analytical solution appears to be the most properly one because of his clarity and the compatibility checking of the problem that is verified at the begining of the calcules.

In the case when there is no solution and the bank slope remains constant, the parameters which can modified are in number of four: flow discarge, Manning coefficient, permissible velocity and the channel slope, that means multiple solutions.

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