Buletinul Ştiinţific al Universităţii "Politehnica" din Timişoara

Seria ELECTRONICĂ şi TELECOMUNICAŢII
TRANSACTIONS on ELECTRONICS and COMMUNICATIONS

Tom 55(69), Fascicola 1, 2010

Interpolation Techniques Applied on Sparsely Sampled ECG Signals – part one: method and kernels

Călin Simu[1]

Abstract – This paper presents the importance of electrocardiographic (ECG) signal interpolation. Also, a way to achieve ECG signals interpolation using cubic interpolation kernels is presented: principles of ECG signal interpolation and four kernels definitions are given. Finally, two examples are presented.
Keywords: ECG, cubic interpolation, interpolation kernels.

I. INTRODUCTION

The electrocardiogram (ECG) reflects the electrical activity of the heart. The 12-lead electrocardiography represents a standard clinical procedure for the investigation of heart diseases. It is a non-invasive diagnostic tool for measuring potentials from the body surface, employs up to 10 electrodes placed on the chest, arms and legs, and offers up to 12 waveforms (leads). The procedure is safe, simple, and reproducible, and the relative cost is minimal.
However, there are several situations in which the ECG signal sampling frequency may be a problem:
- different ECG acquisition systems use different sampling rates, thus a comparison of ECG signals may be not easy to accomplish;
- digital information flow within the intensive care unit (ICU) continues to grow, with advances in technology and computational biology; a data amount reduction may be useful;
- within the frame of telemedicine, mobile e-health applications are using heterogeneous channels for data transmission; in isolated areas, no high bit rates are available, thus using low sampling rates in ECG telemonitoring may be a necessity;
- also, in mobile e-health applications, lost sample recovery may be imposed;
- sometimes in ambulatory ECG measurements are used low sampling rates and an adequate signal processing allows reasonably accurate measurements of the QRS complex;
- in heart rate variability (HRV) analysis, high sampling frequencies are needed.

A solution for those problems is to use digital decimation and/or interpolation. Researches in this direction were made starting in the ’80s and the subject is still of interest [1-7]. Principles of ECG signal decimation process are given in a previous paper [8].

II. DIGITAL INTERPOLATION PRINCIPLES

A. *ECG signal*

ECG signal is a quasi-periodical irregular signal with magnitudes between $0.1\ mV_{pp}$ and $2\ mV_{pp}$ and normal rhythms between *60 bpm* and *100 bpm*.
ECG relative power spectrum experimentally obtained from resting healthy subjects (normal ECG’s) and from subjects with arrhythmia (abnormal ECG’s) is shown in Fig.1 [9]. The spectrum of interest is between *0.05 Hz* and *40 Hz*. It may be observed that the spectrum may be approximated with a monotonously decreasing one.

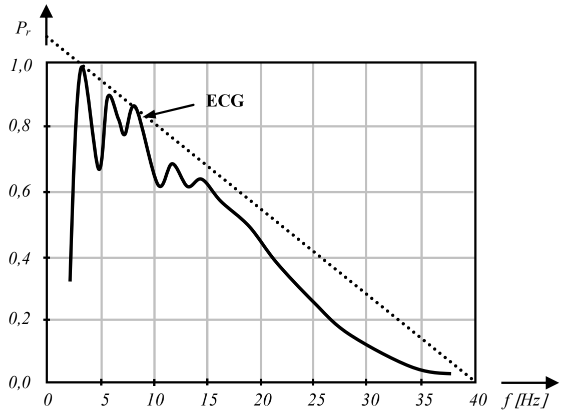


Fig.1. ECG relative power spectrum.

B. *Interpolation considerations*

Interpolation represents a sampling rate increase. The process of interpolating a signal *z[n]*, with an original sampling rate f_d, by an integer ratio R is depicted in Fig.2. The final rate is $f_s = f_d \cdot R$. The sampling rate is increased by inserting $R-1$ samples between each two samples of *z[n]*.
It is easier to use a multistage implementation, calculating the intermediate new sample between two

[1] Facultatea de Electronică şi Telecomunicaţii, Departamentul Comunicaţii Bd. V. Pârvan Nr. 2, 300223 Timişoara, e-mail calin.simu@etc.upt.ro

Fig.2. Interpolating process of *z[n]*.

consecutive initial samples (step 1), then the intermediate new sample between two consecutive known (initial or previously determined) samples (step 2), and so on. Thus, $R = 2$.

For a good compromise between performances and complexity, interpolation uses four consecutive initial samples (z_1^0, z_2^0, z_3^0, z_4^0) to generate a new one (z^1), as depicted in Fig.3.a, where $T_d = 1/f_d$ represents the period of *z[n]* signal; t_1, t_2, t_3, and t_4 represents time intervals between the new sample and the initial ones. The new sample is obtained using the formula:

$$z^1 = h(t_1)\cdot z_1^0 + h(t_2)\cdot z_2^0 + h(t_3)\cdot z_3^0 + h(t_4)\cdot z_4^0, \quad (1)$$

where $h(t_1)$, $h(t_2)$, $h(t_3)$, and $h(t_4)$ are the kernel elements.

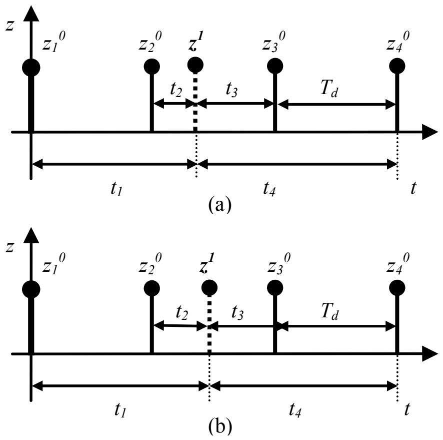


Fig.3. Initial samples used to obtain a new one.

III. INTERPOLATION KERNELS

Several interpolation techniques have been developed and can be found in the literature [10]. The most commonly used methods are the nearest neighbor, linear and spline interpolation techniques. Less common are the polynomial and Lagrange interpolation methods.

Cubic convolution is a third degree interpolation algorithm that fairly well approximates the theoretically optimum *sinc* interpolation function.

The kernel is of form:

$$u_{CCIK}(x) = \begin{cases} \dfrac{(a+2)\left|x^3\right| - (a+3)dx^2 + d^3}{d^3} & \text{for } 0 \le |x| < d \\[2ex] \dfrac{a\left|x^3\right| - 5adx^2 + 8ad^2|x| - 4ad^3}{d^3} & \text{for } d \le |x| < 2\cdot d \\[2ex] 0 & \text{for } 2\cdot d \le |x| \end{cases} \quad . (2)$$

From this relation are obtained the expressions for the interpolation kernels proposed to be used.

Cubic Convolution Interpolation Kernel (CCIK) is obtained from (2) using $a = -0.5$:

$$u_{CCIK}(z) = \begin{cases} \dfrac{3\left|z^3\right| - 5T_d z^2 + 2T_d^{\,3}}{2T_d^{\,3}} & \text{for } |z| \le T_d \\[2ex] \dfrac{-\left|z^3\right| + 5T_d \cdot^2 z^2 - 8T_d^{\,2}|z| + 4T_d^{\,3}}{2T_d^{\,3}} & \text{for } T_d \le |z| \le 2T_d \\[2ex] 0 & \text{for } |z| \ge 2T_d \end{cases} \quad .(3)$$

Cubic Continual (CC) is obtained from (2) using $a = -0.75$:

$$u_{CC}(z) = \begin{cases} \dfrac{5\left|z^3\right| - 9T_d z^2 + 4T_d^{\,3}}{4T_d^{\,3}} & \text{for } 0 \le |z| < T_d \\[2ex] \dfrac{-3\left|z^3\right| + 15T_d z^2 - 24T_d^{\,2} z + 12T_d^{\,3}}{4T_d^{\,3}} & \text{for } T_d \le |z| < 2T_d \\[2ex] 0 & \text{for } |z| \ge 2T_d \end{cases} \quad .(4)$$

Cubic Spline (CS) is obtained from (2) using $a = -1$:

$$u_{CS}(z) = \begin{cases} \dfrac{\left|z^3\right| - 2T_d z^2 + T_d^{\,3}}{T_d^{\,3}} & \text{for } 0 \le |z| < T_d \\[2ex] \dfrac{-\left|z^3\right| + 5T_d z^2 - 8T_d^{\,2} z + 4T_d^{\,3}}{T_d^{\,3}} & \text{for } T_d \le |z| < 2T_d \\[2ex] 0 & \text{for } |z| \ge 2T_d \end{cases} \quad . (5)$$

Another interpolation kernel is represented by **Cubic Lagrange** (CL), given by:

$$u_{CL}(z) = \begin{cases} \dfrac{\left|z^3\right| - 2T_d z^2 - T_d^{\,2}|z| + 2T_d^{\,3}}{T_d^{\,3}} & \text{for } |z| \le T_d \\[2ex] \dfrac{-\left|z^3\right| + 6T_d \cdot^2 - 11T_d^{\,2}|z| + 6T_d^{\,3}}{T_d^{\,3}} & \text{for } T_d \le |z| \le 2T_d \\[2ex] 0 & \text{for } |z| \ge 2T_d \end{cases} \quad . (6)$$

If the new sample is always placed in the middle of the interval between two consecutive known samples, as presented in Fig.3.b, the interpolation kernels will have constant coefficients, given in table 1.

Table 1

coeff. kernel	$h(t_1)$	$h(t_2)$	$h(t_3)$	$h(t_4)$
CCIK	*-1/16*	*9/16*	*9/16*	*-1/16*
CC	*-3/32*	*19/32*	*19/32*	*-3/32*
CS	*-1/8*	*5/8*	*5/8*	*-1/8*
CL	*-1/16*	*9/16*	*9/16*	*-1/16*

IV. EXAMPLES OF ECG INTERPOLATION

In the following part, two examples of ECG interpolation process are presented.

A. *Results for an interpolation process*

The starting set is represented by a set of *64-samples* introduced by the user (for generating a simulated ECG signal) into the interpolator (or a sparsely sampled ECG recording). The set represents a *0.8 seconds* cardiac cycle, depicted in Fig.4.

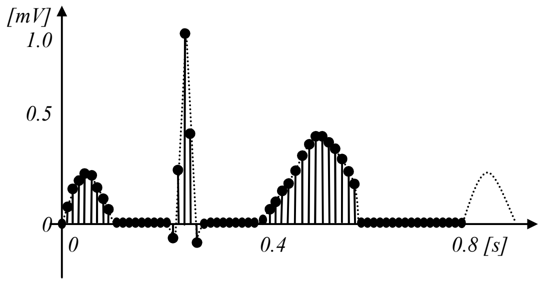
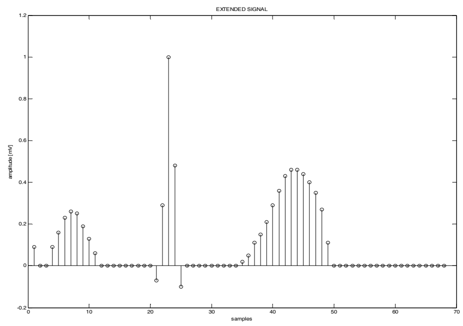
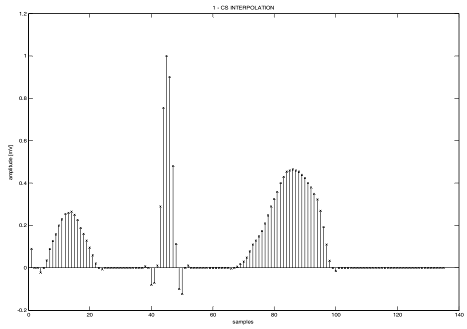


Fig.4. Initial samples, choosed by user.

Three stages of interpolation were used: the first step uses a CS kernel, and the following two steps use a CL kernel. According to [11], this is the best choice.
The interpolator was implemented in MATLAB. In order to obtain all the new samples between the initial ones, an extension of the original set was needed. For this, the first two and the last two sampled were copied "in mirror".
In Fig.5 are presented: (a) the extended set and (b) the result of the first interpolation stage (CS).



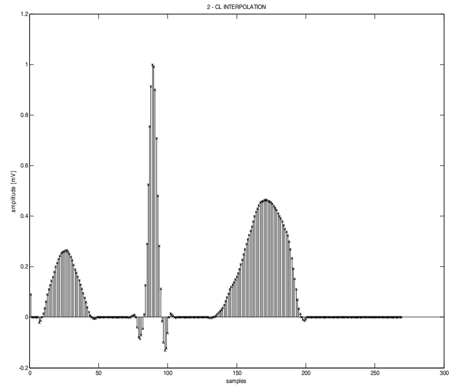
(a)



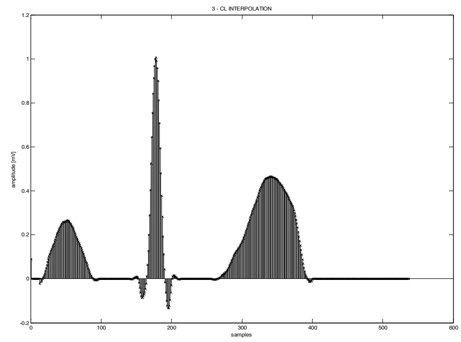
(b)

Fig.5. Initial samples and 1st stage results.

In Fig.6 are presented the results for: (a) the second (CL) and (b) the third (CL) interpolation stages.



(a)



(b)

Fig.6. Results for 2nd and 3rd stages.

Starting from *64 samples*, after the first stage were obtained *68 + 67 = 135 samples*; after the second stage were obtained *135 + 134 = 269 samples*; and after the third stage were obtained *269 + 268 = 537 samples*.

B. *Results for a decimation - interpolation process*

This example refers to the case: accomplish a reduction of ECG data amount / ECG recovery. The first *100 samples* of a real ECG signal are considered (Fig.7). Sampling frequency for the real signal is $f_s = 250\ Hz$, and maximum frequency for the ECG signal is $f_{max} = 40\ Hz$.

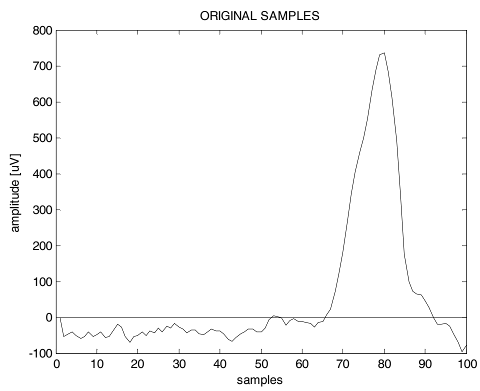
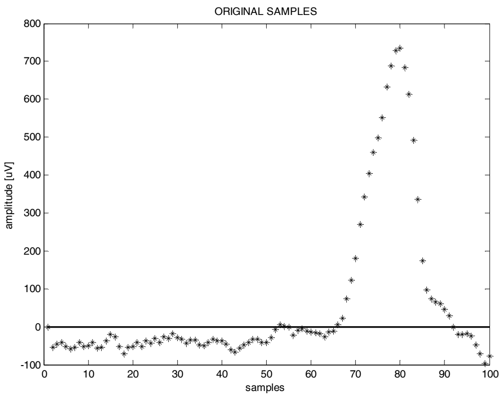


Fig.7. Real ECG signal (linear interpolation).

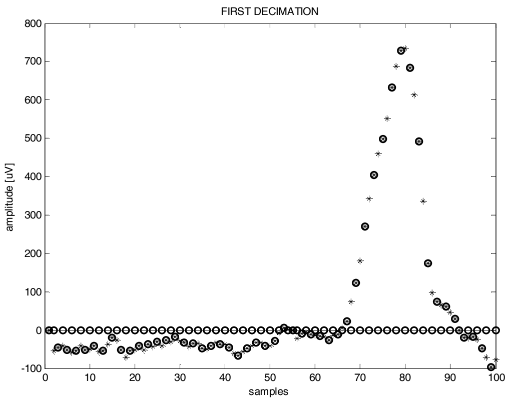
For one decimation step, the new signal will be sampled at $f_{d1} = f_s / 2 = 125\ Hz$; for a second decimation step, the new signal will be sampled at $f_{d2} = f_{d1} / 2 = 62.5\ Hz < 2 \times 40\ Hz$ (sampling theorem condition); this means that only one decimation step can be completed.

Next, one CL interpolation stage is performed, best choice according to [11].

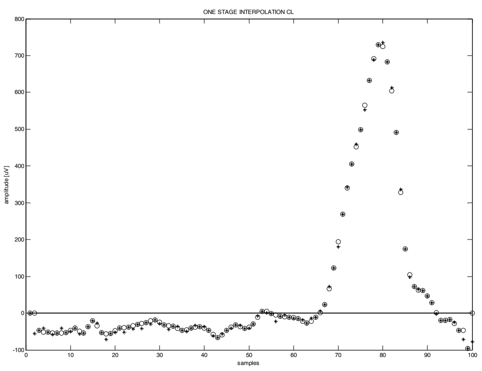
In Fig.8 are represented: (a) initial (real) samples; (b) decimated signal compared with the initial one; and (c) CL interpolated signal compared with the initial one.



(a)



(b)



(c)

Fig.8. Results for one decimation – interpolation step for a real ECG signal.

V. CONCLUSIONS

In the first part are presented some major motives to justify the interest for ECG signal interpolation.

Next, ECG signal features of interest and interpolation principles are presented. Cubic interpolation was preferred, due to a simple implementation and good performances. Four cubic interpolation kernels are presented: Cubic Convolution Interpolation Kernel, Cubic Continual, Cubic Spline and Cubic Lagrange.

Finally, two examples of ECG signal interpolation are given: interpolation of a sparsely sampled ECG recording; decimation (in order to reduce the amount of transmitted / stored data) and interpolation (in order to reconstruct the initial signal). Also, when different ECG acquisition systems use different sampling rates, interpolation may be used in order to obtain similar sampling frequencies, in order to compare ECG recordings.

A comparison between Fig.5.a and Fig.6.b demonstrates interpolation capabilities.

As it can be observed in Fig.8.c, some errors occur. A study of those errors and a comparison between the four kernels performances (and with other interpolation methods) must be accomplished.

REFERENCES

[1] A. Singh, S. Gill, “Interpolation for ambulatory ECG measurements”, *Engineering in Medicine and Biology Society*, Vol.5, 1989, pp. 1397 – 1398.

[2] I. Young, R. Young, R. Hyung, “ECG data compression using fractal interpolation”, *Engineering in Medicine and Biology Society*, Engineering Advances: New Opportunities for Biomedical Engineers. Proceedings of the 16th Annual International Conference of the IEEE, Vol.1, 1994, pp. 161 - 162.

[3] P. Laguna, B. Simon, L. Sornmo, “Improvement in High-Resolution ECG Analysis by Interpolation Before Time Alignment”, *Computers in Cardiology*, Vol.24, 1997, pp. 617-620.

[4] P. Castiglioni, L. Piccini, M. Di Rienzo, “Interpolation Technique for Extracting Features from ECG Signals Sampled at Low Sampling Rates”, *Computers in Cardiology*, Vol.30, 2003, pp.481-484.

[5] M. Manikandan, S. Dandapat, “ECG Signal Compression using Discrete Sinc Interpolation”, *Proceedings of the 2005 3rd International Conference on Intelligent Sensing and Information Processing*, 2005, pp. 14-19.

[6] A. Prieto-Guerrero, C. Mailhes, F. Castanié, “Lost Sample Recovering of ECG Signals in e-Health Applications”, *Proceedings of the 29th Annual International Conference of the IEEE EMBS Cité Internationale*, Lyon, France August 23-26, 2007, pp. 31-34.

[7] G. Clifford, W. Long, G.Moody, P. Szolovits, “Robust parameter extraction for decision support using multimodal intensive care data”, *Phil. Trans. R. Soc.*, A 2009 367, 2009, pp. 411-429.

[8] C. Simu, E. Mârza, “On the Interpolation Error of Decimated ECG Signals”, Sesiunea anuală de comunicări ştiinţifice, Oradea, 1995, *Analele Universităţii din Oradea*, Fascicola Electrotehnică, 1995, pp. 338 – 343.

[9] N. Thakor, J. Webster, W. Tompkins, “Estimation of QRS Complex Power Spectra for Design of a QRS Filter”, IEEE Transactions on Biomedical Engineering, Vol. BME-31, 1984, pp. 702-706.

[10] M. Poth, “Image interpolation techniques”, Uni-obuda C., 2004, http://uni-obuda.hu/conferences/sisy2004/Poth.pdf.

[11] C. Simu, “Interpolarea numerică a semnalelor ECG decimate”, *Proceedings: Al III-lea Simpozion Internaţional "Tinerii şi cercetarea multidisciplinară"*, Ed. Sudura, Timişoara, ISBN 973-99425-9-8, 2001, pp. 128 – 135.