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Windows for Dynamic Testing of High-Resolution A/D Converters by Means of the Energy-Based Method

Daniel Belega[1]

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Keywords: cosine windows, leakage effect, energy-based method, estimation of the ADC effective number of bits (ENOB).

I. INTRODUCTION

For dynamic testing of an analog-to-digital converter (ADC) the spectral analysis approach is often used. This usually uses a sine wave as test signal. Then, a certain algorithm, based on the discrete Fourier transform (DFT) of the ADC output signal is used to estimate the ADC dynamic parameters [2] – [5]. In non-coherent sampling mode, most encountered in practice, the ADC output signal spectrum is affected by spectral leakage phenomena. This means that the energy of the original spectral lines is spread over the whole frequency axis [6], [7]. The way used to suppress the leakage errors is called “windowing”, which supposes an *a priori* multiplication of the ADC output signal by a suitable sequence, called window [6], [7]. Due to their simple implementation the cosine windows are ones of the most widely used windows in dynamic testing of ADCs by spectral analysis.

The energy-based method is often used in testing ADCs by spectral analysis in non-coherent sampling mode. This method provides high estimation accuracy of ADC dynamic parameters and can be easily implemented [4], [5]. By energy-based method the most important ADC dynamic parameters can be accurately estimated: signal-to-noise and distortion ratio (SINAD), total harmonic distortion (THD), signal-to-non harmonic ratio (SNHR), spurious free dynamic range (SFDR). Based on the SINAD value the ADC dynamic parameter – effective number of bits (ENOB) is calculated [2].

The IEEE standard 1241 for ADCs [2] only recommends the use of windows for accurate estimation of ADC dynamic parameters, without given any window or criterion for selecting the windows.

The DYNAD European draft standard [3] recommends the 7-term Blackman-Harris window for dynamic testing of high-resolution ADCs (up to 24 bits). However, in DYNAD any 5 or 6-term cosine windows for dynamic testing of ADCs with resolutions higher or equal to 14 bits are not given.

Another cosine window which can be used for dynamic testing of high-resolution ADCs (up to 20 bits) by means of energy-based method is the 7-term Blackman-Harris-Hodie window [1].

In this paper in order to replace the 7-term cosine windows in dynamic testing of high resolution ADCs (14-20 bits) two new cosine windows of 5 and 6 orders are proposed. The effectiveness of each window in spectral leakage reduction is evaluated by means of the parameter n_{eff} defined in [1]. For each window the frequency response and the main characteristics are presented. Moreover, the effectiveness of each window is verified by means of computer simulation.

II. THEORETICAL BACKGROUND

Let us consider a n-bit ADC under test fed by a sine wave test signal with amplitude A, frequency f_{in}, phase φ and offset d defined as:

$$x(t) = A\sin\left(2\pi f_{in} t + \varphi\right) + d. \qquad (1)$$

To test all the ADC output codes, A must be equal to FSR/2, where FSR is the ADC full-scale range. The offset d is equal to 0 for a bipolar ADC and to FSR/2 for an unipolar ADC.

The signal obtained at the ADC output is:

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$$y(m) = A\sin\left(2\pi \frac{f_{in}}{f_s} m + \varphi\right) + d + e(m), \qquad (2)$$
$$m = 0, 1, 2, \ldots$$

where f_s is the ADC sampling frequency and $e(m)$ is the ADC noise which includes harmonic components, spurious components, effect of sampling time errors, the quantization noise and random noises.

When M samples of ADC output signal are acquired, the relationship between the frequencies f_{in} and f_s is:

$$\frac{f_{in}}{f_s} = \frac{\lambda_0}{M} = \frac{l+\delta}{M}, \qquad (3)$$

where λ_0 is the number of recorded sine wave cycles, l and δ are respectively the integer part and the fractional part of λ_0 ($0 \le \delta < 1$). $\delta = 0$ corresponds to the coherent sampling mode [8], [9]. When $\delta \neq 0$, the sampling process is in non-coherent mode and the ADC output signal spectrum is affected by spectral leakage errors. In order to suppress these errors windowing approach is used leading to the spectral analysis of $y_w(m) = y(m) \cdot w(m)$, where $w(m)$ is the window sequence [6], [7]. The cosine-windows are used in ADC dynamic testing by means of energy-based method. These are defined as:

$$w(m) = \sum_{h=0}^{H-1} (-1)^h a_h \cos\left(\frac{2\pi h m}{M}\right), \qquad (4)$$
$$m = 0, 1, \ldots, M-1$$

where H is the window's order and a_h are the window's coefficients.

The discrete-time Fourier transform (DTFT) of $y_w(m)$ is given by:

$$\begin{aligned} Y_w(\lambda) &= d \cdot W(\lambda) \\ &+ \frac{A}{2j}\left[W(\lambda - \lambda_0)e^{j\varphi} - W(\lambda + \lambda_0)e^{-j\varphi}\right] \qquad (5) \\ &+ E_w(\lambda), \qquad \lambda \in [0, M) \end{aligned}$$

where λ represents the normalized frequency expressed in bin, $W(\lambda)$ is the DTFT of $w(m)$ and $E_w(\lambda)$ is the DTFT of the noise signal $e_w(m) = e(m) \cdot w(m)$.

The second term from the square brackets of (5) represents the image part of the spectrum. For large values of l the contribution from the image part can be neglected.

The DTFT of an H-term cosine window can be approximated by [10]:

$$W(\lambda) = \frac{M\lambda \sin(\pi\lambda)}{\pi} \cdot \sum_{h=0}^{H-1} \frac{(-1)^h a_h}{\lambda^2 - h^2}. \qquad (6)$$

The energy-based method estimates the power of the ADC output signal by the power of the signal components centred on the input frequency and situated inside the frequency band covering the window spectrum main lobe. For an H-term cosine-window this frequency band contains $(2H+1)$ spectral components.

By means of the energy-based method the parameter SINAD is estimated by [4], [11]:

$$SINAD_{est}\,[dB] = \frac{2\sum_{k=J-H}^{J+H} \left|Y_w(k)\right|^2}{\sum_{k \in B_r} \left|Y_w(k)\right|^2} \cdot \frac{M - 3(2H+1)}{M}, \qquad (7)$$

where the set B_r includes the integer values from the intervals $[H+1,\ J-H-1] \cup [\ J+H+1,\ M-J-H-1] \cup [M-J+H+1,\ M-H-1]$, in which J is the index corresponding to the maximum of the spectrum $|Y_w(k)|$ ($J = l$ for $0 < \delta \le 0.5$ and $J = l + 1$ for $0.5 < \delta < 1$).

From (7) the ENOB parameter can be estimated by [2]:

$$ENOB = \frac{SINAD_{est}\,[dB] - 1.76}{6.02}. \qquad (8)$$

It is very important to estimate with high accuracy the ENOB parameter since it evaluates the global dynamic performances of the ADC under test.

III. THE PROPOSED WINDOWS

The effectiveness of a window in spectral leakage reduction can be evaluates by means of the resolution of an ideal ADC, n_{eff}, up to which the window is still efficient [1]. The parameter n_{eff} is given by [1]:

$$n_{eff} = 0.5 - 1.66\log(3\mu\, err), \qquad (9)$$

where:

$err = 0.5 - \dfrac{\sum_{k=-H}^{H} \left|W(k - \tilde{\delta})\right|^2}{M^2 NNPG}$, in which NNPG is the normalized noise power gain ($NNPG = \sum_{m=0}^{M-1} w^2(m)/M$) and $\tilde{\delta}$ is equal to δ for $0 < \delta \le 0.5$ and to $(1 - \delta)$ for $0.5 < \delta < 1$;

μ is the ratio between the power of the spectral leakage and the power of the quantization noise (σ_q^2); for small error it is recommended to use $\mu \ge 20$.

The power of the spectral leakage can be approximated by [1]:

$$ERR = \frac{(FSR)^2}{8} err. \qquad (10)$$

The effectiveness of a window in spectral leakage reduction increases when n_{eff} increases. The worst case is obtained for δ equal to 0.5 or very close to 0.5 (case of the minimum error energy windows). For the 3 and 4-term cosine windows the best performances are obtained in the case of minimum error energy windows. These windows were designed for minimizing the error energy outside the frequency band of the main lobe [12]. This ensures that the power $\sum_{k\in B_r}\left|Y_w(k)\right|^2$ is minim. Thus, from (7) it follows a maximum $SINAD_{est}$ and so a high n_{eff}.

A minimum power of $\sum_{k\in B_r}\left|Y_w(k)\right|^2$ implies a minimum power of $\sum_{k\in B'_r}\left|Y_w(k)\right|^2$ where set B'_r includes the integer values from the interval $[J+H+1,\ M/2+J]$. From (5) it can be established the following equality:

$$\sum_{k\in B'_r}\left|Y_w(k)\right|^2=\frac{A^2}{4}\sum_{k=H+1}^{M/2}\left|W\left(k-\tilde{\delta}\right)\right|^2. \qquad (11)$$

Based on the above observations and on (11) a procedure for determining the coefficients of a cosine window (i.e the cosine window) can be established.

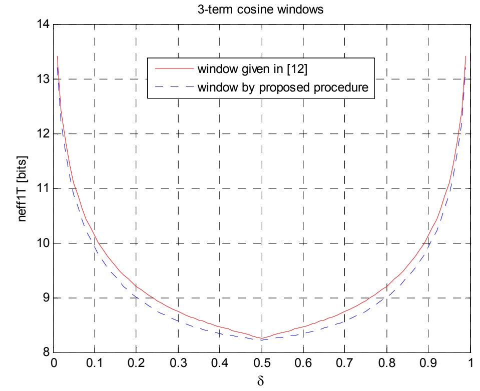
Procedure: The coefficients a_h of a cosine window are determined in order to minimizing the power $\sum_{k=H+1}^{M/2}\left|W(k-0.5)\right|^2$ *corresponding to the worst case.*

For this purpose the coefficients a_h are finding by means of a minimum search algorithm. In this paper the Nelder-Mead simplex algorithm (the function *fminsearch* ( ) from Matlab 5.3) is used.

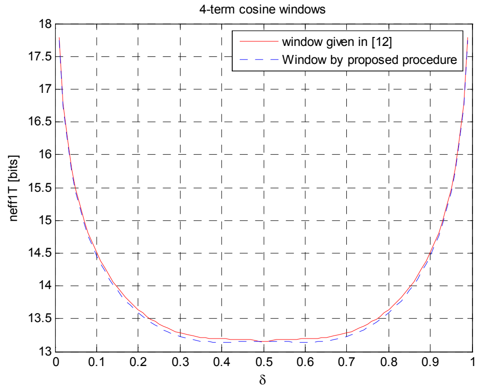
Fig. 1 shows the n_{eff} as a function of δ when μ = 20 for the 3 and 4-term minimum error energy windows presented in [12] and for the ones with the same orders designed by the proposed procedure. M is set to 4096. For the 3-term window the coefficients designed by the proposed procedure are: a_0 = 0.41078768062433, a_1= 0.49917916191364 and a_2 = 0.09003315746203. For the 4-term window the coefficients obtained by the proposed procedure are: a_0 = 0.35039994065015, a_1 = 0.48536072022250, a_2 = 0.14962877167163 and a_3 = 0.01461056745572.

It is clearly evident from both graphics presented in Fig.1 that the performances of the designed windows are very close to the ones of the minimum error energy windows presented in [12].

Fig. 2 shows the n_{eff} as a function of δ when μ = 20 for the 5 and 6-term windows designed by the proposed procedure. M is set to 4096.

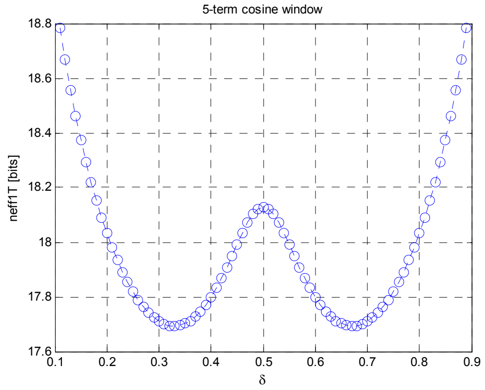


(a) $H = 3$

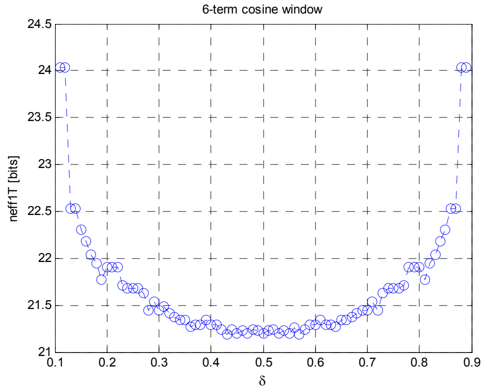


(b) $H = 4$

Fig. 1. n_{eff} as a function of δ when μ = 20 for the windows presented in [12] and the designed one: (a) H =3; (b) H = 4.



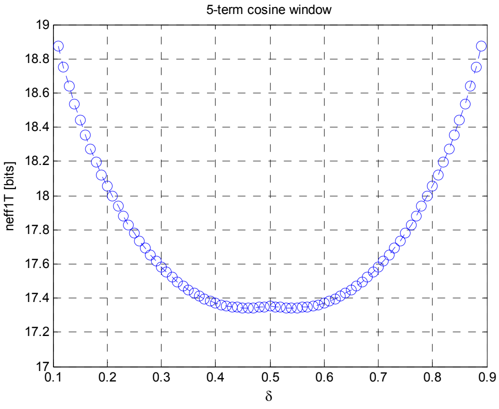
(a) $H = 5$



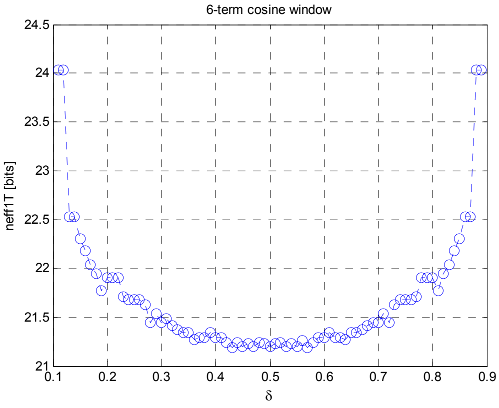
(b) $H = 6$

Fig. 2. n_{eff} as a function of δ when μ = 20 for windows designed by the proposed procedure: (a) H = 5; (b) H = 6.

From Fig. 2 it follows that for coefficients obtained by the proposed procedure the worst case is not obtained for $\delta = 0.5$ as for the 3 and 4-term cosine windows. A possibility to obtain also for these windows the worst case for $\delta = 0.5$ is to set one of the window's coefficient to a fixed value and then apply the proposed procedure. A good behaviour has been obtained for $a_1 = 0.467$ in the case of the 5-term window and for $a_1 = 0.45$ in the case of 6-term maximum window. Fig. 3 shows the n_{eff} as a function of δ when $\mu = 20$ for the 5 and 6-term windows obtained by the proposed procedure with a_1 *a priori* established to the values aforementioned. M is set to 4096.



(a) $H = 5$

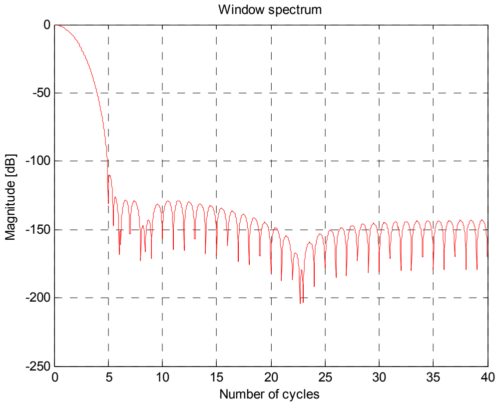


(b) $H = 6$

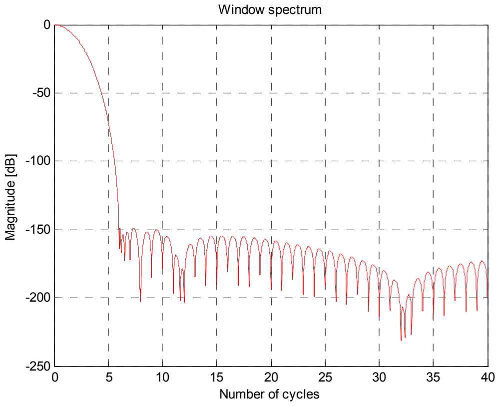
Fig. 3. n_{eff} as a function of δ when $\mu = 20$ for windows designed by the proposed procedure with a_1 a priori fixed:
(a) 5-term window; (b) 6-term window.

For the 5-term window the coefficients obtained are: $a_0 = 0.31516086039782$, $a_1 = 0.467$, $a_2 = 0.18301739393731$, $a_3 = 0.03299781119189$ and $a_4 = 0.00182393447297$.

For the 6-term window the coefficients obtained are: $a_0 = 0.29084692392447$, $a_1 = 0.45$, $a_2 = 0.20368610117409$, $a_3 = 0.04983846249154$, $a_4 = 0.00546711753710$ and $a_5 = 1.613948728039398 \cdot 10^{-4}$. These windows are the ones proposed in this paper. The frequency responses of these windows are presented in Fig. 4 and some important characteristics of these windows are given in Table 1.



(a) $H = 5$



(b) $H = 6$

Fig. 4. Frequency responses of the proposed windows.

Table 1. Windows's characteristics.

H	Peak side lobe [dB]	Decay rate [dB/oct]	NPSG[1]	NNPG	ENBW[2] [bin]
5	-109.96	6	0.3152	0.2257	2.2720
6	-148.48	6	0.2908	0.2078	2.4570

[1] NPSG is the window normalized peak signal gain [1];
[2] ENBW is the window equivalent noise bandwidth [1].

IV. COMPUTER SIMULATION

The aim of this section is to verify by means of computer simulation the effectiveness of the proposed windows. For this purpose the following signal $y_w(m)$ is used in simulation:

$$y_w(m) = \left(x(m) + h(m) + e_q(m) + e_n(m)\right) \cdot w(m), \quad m = 0, 1, \ldots, M-1 \tag{12}$$

where: $x(m)$ is simulated sine wave input signal $\left(x(m) = A_1 \sin\left(2\pi \frac{l+\delta}{M} m + \varphi_1\right)\right)$;

$h(m)$ are simulated harmonic components;

$$\left(h(m) = A_2 \sin\left(4\pi \frac{l+\delta}{M} m + \varphi_2\right) + 0.5 A_2 \sin\left(6\pi \frac{l+\delta}{M} m + \varphi_3\right)\right)$$

$e_q(m)$ are simulated quantization noise;
$e_n(m)$ is simulated normal noise.

The parameters of the sine wave input signal are: A_1 = FSR/2 = 2.5 and φ_1 uniformly distributed on [0, 2π) rad. A_2 is established as a function of ENOB of the ADC under test. The phases φ_2 and φ_3 are uniformly distributed on [0, 2π) rad. The number of samples is set to M = 4096 and l = 123. The quantization noise is modelled by uniformly distributed additive noise. The normal noise is characterized by mean 0 and standard deviation $A_2/4$. δ varies in the range [0, 1) with an increment of 1/40. The ENOB parameter is established by (8). For each δ the worst ENOB_{est} occurring in 100 estimations, during the variations of φ_1, φ_2 and φ_3 and of uniform and normal noises, is retained.

Fig. 5 shows the modulus of the maximum of the absolute error of ENOB, $|\Delta\mathrm{ENOB}|_{max}$ as a function of δ for different values of ENOB. The ADC tested is a bipolar one with 18 bits resolution. The proposed 5-term window is used.

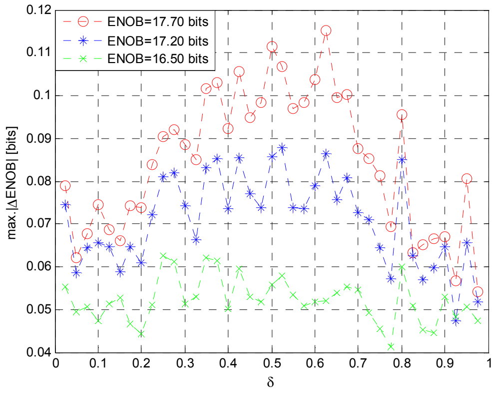


Fig. 5. $|\Delta\mathrm{ENOB}|_{max}$ as a function of δ for different values of ENOB when the proposed 5-term window is used.

From Fig. 5 it is obvious that when the proposed 5-term window is used accurate ENOB estimates are obtained for ENOB smaller than 17 bits ($|\Delta\mathrm{ENOB}|_{max}$ is smaller than 0.1 bits).

Fig. 6 shows also $|\Delta\mathrm{ENOB}|_{max}$ as a function of δ for different values of ENOB when the ADC tested is a bipolar one with 22 bits resolution and the proposed 6-term window is used.

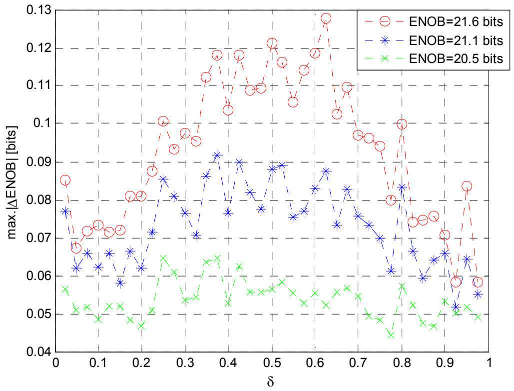


Fig. 6. $|\Delta\mathrm{ENOB}|_{max}$ as a function of δ for different values of ENOB when the proposed 6-term window is used.

Based on the Fig, 6 it follows that when the proposed 6-term window is used ENOB is accurately estimated if it is smaller than 21 bits ($|\Delta\mathrm{ENOB}|_{max}$ is smaller than 0.1 bits).

V. CONCLUSION

In this paper two cosine windows for dynamic testing of high resolution ADCS by means of the energy-based method have been proposed. The fist one is a 5-term window which can be used for dynamic testing of ADCs with resolutions up to 16 bits. This is very suited to be use for testing ADCs with 14 and 16 bits resolutions. The second is a 6-term window which can be used for dynamic testing ADCs with resolutions up to 20 bits. This is very suited to be use for testing ADCs with 18 and 20 bits resolutions. Carried out simulations confirm the effectiveness of each proposed window.

The main objective of the proposed windows is to replace the 7-term cosine windows which are generally used in the dynamic testing of high-resolution ADCs.

In the author's opinion these windows must be considered by the existent Standards for ADCs concerning the dynamic testing of ADCs by spectral analysis approach in non-coherent sampling mode.

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