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Adaptive Beamforming Applied for Signals Estimated with MUSIC Algorithm

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Abstract – In this paper, a method to combine the adaptive beamforming with an algorithm for estimation of Direction-of-Arrival (DoA) is investigated. It was used the Multiple Signal Classification (MUSIC) algorithm for estimation of DoA. All the simulations are implemented in Matlab, for different number of antenna elements and for different values of SNR.

Keywords: Adaptive Beamforming, Direction – of – Arrival estimation, MUSIC pseudospectrum

I. INTRODUCTION

The potential for using adaptive beamforming to improve the performance of sensor arrays was recognized in the early 1960's in the fields of sonar [1], radar [2], and seismic [3] signal processing. The goal of the adaptive beamforming is to change the directionality of the array. When transmitting, a beamformer controls the phase and relative amplitude of the signal at each transmitter, in order to create a pattern of constructive and destructive interference in the wavefront. When receiving, information from different sensors is combined in a way where the expected pattern of radiation is preferentially observed.

Adaptive beamforming is a subject of recent interest as it is applicable in sensor networks and other distributed systems, where cooperation among antennas is needed. A system with transmitter cooperation or transmit–beamforming includes a number of antennas transmitting similar narrowband signals, each using a different complex weight. The beamforming weights are determined so that the different transmitted waves add constructively in the direction of the receiver. In a system with receiver cooperation or receive beamforming, the different weights are applied to the signals received at the different antennas, so that the weighted signals coherently add for waves received from a desired physical direction, where the transmitter is located.

Transmit and receive beamforming are mathematically very similar and their optimal weights are identical. When applied to sensor networks, receive beamforming is more challenging because it requires the collection of signals received at different antennas.

II. SIGNAL MODEL AND PROBLEM FORMULATION

A. *MUSIC algorithm*

In wireless transmission, the receiving antennas can collect more signals that can be emitted by several sources, as shown in Fig. 1. An important fact is the direction of arrival estimation of signals received from different sources.

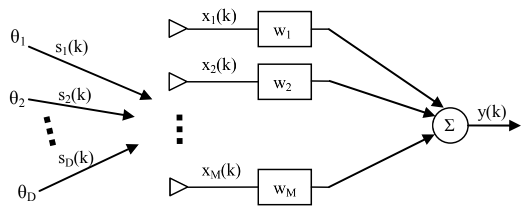

Fig. 1. Uniform Linear Array (ULA)

We define the *M x M* array correlation matrix R_{xx} as [4]:

$$\begin{aligned} R_{xx} &= E\left[\bar{x}\cdot\bar{x}^H\right] = E\left[\left(\bar{A}\bar{s}+\bar{n}\right)\left(\bar{s}^H\bar{A}^H+\bar{n}^H\right)\right] \\ &= \bar{A}E\left[\bar{s}\cdot\bar{s}^H\right]\bar{A}^H + E\left[\bar{n}\cdot\bar{n}^H\right] \\ &= \bar{A}R_{ss}\bar{A}^H + R_{nn}. \end{aligned} \quad (1)$$

where R_{ss} represents the source correlation matrix (*D x D* elements), $R_{nn}=\sigma^2_n I$ represents the noise correlation matrix (*M x M* elements), and *I* represents the identity matrix (*N x N* elements).

The array correlation matrix has *M* eigenvalues ($\lambda_1, \lambda_2, \ldots, \lambda_M$) along with associated eigenvectors $\bar{E} = \left[\bar{e}_1\,\bar{e}_2 \cdots \bar{e}_M\right]$. If the eigenvalues are sorted from smallest to largest, we can divide the matrix $\bar{E}$ into two subspaces: $\bar{E} = \left[\bar{E}_N\,\bar{E}_S\right]$ (the noise subspace and the signal subspace respectively).

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The MUSIC algorithm is based on the assumption that the noise subspace eigenvectors are orthogonal to the array steering vectors, $\bar{a}(\theta)$, at the angles of arrival $\theta_1, \theta_2, \ldots, \theta_D$. Because of this orthogonality condition, one can show that the Euclidian distance $d^2 = \bar{a}^H(\theta)\bar{E}_N\bar{E}_N^H\bar{a}(\theta) = 0$ for each and every arrival angle $\theta_1, \theta_2, \ldots, \theta_D$. Placing this distance expression in the denominator creates sharp peaks at the angles of arrival. The MUSIC pseudospectrum is

$$P_{MUSIC}(\theta) = \frac{1}{\bar{a}(\theta)^H \bar{E}_N \bar{E}_N^H \bar{a}(\theta)}. \qquad (2)$$

B. *Adaptive beamforming*

The basic idea behind beamforming techniques [5] is to “steer” the array in one direction at a time and measure the output power. When the “steered” direction coincides with a DOA of a signal, the maximum output power will be observed. The development of the DOA estimation schemes is essentially the design of an appropriate form of output power that will be strongly related to the DOA.

Given the knowledge of array steering vector, an array can be steered electronically just as a fixed antenna can be steered mechanically. However, the array pattern can change shape in addition to changing orientation. A weight vector w can be designed and then used to linearly combine the data received by the array elements to form a single output signal *y(t)*,

$$y(t) = w^H x(t). \qquad (3)$$

The total averaged output power out of an array over *K* snapshots can be expressed as [6]:

$$P(w) = \frac{1}{K}\sum_{k=1}^{K} w^H x(t_k) x^H(t_k) w = w^H R_{xx} w. \qquad (4)$$

Different beamforming techniques have been developed by measuring the above output power with different choices of the weighting vector w. In the conventional beamforming approach [7], $w = \bar{a}(\theta)$ with θ being a scanning angle that is scanned over the angular region of interest for a ULA of *M* elements. The vector $w = \bar{a}(\theta)$ is defined similarly as the steering vector [4], but with an arbitrary scanning angle θ:

$$\bar{a}(\theta) = [e^{-jkd\sin\theta} \; 1 \; e^{jkd\sin\theta}]^T. \qquad (5)$$

For each look or scanned direction θ, the average power output *P(θ)* of the steered array is then measured or computed with (4).

In other words, the output power versus θ is recorded with (4). It can be shown that when $\theta = \theta_i$, an impinging angle of the signal from source i, the output power *P(θ)* will reach a peak or maximum point. At this moment, $w = \bar{a}(\theta - \theta_i)$ aligns the phases of the signal components received by all the elements of the array, causing them to add constructively and produce a maximum power.

In practical computations, $w = \bar{a}(\theta)$ is normalized as:

$$w = \frac{\bar{a}(\theta)}{\sqrt{\bar{a}^H(\theta)\bar{a}(\theta)}}. \qquad (6)$$

By inserting the weight vector equation (6) into (4), the output power as a function of angle of arrival, or termed as spatial spectrum, is obtained as:

$$P(\theta) = \frac{\bar{a}^H(\theta) R_{xx} \bar{a}(\theta)}{\bar{a}^H(\theta)\bar{a}(\theta)}. \qquad (7)$$

The weight vector (6) can be interpreted as a spatial filter; it is matched to the impinging spatial angles of the incoming signal to produce a peak but attenuate the output power for signals not coming from the angles of the incoming signals. Intuitively, it equalizes the different signal delays experienced by the array elements and maximally combine their respective contributions to form a peak in output power at the angles of the incoming signals.

III. SIMULATION RESULTS

In this section, computer simulations are provided to substantiate the performance analysis. In all cases, the impinging angles of the sources are relative to the broadside of a linear uniform array. The space between two adjacent array elements is one half of a wavelength. The additive background noise is assumed to be spatially and temporally white complex Gaussian with zero – mean.

It is considered that two signals are emitted from two sources placed on two directions. The direction for desired signal is 0°, and the direction for interference signal is +10° and +50° respectively. These two signals are considered to have equal amplitudes.

In the first case, we consider an array formed by 8 antenna elements and the desired signal is supposed to be received from 0° and the interference signal from +50°. The MUSIC pseudospectrum obtained for a value of SNR equal to 0 dB is presented in Fig. 2.

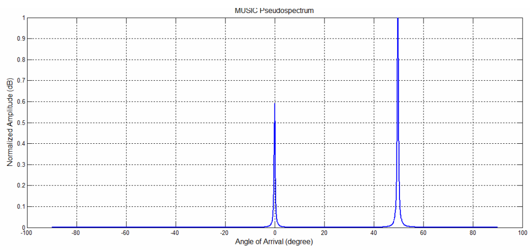

Fig. 2. MUSIC pseudospectrum obtained in case of 8 antenna elements, for SNR=0dB and the interference signal arrival from +50°.

Using these estimations of the received signals we apply the beamforming technique for maximize the desired source direction and for minimize the interference source direction. The result is presented in Fig. 3.

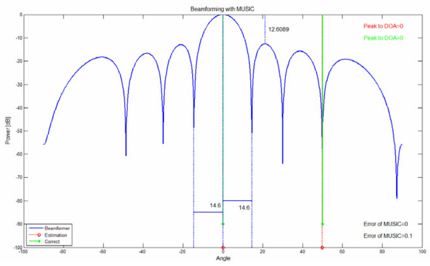

Fig. 3. The output of beamformer obtained in case of 8 antenna elements, for SNR=0dB and the interference signal arrival from +50°.

The first minimum output of beamformer appears at 14.6° relative to the peak, and the value of RSL is 12.6 dB. We can observe that the peak output of beamformer is centered to the desired direction and the interference direction is strongly diminished.

The MUSIC pseudospectrum obtained for the desired source placed at 0° and for the interference source placed at +10° is presented in Fig. 4.

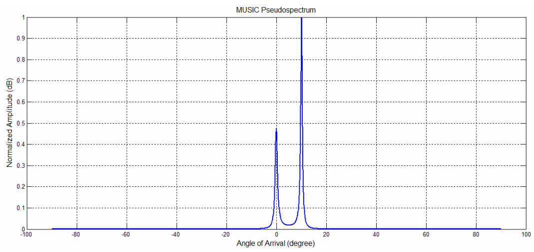

Fig. 4. MUSIC pseudospectrum obtained in case of 8 antenna elements, for SNR=0dB and the interference signal arrival from +10°.

The output of beamformer is presented in Fig. 5. In this case the value of RSL is 7.01 dB and the peak value of the output is not centered to the desired source direction (2.2°). The first minimum of the output appears at 12.4° in the right side of the peak value and at 13.2° in the left side of the peak value. This is happened because the beamforming technique minimizes the interference source direction.

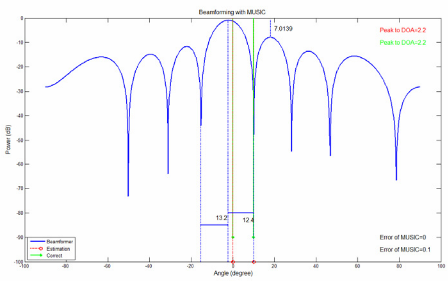

Fig. 5. The output of beamformer obtained in case of 8 antenna elements, for SNR=0dB and the interference signal arrival from +10°.

In the second case, we consider an array formed by 16 antenna elements and the interference source placed at +10° and +50° respectively. In Fig. 6 the MUSIC pseudospectrum obtained for an interference source placed at +50° and a value of SNR equal to 0 dB is presented.

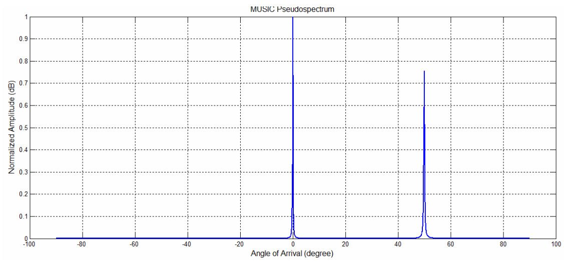

Fig. 6. MUSIC pseudospectrum obtained in case of 16 antenna elements, for SNR=0dB and the interference signal arrival from +50°.

The output of beamformer obtained in this case is presented in Fig. 7.

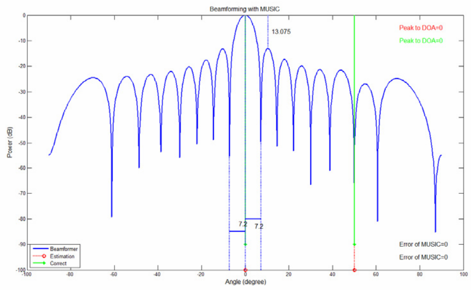

Fig. 7. The output of beamformer obtained in case of 16 antenna elements, for SNR=0dB and the interference signal arrival from +50°.

The value of RSL is 13.07 dB and the first minimum of the output appears at 7.2° relative to the peak value. We observe that the main lobe of the output is much narrower and it is centered to the desired source direction.

If the interference signal is considered to be received from +10° direction, the MUSIC pseudospectrum obtained for SNR= 0 dB is presented in Fig. 8.

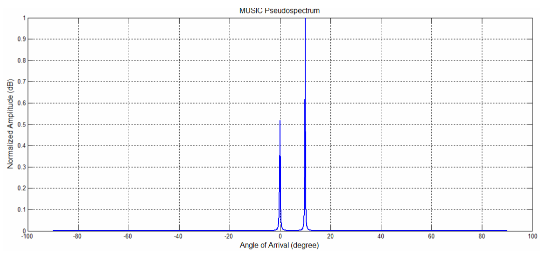

Fig. 8. MUSIC pseudospectrum obtained in case of 16 antenna elements, for SNR=0dB and the interference signal arrival from +10°.

Because of the greater number of antenna elements, the output of beamformer stay centered of the desired source direction, as is shown in Fig. 9. The RSL value is equal to 13.23 dB and the first minimum of the output appears at 10° in the right side of the peak value and at 7.4° in the left side of the peak value.

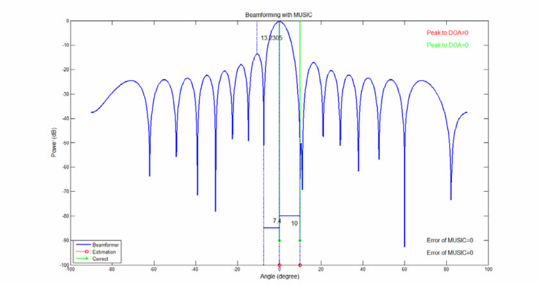

Fig. 9. The output of beamformer obtained in case of 16 antenna elements, for SNR=0dB and the interference signal arrival from +10°.

The difference between the peak value of the output of beamformer and the estimation of the desired source direction according to the number of antenna elements, for SNR=0dB, is presented in Table 1.

Table 1

Number of array elements	Displacement
4	14.8°
6	5.4°
8	2.2°
10	0.6°
12	0.2°
14	0.2°
16	0°

It is observed that the main lobe of the output is centered to the desired source direction if the number of antenna elements is increased.
Also, the difference between the peak value of the output of beamformer and the estimation of the desired source direction according to the position of the interference source, for SNR=0dB, is presented in Table 2.

Table 2

Position	Displacement
+10°	2.2°
+20°	0.3°
+30°	0°
+40°	0.1°
+50°	0.1°
+60°	0.1°
+70°	0.2°
+80°	0.1°

It is observed that if the sources are much closed the difference between the peak value of the output of beamformer and the estimation of the desired source direction is considerable.
The same simulations are made for the desired source placed at 0° and the interference source at +10°, but for a value of SNR equal to -5 dB. The results obtained for an array antenna formed by 8 elements are shown in Fig. 10 and Fig. 11 respectively. The value of RSL is 5.67 and the difference between the peak output and the estimation of the desired direction is 3.1°. We observe that the MUSIC algorithm introduces an error (0.5°) in estimation of desired direction, which is kept by beamforming technique.

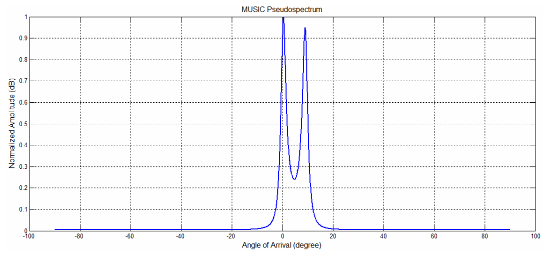

Fig. 10. MUSIC pseudospectrum obtained in case of 8 antenna elements, for SNR=-5dB and the interference signal arrival from +10°.

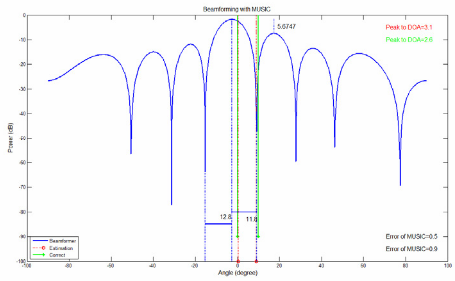

Fig. 11. The output of beamformer obtained in case of 8 antenna elements, for SNR=-5dB and the interference signal arrival from +10°.

Fig. 12 and Fig. 13 contain the MUSIC pseudospectrum and the output of the beamformer in case of an array antenna composed by 16 elements and the value of SNR equal to -5 dB.

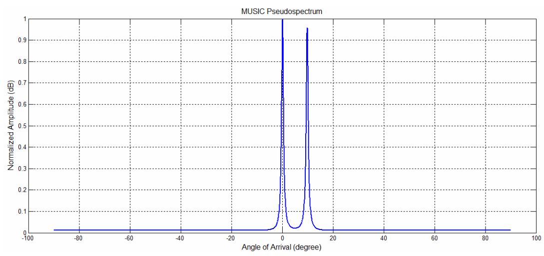

Fig. 12. MUSIC pseudospectrum obtained in case of 16 antenna elements, for SNR=-5dB and the interference signal arrival from +10°.

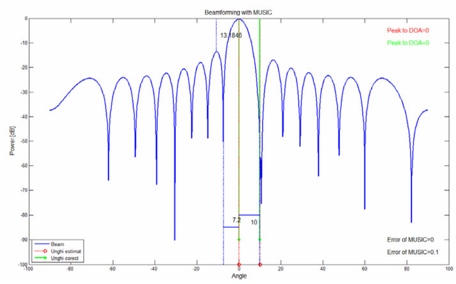

Fig. 13. The output of beamformer obtained in case of 16 antenna elements, for SNR=-5dB and the interference signal arrival from +10°.

The MUSIC algorithm introduces an error in estimation of desired direction equal to 0.2°, but the difference between the peak output of beamformer and the estimation of the desired direction is 0.2°.

IV. CONCLUSIONS

Using the method presented in this paper, the signal received from the desired source direction is maximized and the signal received from the interference source direction is minimized. If the sources are placed much closed, a displacement of the peak value output of beamformer occurs according to the estimation of the desired source direction. With a greater number of antenna elements this inconvenience is avoided only if the value of SNR is not decreased.

REFERENCES

[1] V. Vanderkulk, "Optimum Processing for Acoustic Arrays," *IEEE Radio and Electronic Engineer,* vol. 26, no. 4, Oct. 1963, pp. 285–292.

[2] S. P. Applebaum, "Adaptive Arrays," *IEEE Transactions on Antennas and Propagation,* vol. 24, Sept. 1976, pp. 585–598.

[3] J. Capon, R. J. Greenfield, and R. J. Kolker, "Multidimensional Maximum-likelihood processing of a large aperture seismic array," *Proceedings of the IEEE*, vol. 55, Feb. 1967, pp. 199–211.

[4] F. Gross, *Smart Antenna for Wireless Communications – with MATLAB*, Ed. New York: McGraw-Hill, 2005.

[5] Z. Chen, G. Gokeda, and Y. Yu, *Introduction to Direction-of-Arrival Estimation,* Ed. Artech House, 2010.

[6] H. Cox, R. M. Zeskind, and M. M. Owen, "Robust Adaptive Beamforming," *IEEE Trans. Acoustic, Speech and Signal Proc.*, vol. ASSP-35, no. 10, Oct. 1987, pp. 1365–1376.

[7] M. Haardt, et al., "2D Unitary ESPRIT for Efficient 2D Parameter Estimation," *Proc. IEEE Int. Conf. Acoust., Speech, Signal Proc.*, vol. 3, May 1995, pp. 2096–2099.