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Asymmetric Turbo Coded Modulation with Transmit and Receive Antenna Diversity

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Abstract – This paper analyses the performances of turbo coded modulation with antenna diversity at transmission and reception, considering asymmetric turbo codes. We studied the cases when the component convolutional codes have memory 2 and 3, respectively, and their generator polynomials are both primitive and non-primitive. Simulations were performed to study these cases, considering both quasi-static and block fading, and the bit error rate (BER) and the frame error rate (FER) performances of asymmetric turbo coded modulation were evaluated. Based on this analysis, we note that primitive polynomials lead to better performances for FER, whereas the non-primitive ones lead to slightly improvements of BER in low SNR range.
Keywords: space-time modulation, antenna diversity, asymmetric turbo codes

I. INTRODUCTION

Even if wireless communications experienced exponential growth during the last two decades, obtaining reliable high speed data services continues to be a major goal for the research community. The main challenge consists in obtaining robust communications under difficult channel conditions. It has been proven that the capacity of a system encountering block Rayleigh fading significantly improves when using multiple transmission and reception antennas [1], [2].

The block fading channel model [3] uses a codeword of length $N=F\cdot L$ with F blocks of length L. The group of F blocks is named a frame. The fading value for each block is assumed constant and each block is sent through an independent channel. Moreover, symbols from the F blocks can be spread using an interleaver, resulting in independent fades. Such an example is the slow frequency hopping technique used in GSM systems.

The error correcting codes have become an indispensable tool for digital communication over noisy channels. Among the error correcting codes, turbo codes with iterative decoding have become an area of maximum interest in the past decade [4]. These codes have been discovered by Berrou, Glavieux and Thitimajshima in 1993 [4].

Lately, a great interest has been shown for space-time coding. Stefanov and Duman [5] introduced turbo-coded modulation for transmission and reception systems with antenna diversity over block fading channels. They only considered symmetric turbo codes with component convolutional codes with memory 2. This paper will consider asymmetric turbo codes for the turbo-coded modulation.

The paper is structured in five sections. Section 2 presents the used system model and the relations specific to the fading channel. Section 3 presents the block schemes of the transmission and reception systems and the relation that governs demodulation for the antenna diversity case. An overview of asymmetric turbo codes is presented and several examples composed by convolutional codes of memory 2 and 3 and different generating polynomials are provided. Section 4 shows simulation results and Section 5 concludes the paper.

II. SYSTEM MODEL

We consider a mobile communication system with N_t transmitting antennas and N_r receiving ones. The information bits are turbo coded, serial to parallel converted and transformed into a constellation symbol. At each time instant, the signal at the modulator output is $c_{t,i}$, transmitted using antenna i, for $1 \le i \le N_t$. All signals have the same transmitting period T and are simultaneously transmitted by a different antenna.

The received signal is a noisy superposition of the transmitted signals corrupted by Rayleigh fading. The $\alpha_{i,j}$ coefficient is the path gain from the transmit antenna i, $1 \le i \le N_t$, to the receive antenna j, $1 \le j \le N_r$. As we assumed block Rayleigh fading, the path gains are modeled by realizations of complex Gaussian random variables, with zero mean and variance 0.5 for each dimension. In addition, the path gains are constant over blocks of L symbols corresponding to $R_c\cdot L$ information bits, and independent from one block to another. R_c is the system spectral efficiency.

At time instant t, the signal received by antenna j, $r_{t,j}$, is given by:

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$$r_{t,j}=\sum_{i=1}^{N_t}\alpha_{i,j}c_{t,i}+\eta_{t,j} \quad (1)$$

where the noise samples $\eta_{t,j}$ are modeled as independent realizations of a complex Gaussian random variable with zero mean and variance $N_0/2$ for each dimension. The signal-to-noise ratio (SNR) is defined as E/N_0, where E is the total energy corresponding for each transmission interval. More precisely, $E=\sum_{i=1}^{N_t}E_i$, where E_i is the constellation symbol energy at transmission antenna *i*.

Equivalently, we can write

$$r_t=Hc_t+\eta_t \quad (2)$$

where

$$\begin{aligned} r_t&=\begin{bmatrix} r_{t,1}, & r_{t,2}, & \dots & r_{t,N_r}\end{bmatrix}^T\\ c_t&=\begin{bmatrix} c_{t,1}, & c_{t,2}, & \dots & c_{t,N_t}\end{bmatrix}^T\\ \eta_t&=\begin{bmatrix} \eta_{t,1}, & \eta_{t,2}, & \dots & \eta_{t,N_r}\end{bmatrix}^T\end{aligned} \quad (3)$$

and

$$H=\begin{bmatrix}\alpha_{1,1} & \alpha_{2,1} & \cdots & \alpha_{N_t,1}\\ \alpha_{1,2} & \alpha_{2,2} & \cdots & \alpha_{N_t,2}\\ \vdots & \vdots & \ddots & \vdots\\ \alpha_{1,N_r} & \alpha_{2,N_r} & \cdots & \alpha_{N_t,N_r}\end{bmatrix} \quad (4)$$

The obtained results are compared to the outage probability corresponding to the capacity of a channel with multiple transmitting and receiving antennas. For a system with no delay constraints, where the number of blocks F is not bounded and the channel is perfectly known at the receiver, this value is

$$C=E\left\{\log_2\det\left[I_{N_r}+\frac{\rho}{N_t}HH^+\right]\right\} \quad (5)$$

where ρ is the signal-to-noise ratio, I_{N_r} is the $N_r\times N_r$ identity matrix and H^+ is the transposed conjugate of *H*. To compute the channel capacity we will assume an ergodic channel and use Monte-Carlo integration method, which averages over a large number of channel realizations.

III. ASYMMETRIC TURBO CODES FOR ANTENNA DIVERSITY SYSTEMS

We will use the block scheme in [5], using asymmetric turbo codes [6] instead of the symmetric ones. The transmitter and receiver structures are unchanged (Fig. 1 and Fig. 2). Data is divided into blocks of *N* bits and coded with a binary asymmetric turbo code [6]. The coded bits are interleaved, serial to parallel converted and transformed into a modulation symbol. Different spectral efficiencies can be obtained by modifying the coding rate and constellation dimensions. As we assumed block fading, the turbo code interleaver dimension will be a multiple of $R_c\cdot L$. The additional interleaver is used to decorrelate successive bits. Its dimension is chosen such that no additional system delay is introduced. The additional interleaver is needed to decorrelate LLR (Logarithm Likelihood Ratio) of adjacent bits. Moreover, it disperses error groups caused by strong fading over the whole frame, leading to increased diversity.

The encoded modulation scheme is obtained by concatenating N_t memoryless modulators through the additional bit interleaver, representing a bit interleaved coded modulation [7] with antenna diversity.

The reception scheme involves a sub-optimal algorithm which first computes LLRs for transmitted bits and then uses them as LLRs for observations from a BPSK modulation over a AWGN channel.

The chosen constellation is bi-dimensional and has the size 2^M, therefore each symbol at the transmission antennas will be represented by *M* bits.

If in (1) and (2) we eliminate the *t* index representing the time, we obtain

$$r_j=\alpha_{1,j}c_1+\alpha_{2,j}c_2+\ldots+\alpha_{N_t,j}c_{N_t}+\eta_j \quad (6)$$

The received signals $r_1,\ldots,r_{N_r}$ correspond to $N_t\cdot M$ encoded bits. Let **b** be the bit vector representing symbols $c_1,c_2,\ldots,c_{N_t}$

$$\mathbf{b}=\left(b_1,\ldots,b_M,b_{M+1},\ldots,b_{N_tM}\right) \quad (7)$$

The group of bits $b_{(i-1)M+1},\ldots,b_{iM}$ is used to determine the constellation symbol for transmission antenna *i*, noted c_i, $i=1,2,\ldots,N_t$. Then, the LLR for l^{th} element of **b**, b_l is given by

$$\Lambda\left(b_l\right)=\log\frac{\displaystyle\sum_{\mathrm{c:c}=f(\mathrm{b}),b_l=1}\prod_{j=1}^{N_r}\exp\left(-\frac{\left|r_j-\sum_{i=1}^{N_t}\alpha_{i,j}c_i\right|^2}{N_0}\right)}{\displaystyle\sum_{\mathrm{c:c}=f(\mathrm{b}),b_l=0}\prod_{j=1}^{N_r}\exp\left(-\frac{\left|r_j-\sum_{i=1}^{N_t}\alpha_{i,j}c_i\right|^2}{N_0}\right)} \quad (8)$$

where $f(\cdot)$ is the modulator function.

An asymmetric turbo code is composed of two recursive convolutional codes with different generator polynomials [6]. In order to improve the BER, in [8], one of the component codes was “weak” (non-primitive feedback polynomial), and the second code was “strong” (primitive feedback polynomial). The weak component code leads to the improvement of the BER at low SNR values, while the strong component code, at high SNR values, being responsible for creating a larger minimum distance of the asymmetric turbo code.

Different combinations of primitive and non-primitive polynomials will be used. The primitive polynomial leads to a maximum cycle length in the states diagram.

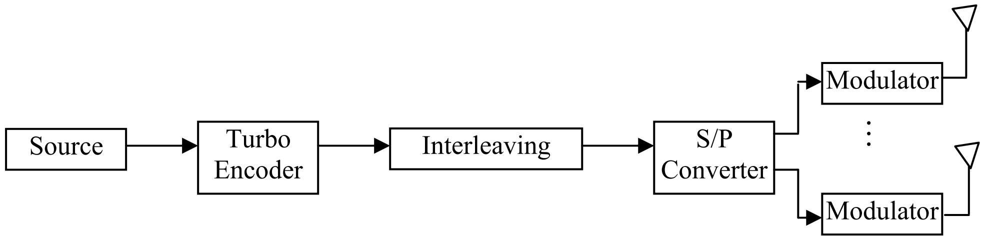


Fig. 1. Turbo encoder transmitter block scheme

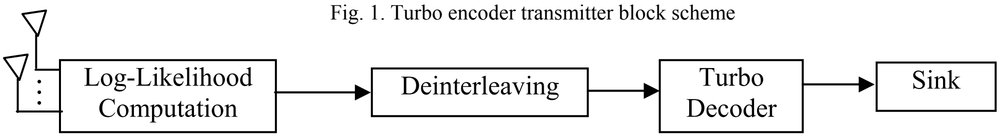


Fig. 2. Receiver block scheme

The turbo codes which have parallel concatenated two systematic recursive convolutional codes (SRCC), c_1 and c_2 are noted with $C_T[c_1 , c_2]$, where the first trellis is terminated and second is not. We have to mention that in literature the focus was on the premise that the component codes are identical. The convolutional codes will be denoted by (FF_{oct}, FB_{oct}), where FF_{oct} represents the feed forward encoding polynomial and FB_{oct} is the feedback encoding polynomial. Both scenarios will be studied, when the feedback polynomial is primitive and, also, when the feedback polynomial is non-primitive.

The simulations were performed for a turbo code having the global coding rate of 1/2, with puncturing, over a block Rayleigh fading channel, with a M-PSK (M- Phase Shift Keying) modulation. The criteria for stopping the iterations are of the type of genie stopper, meaning that the iterations in turbo decoding are stopped when the decoded bit frame is identical to the information bit frame originally coded.

The interleaver used was the 260 length *S*-random or 1300 length QPP (Quadratic Permutation Polynomial) with largest spread. The development of the *S*-random interleaver is performed based on the random choosing of permutation elements, with a restriction over the magnitude of the spreading [9]. The *S* parameter must fulfill the requirement:

$$(\forall) i, j \in \{0,1,\ldots,L-1\} \text{ , with } |i-j| \le S \text{ , we have}$$

$$|\pi(i)-\pi(j)| > S \text{ ,} \qquad (9)$$

where π represents the permutation describing the interleaver.

The increased value of the *S* parameter together with the high normalized dispersion ($\gamma \cong 0.81$) lead to the fact that the use of this interleaver can determine very good performances for lots of applications, despite the used constitutive codes.

QPP interleavers with largest spread ant their advantages are described in [10].

It has been proven that the feedback polynomial must be primitive, in order for the effective free distance (the minimum distance obtained for the input sequence of weight 2) to be high. This applies to AWGN channel.

In Table 1 the component convolutional codes for the turbo codes are presented. Px denotes the primitive generator polynomial of the x state component code and NPx denotes the non-primitive generator polynomial of the x state component code.

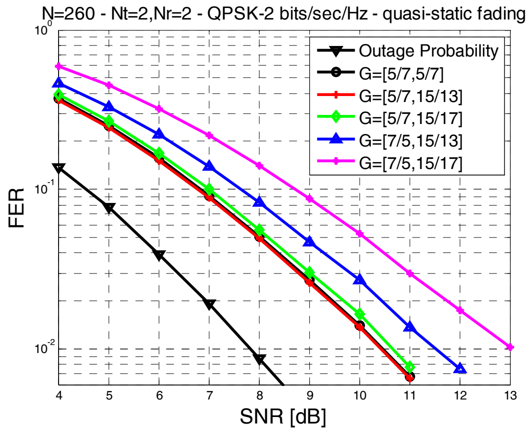
Table1: Table of Turbo-Code Notation

Asymmetric Turbo Code	Short Notation
$C_T[(5,7) , (5,7)]$	P4 – P4
$C_T[(5,7) , (15,13)]$	P4 - P8
$C_T[(5,7) , (15,17)]$	P4 - NP8
$C_T[(15,17) , (5,7)]$	NP8 – P4
$C_T[(7,5) , (15,13)]$	NP4 - P8
$C_T[(15,13) , (7,5)]$	P8 - NP4
$C_T[(7,5) , (15,17)]$	NP4 - NP8

IV. SIMULATION RESULTS

Simulations were performed for turbo code interleaver lengths of 260 and 1300. The coding rate is 1/2, obtained by alternatively puncturing parity bits and the decoding algorithm is Maximum A Posteriori Probability (APP) given in [11]. The used modulation is QPSK (M=2). There are two transmit antennas and two receive antennas, leading to a spectral efficiency of 2 bits/sec/Hz. The fading model was quasi-static Rayleigh fading for the 260 length and block fading for both lengths. For the block fading, the path gains are constant for L=65 successive transmissions, corresponding to 130 information bits for the 260 length (F=2) and for L=130 successive transmissions, corresponding to 260 information bits for the 1300 length (F=5). For the quasi-static fading, the path gains are constants for the whole bit frame corresponding to the interleaver length.

Fig. 3 presents FER and BER curves for the above mentioned cases and five codes from table 1 for quasi-static fading. It can be noticed that, for the FER curves, the codes P4-P4 and P4-P8 lead to identical performances. The performances of the P4-NP8 are closed to the previous ones. The performances degrade for the turbo codes NP4-P8 and NP4-NP8, in this order. We can also observe that the memory 2 code has a stronger influence on the BER performance. The BER performances are relatively close for the simulated codes. At low SNR we can notice a slight performance improvement for codes with non-primitive polynomial component codes.



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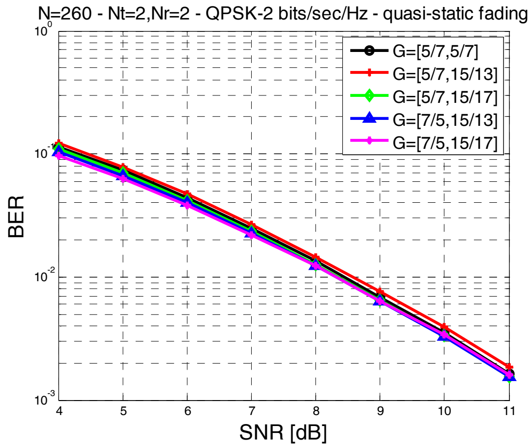
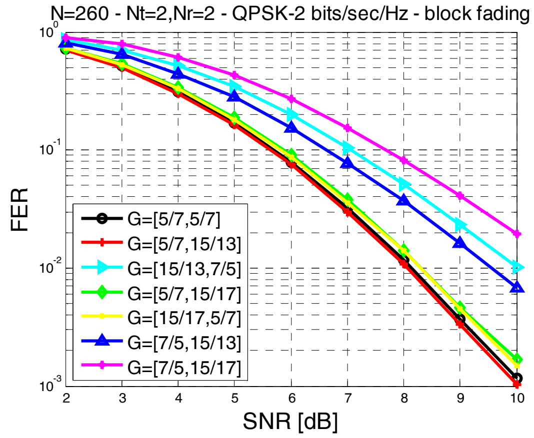


b)

Fig.3. a) FER and b) BER curves for N=260 length interleaver and quasi-static fading (N_t=2, N_r=2)



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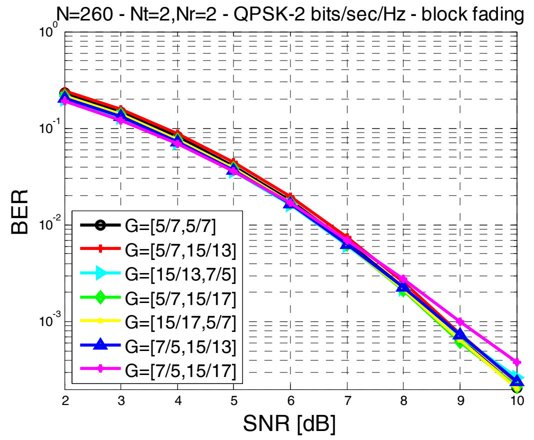


b)

Fig.4. a) FER and b) BER curves for N=260 length interleaver and block fading(N_t=2, N_r=2)

Fig. 4 presents FER and BER curves for the above mentioned cases and codes from Table I for block fading. The performance order for the simulated codes is the same as for quasi-static fading, but the SNR values required to obtain the FER=10^{-2} are about 2.5dB lower for the best codes. The performances degrade for the turbo codes NP4-P8, NP8-NP4 and NP4-NP8, in this order.

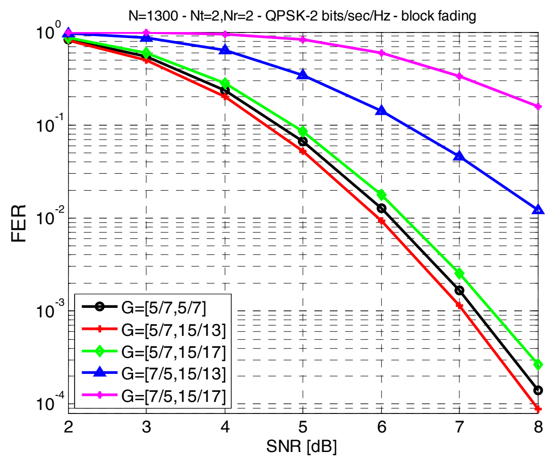
The simulation results for length 1300 are given in Fig. 5 for N_t=2, N_r=2. The codes, in the FER performance order are P4-P8, P4-P4, P4-NP8 with similar performances and NP4-P8 and NP4-NP8 with considerably lower performances. Again, we notice the strong influence of the memory 2 code with primitive polynomial on the performance.

From a BER point of view, the first three codes have similar performances, NP4-P8 has a slightly lower performance and NP4-NP8 the lowest performance. At low SNR we notice a small improvement for codes with non-primitive polynomials.

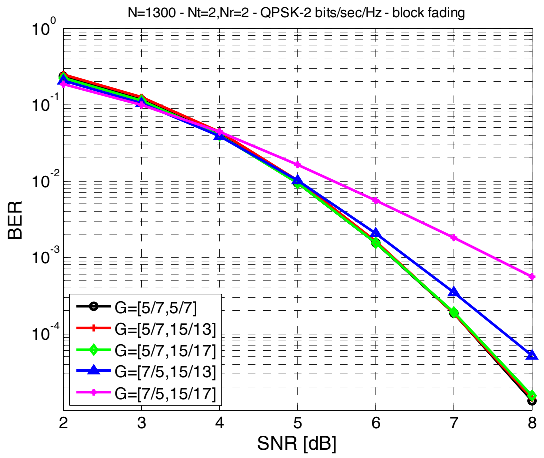
V. CONCLUSIONS

A simulation based analysis of performances of turbo-coded modulation with transmit and receive antenna diversity was performed, considering both quasi-static and block fading. The turbo code component codes are not identical. We considered the cases when the memory of the component encoders is 2 and 3 for primitive and non-primitive polynomials. The turbo-code interleaver lengths are 260 and 1300, respectively.

For both lengths, the performance difference between codes with primitive and non-primitive polynomials is more visible in the FER domain.



a)



b)

Fig. 5. a) FER and b) BER curves for N=1300 length interleaver and block fading (N_t=2, N_r=2)

The codes with primitive polynomials lead to better results and those with memory 2 have a higher influence on the system performance when they are upper codes in turbo codes. This can be explained by the fact that only the first trellis in the turbo code is terminated and second one is not. In the BER domain the performances are similar for length 260 and lower for non-primitive polynomials for length 1300. A slight improvement can be noticed for the codes with non-primitive polynomials at low SNR.

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