Buletinul Ştiinţific al Universităţii "Politehnica" din Timişoara

Seria ELECTRONICĂ şi TELECOMUNICAŢII
TRANSACTIONS on ELECTRONICS and COMMUNICATIONS

Tom 53(67), Fascicola 1, 2008

A new approach for steady-state analysis of the circuits with strong nonliniarities

Mihai Iordache[1], Lucia Dumitriu[1], Dragos Niculae[1]

Abstract – The paper presents a new version of the generalised hybrid method for the analysis of the analog circuits with strong nonlinearities, driven by signals with widely separated frequencies. The key idea is to formulate the circuit equations by using multiple time variables, which enable an efficient representation of this kind of signals. Using multiple time-scale the differential algebraic equations (DAE) describing the nonlinear analog circuits are transformed into multi-time partial differential equations (MPDE). An illustrative example is presented.

Keywords: Hybrid method, Nonlinear analog circuits, Steady-state analysis

I. INTRODUCTION

The systems that work with signals in a large range of frequencies from the kHz (for the modulated signal) to GHz (for the carrier) as, for example, RF-IC applications, are called systems with *multi-rate signals*. These systems are typically difficult to analyze using traditional numerical integration algorithms, such as those in SPICE like programs [1]: following fast-varying signal components long enough to obtain information about the slowly-varying ones is computationally expensive, and can also be inaccurate. So finding the steady-state by the brute-force method is, in this case, time-consuming [1, 2]. The multi-rate signals, can be represented efficiently as functions of two or more time variables, i.e., as *multivariate functions*. If a circuit is described with differential-algebraic equations (DAE), using multivariate functions for the unknowns naturally leads to a partial differential equation (PDE) form, called *Multi-rate Partial Differential Equations* (*MPDE*). Applying time-domain numerical methods to solve the MPDE directly for the multivariate forms of the unknowns, we are able to analyze the combination of strong nonlinearities and multi-rate signals. In order to reduce the computing time and the memory it is necessary to separate the circuit into a linear part and a nonlinear one. This "separation" allows the calculation of those terms in the circuit equations depending on the parameters of the linear circuit elements, only once, at the beginning of the computing process.

Among the many distinct methods for formulating circuit equations is the *m-port hybrid-analysis approach* [2-4, 11 and 14]. This method is more general and more efficient when the under-consideration circuit contains a large percentage of linear circuit elements and controlled sources. Another advantage of the hybrid-analysis approach is that it allows the nonlinearities to be either voltage-controlled (c. v.) or current-controlled (c. c.), a flexibility not shared by nodal-analysis method. The basic philosophy is to form an *m*-port $\hat{C}$ from the given circuit by extracting an appropriate set of two-terminal elements so that the resultant *m*-port contains only linear capacitors, inductors, resistors, and linear controlled sources. Our objective in this paper is to develop a general algorithm for formulating the hybrid equations of a very large class of nonlinear analog circuits. This method - called **hybrid analysis** - allows a mixture of both current and voltage-controlled resistors, voltage-controlled (v. c.) nonlinear capacitors, current-controlled (c. c.) nonlinear inductors, linear capacitors, inductors (magnetic coupled or not), resistors, independent sources, and all four types of linear controlled sources

Replacing each capacitor and inductor (magnetic coupled or not) by a discrete resistive model associated with a preselected implicit numerical integration algorithm (backward Euler algorithm, trapezoidal algorithm, or Gear's second-order algorithm), efficiency in numerical computing of the associated MPDE is obtained.

The hybrid equations (HEs) are very easy to formulate and to implement into a program [14]. The characteristics of the nonlinear circuit elements are approximated by piecewise-linear continuous curves [5-11].

II. NUMERICAL METHOD TO SOLVE MPDE

In order to analyze the nonlinear analog circuit, driven by multi-tone signals, we shall use the hybrid equations.

[1] "Politehnica" University of Bucharest, Electrical Engineering Department,
Spl. Independentei 313, Cod 06 0042, Bucharest, Romania
Phone/Fax (+4021) 318 10 16, iordache@elth.pub.ro, dlucia@elth.pub.ro,dragosn@elth.pub.ro

To avoid dealing with certain types of circuits whose hybrid equations either do not exist or are pathological, in the sense that hybrid equation solutions depend on the precise value of some element parameters, we shall assume that our circuits meet the following requirements:

1. **Consistency assumptions**: a) There does not exist any loop made up of only independent and/or controlled voltage sources (*E loop*); b) There does not exist any cutest made up of only independent and/or controlled current sources (*J cuset*).

2. **Assumption on controlling variables:** the controlled sources can be depended only the currents (the voltages) of the c. c. (c. v.) nonlinear circuit elements and of linear resistors (inclusively the linear resistors from the companion models corresponding to the linear dynamic elements).

3. **Normal-tree assumptions:** We choose a special tree – called the *normal tree* (NT)– whose elements are selected according to the following priority: a) all independent and controlled voltage sources; b) all c.v. nonlinear circuit elements (capacitors and/or resistors); c) as many linear resistors (inclusively the linear resistors from the companion models corresponding to the linear dynamic elements). NT does not contain any independent and controlled current source and any c.c. nonlinear element (inductor and/or resistor).

We consider the two-rate case. The MPDE have the periodic boundary conditions (BCs) $\hat{x}(t_2+T_2,t_1+T_1)=\hat{x}(t_2,t_1)$. We take a uniform grid $\{\bar{t}(j,i)\}$ of size (p_2+1)x(n_1+1) on the rectangle [0, T_2]x[0, m_1T_1] (Fig.1), where $\bar{t}(j,i)=(t_{2\_j},t_{1\_i})$, with t_{2_j} = $(i\text{-}2)T_2+(j\text{-}1)h_2$, $t_{1-i}=(i-1)h_1$, $j=\overline{1,p_2+1}$, $i=\overline{1,n_1+1}$; $h_2 = = T_2/p2$, and $h_1 = m_1T_1/n_1 = T_1/p_1$. Consider that the slow components of variables depend on t_1 and the fast components depend on t_2. The periodic boundary conditions and the integration algorithm are described in Fig. 1 [13]. At each time $\bar{t}\,(j,i)$ we have to solve a nonlinear algebraic equation system. For this, we can use the Newton-Raphson algorithm or other efficient numerical iteration algorithms [2-4, 9-14].

Let C be a circuit that satisfies the standing assumptions, and let T be a normal tree and L its corresponding co-tree. Our first task is to form a linear *m*-port $\hat{C}$, with $m = n_e+n_j+n_v+n_c$ (where, for example, n_v (n_c) is the c.v. (c.c.) nonlinear element number), obtained by extracting from C all independent sources, all c.v. nonlinear elements (capacitors and resistors), and all c.c. nonlinear elements (inductors and resistors), as shown in Fig. 2, a. In view of our procedure for selecting the NT, all extracted elements on the left of $\hat{C}$ are tree branches and therefore constitute a part of NT and all elements on the right of $\hat{C}$ are co-tree branches (links) and belong to the co-tree L. The remaining elements in the *m*-port $\hat{C}$ consist only of linear capacitors, inductors (magnetic coupled or not), resistors, and linear controlled sources. Substituting all nonlinear elements from the left side in Fig. 2, a by ideal voltage sources and all nonlinear elements from the right side by ideal current sources, and replacing the linear capacitors and inductor by their resistive discrete circuit models associated with a given integration algorithm (for example, the backward Euler algorithm), we obtain the linear and time-invariant circuit in Fig. 2, b.

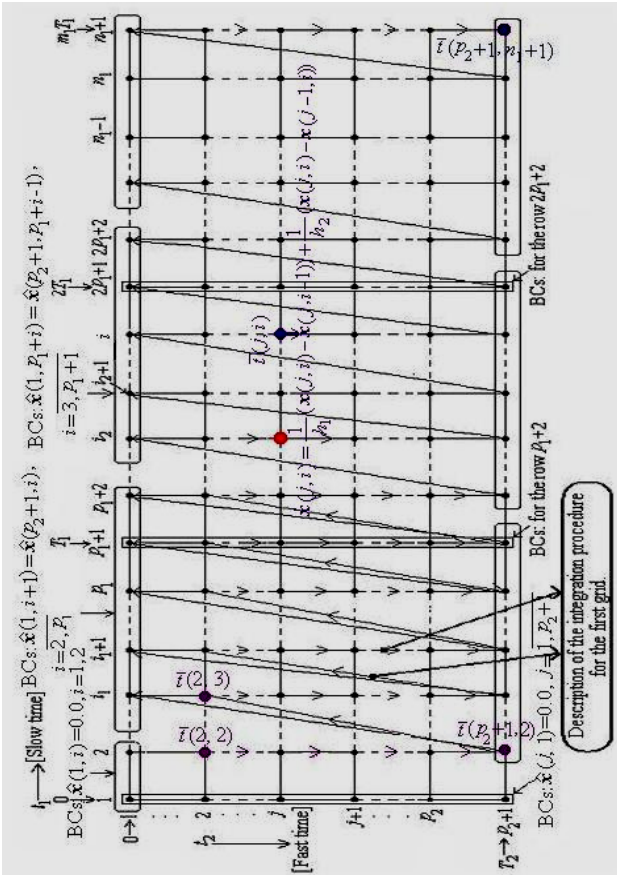


Fig. 1. A uniform grid $\{\bar{t}(j,i)\}$ of size $(p_2+1)\times(n_1+1)$.

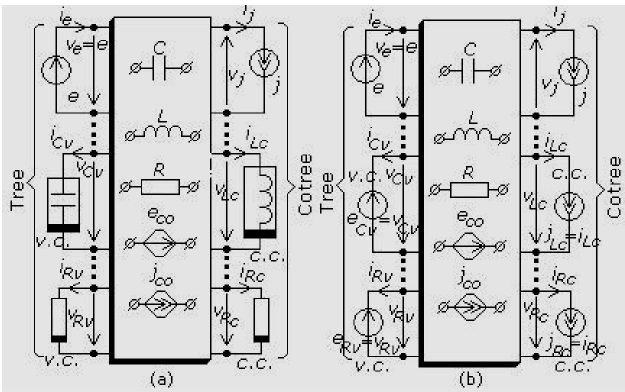


Fig.2. a) The linear *m*-port $\hat{C}$ created by extracting all independent sources and all nonlinear elements;

b) Linear *m*-port $\hat{C}$ with the tree voltage ports appearing on the left side and the cotree current ports appearing on the right side.

Applying the superposition theorem to the linear m-port $\hat{C}$ in Fig. 2, b, for the time moment $\bar{t}(i,j)$, when all linear capacitors and all linear inductors are replaced by discrete resistive models associated to backward Euler algorithm, and at the $(k+1)^{\text{th}}$ iteration of the Newton-Raphson algorithm, it results:

$$\begin{bmatrix} \boldsymbol{i}_{v,(j,i)}^{(k+1)} \\ \boldsymbol{v}_{c,(j,i)}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_{v,v} & \boldsymbol{B}_{v,c} \\ \boldsymbol{A}_{c,v} & \boldsymbol{R}_{c,c} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_{v,(j,i)}^{(k+1)} \\ \boldsymbol{i}_{c,(j,i)}^{(k+1)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{G}_{v,E} & \boldsymbol{B}_{v,J} \\ \boldsymbol{A}_{c,E} & \boldsymbol{R}_{c,J} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{e}_{(j,i)} \\ \boldsymbol{j}_{(j,i)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{G}_{v,L} & \boldsymbol{B}_{v,C} \\ \boldsymbol{A}_{i,L} & \boldsymbol{R}_{i,C} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{L(j-1,i-1)} \\ \boldsymbol{j}_{C(j-1,i-1)} \end{bmatrix}. \tag{1}$$

In relation (1) $\boldsymbol{B}_{v,c}\left(\boldsymbol{A}_{c,v}\right)$ represents the current (voltage) transfer coefficient matrix of the tree-branch (link) v.c. (c.c.) nonlinear elements in respect of the link (tree-branch) c.c. (v.c.) nonlinear elements; $\boldsymbol{v}_{v,(j,i)}^{(k+1)}\left(\boldsymbol{i}_{c,(j,i)}^{(k+1)}\right)$ is the voltage (current) vector of the v.c. (c.c.) tree-branch (link) nonlinear elements from the time moment $\bar{t}(j,i)$ and the $(k+1)^{\text{th}}$ iteration and $\boldsymbol{e}_{L(j-1,i-1)} = \dfrac{\boldsymbol{L}\boldsymbol{i}_{L(j-1,i)}}{h_2} + \dfrac{\boldsymbol{L}\boldsymbol{i}_{L(j,i-1)}}{h_1}$ $\left(\boldsymbol{j}_{C(j-1,i-1)} = \dfrac{\boldsymbol{C}\boldsymbol{u}_{C(j-1,i)}}{h_2} + \dfrac{\boldsymbol{C}\boldsymbol{u}_{C(j,i-1)}}{h_1}\right)$ represents the voltage (current) vector of the ideal independent voltage (current) sources from the companion scheme of linear inductors (capacitors) at the time moments $\bar{t}(j-1,i)$ and $\bar{t}(j,i-1)$.

If we denote by:

$$\boldsymbol{X}_{(j,i)}^{(k+1)} = \begin{bmatrix} \boldsymbol{i}_{v,(j,i)}^{(k+1)} \\ \boldsymbol{v}_{c,(j,i)}^{(k+1)} \end{bmatrix};\ \boldsymbol{H} = \begin{bmatrix} \boldsymbol{G}_{v,v} & \boldsymbol{B}_{v,c} \\ \boldsymbol{A}_{c,v} & \boldsymbol{R}_{c,c} \end{bmatrix};\ \boldsymbol{x}_{(j,i)}^{(k+1)} = \begin{bmatrix} \boldsymbol{v}_{v,(k+1)}^{(k+1)} \\ \boldsymbol{i}_{c,(j,i)}^{(k+1)} \end{bmatrix}; \tag{2,a}$$

$$\boldsymbol{S}_{(j,i)} = \begin{bmatrix} \boldsymbol{G}_{v,e} & \boldsymbol{B}_{v,J} \\ \boldsymbol{A}_{c,e} & \boldsymbol{R}_{c,J} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{(j,i)} \\ \boldsymbol{j}_{(j,i)} \end{bmatrix};$$

$$\boldsymbol{S}_{LC(j-1,i-1)} = \begin{bmatrix} \boldsymbol{G}_{v,L} & \boldsymbol{B}_{v,C} \\ \boldsymbol{A}_{c,L} & \boldsymbol{R}_{c,C} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{L(j-1,i-1)} \\ \boldsymbol{j}_{C(j-1,i-1)} \end{bmatrix}, \tag{2,b}$$

the equations (1) become

$$\boldsymbol{X}_{(j,i)}^{(k+1)} = \boldsymbol{H}\boldsymbol{x}_{(j,i)}^{(k+1)} + \boldsymbol{S}_{(j,i)} + \boldsymbol{S}_{LC(j-1,i-1)}, \tag{3}$$

where: $\boldsymbol{H}$ is circuit hybrid matrix; $\boldsymbol{S}_{(j,i)}$ represents the source vector corresponding to the independent voltage and current sources from the time $\bar{t}(i,j)$ and $\boldsymbol{S}_{LC(j-1,i-1)}$ is the source vector corresponding to the companion schemes of the linear inductors and capacitors, at the previous time steps $\bar{t}(j-1,i)$ and $\bar{t}(j,i-1)$.

The nonlinear resistor characteristics approximated by piecewise linear continuous curves have, for the time moment $\bar{t}(i,j)$ and the $(k+1)^{\text{th}}$ iteration, the following expressions:

$$\boldsymbol{i}_{Rv(j,i)}^{(k+1)} = \boldsymbol{G}_{dv}\left(s_{(j,i)}^{(k)}\right)\boldsymbol{v}_{Rv,(j,i)}^{(k+1)} + \boldsymbol{j}_{Rv}\left(s_{(j,i)}^{(k)}\right), \tag{4}$$

for the v. c. nonlinear resistors, and

$$\boldsymbol{v}_{Rc(j,i)}^{(k+1)} = \boldsymbol{R}_{dc}\left(s_{(j,i)}^{(k)}\right) \cdot \boldsymbol{i}_{Rc,(j,i)}^{(k+1)} + \boldsymbol{e}_{Rc}\left(s_{(j,i)}^{(k)}\right), \tag{5}$$

for the c. c. nonlinear resistors.

According to the equation (4) ((5)) each v. c. (c. c.) nonlinear resistor, for any arbitrary segment $s_{(j,i)}^{(k)}$ (for the moment $\bar{t}(i,j)$, and the $(k+1)^{\text{th}}$ iteration), can be substituted by the equivalent circuit shown in Fig. 3,a (Fig. 3,b).

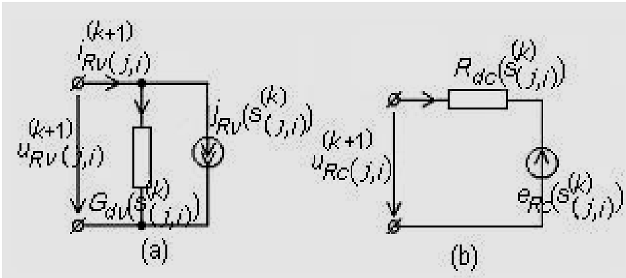


Fig. 3. Equivalent circuits for the piecewise linear nonlinear resistors.

The current expression of a v. c. nonlinear capacitor, when it's characteristic is approximated by piecewise linear continuous curve, and for the time moment $\bar{t}(i,j)$, and the $(k+1)^{\text{th}}$ iteration (using the backward Euler integration algorithm), has the following expression:

$$i_{Cv(j,i)}^{(k+1)} = G_{dCv}\left(s_{(j,i)}^{(k)}\right)v_{Cv(j,i)}^{(k+1)} - \hat{j}_{Cv(j,i)}\left(s_{(j,i)}^{(k)}\right) - j_{Cv(j-1,i-1)},$$

$$G_{dCv}\left(s_{(j,i)}^{(k)}\right) = \frac{(h_2+h_1)}{h_2h_1}C_{dv}\left(s_{(j,i)}^{(k)}\right),\ \hat{j}_{Cv(j,i)}\left(s_{(j,i)}^{(k)}\right) = \tag{6}$$

$$= \frac{(h_2+h_1)}{h_2h_1}Q_{Cv}\left(s_{(j,i)}^{(k)}\right),\ j_{Cv(j-1,i-1)} = \frac{q_{Cv(j-1,i)}}{h_2} + \frac{q_{Cv(j,i-1)}}{h_1}.$$

According to the equation (6) each v. c. nonlinear capacitor can be substituted by a discrete resistive model associated with: backward Euler algorithm shown in Fig. 4. The voltage expression of a c.c. nonlinear inductor, when it's characteristic is approximated by piecewise linear continuous curve, and for the time moment $\bar{t}(i,j)$, and the $(k+1)^{\text{th}}$ iteration (using the backward Euler integration algorithm), has the following expression:

$$v_{Lc(j,i)}^{(k+1)} = R_{dLc}\left(s_{(j,i)}^{(k)}\right) \cdot i_{Ld(j,i)}^{(k+1)} - \hat{e}_{Ld(j,i)}\left(s_{(j,i)}^{(k)}\right) - e_{Ld(j-1,i-1)},$$

$$R_{dLc}\left(s_{(j,i)}^{(k)}\right) = \frac{(h_2+h_1)}{h_2h_1}L_{dc}\left(s_{(j,i)}^{(k)}\right),\ \hat{e}_{Ld(j,i)}\left(s_{(i,j)}^{(k)}\right) = \tag{7}$$

$$= -\frac{(h_2+h_1)}{h_2h_1}\Phi_{Lc}\left(s_{(j,i)}^{(k)}\right),\ e_{Ld(j-1,i-1)} = \frac{\varphi_{Ld(j-1,i)}}{h_2} + \frac{\varphi_{Ld(j,i-1)}}{h_1}.$$

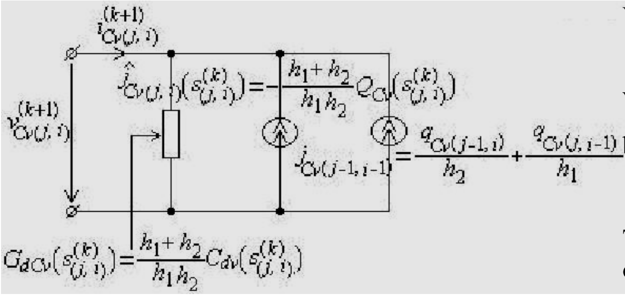


Fig.4. Discrete resistive model associated with: backward Euler algorithm for a v. c. nonlinear capacitor.

$$v^{(k+1)}_{Lc(j,i)}=R_{dLc}\left(s^{(k)}_{(j,i)}\right)\cdot i^{(k+1)}_{Ld(j,i)}-\hat{e}_{Ld(j,i)}\left(s^{(k)}_{(j,i)}\right)-e_{Ld(j-1,i-1)},$$
$$R_{dLc}\left(s^{(k)}_{(j,i)}\right)=\frac{(h_2+h_1)}{h_2h_1}L_{dc}\left(s^{(k)}_{(j,i)}\right),\quad \hat{e}_{Ld(j,i)}\left(s^{(k)}_{(i,j)}\right)= \tag{7}$$
$$=-\frac{(h_2+h_1)}{h_2h_1}\Phi_{Lc}\left(s^{(k)}_{(j,i)}\right),\ e_{Ld(j-1,i-1)}=\frac{\varphi_{Ld(j-1,i)}}{h_2}+\frac{\varphi_{Ld(j,i-1)}}{h_1}.$$

Equation (7) leads to the discrete resistive model associated with: backward Euler algorithm shown in Fig. 5.

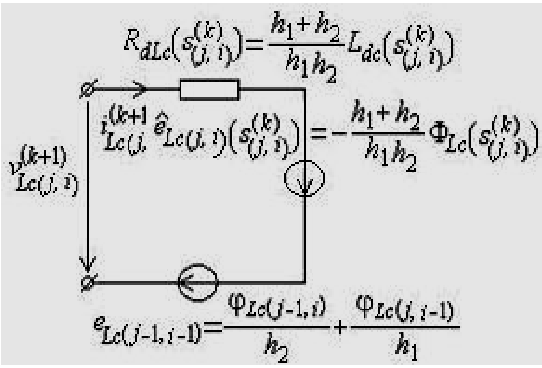


Fig.5. Discrete resistive model associated with: backward Euler algorithm for a c. c. nonlinear inductor.

Introducing into equations (1) the linear piecewise characteristic of the nonlinear circuit elements, we obtain the circuit equations from the moment $\bar{t}(i,j)$, and the $(k+1)^{\text{th}}$ iteration

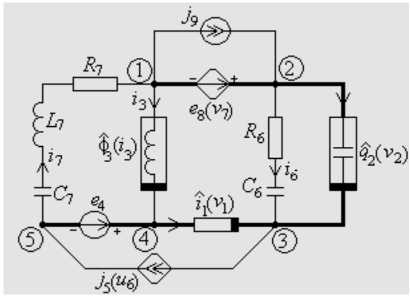
$$\begin{bmatrix} \boldsymbol{G}_{dv}\left(s^{(k)}_{(j,i)}\right)-\boldsymbol{G}_{v,v} & -\boldsymbol{B}_{v,c} \\ -\boldsymbol{A}_{c,v} & \boldsymbol{R}_{dc}\left(s^{(k)}_{(j,i)}\right)-\boldsymbol{R}_{c,c} \end{bmatrix}\begin{bmatrix} \boldsymbol{v}^{(k+1)}_{v(j,i)} \\ \boldsymbol{i}^{(k+1)}_{c(j,i)} \end{bmatrix}=$$
$$=\begin{bmatrix} \hat{\boldsymbol{j}}_{v}\left(s^{(k)}_{(j,i)}\right) \\ \hat{\boldsymbol{e}}_{c}\left(s^{(k)}_{(j,i)}\right) \end{bmatrix}+\boldsymbol{S}_{(j,i)}+\boldsymbol{S}_{LcCv(j-1,i-1)}+\boldsymbol{S}_{LC(j-1,i-1)}. \tag{8}$$

Equation (8) constitute $m=m_1+m_2$ independent equations in m_1 unknown voltages – voltage vector $\boldsymbol{v}^{(k+1)}_{v(j,i)}$, and m_2 unknown currents – current vector $\boldsymbol{i}^{(k+1)}_{c(j,i)}$, and are called the *hybrid equations* of the nonlinear circuit *C*.

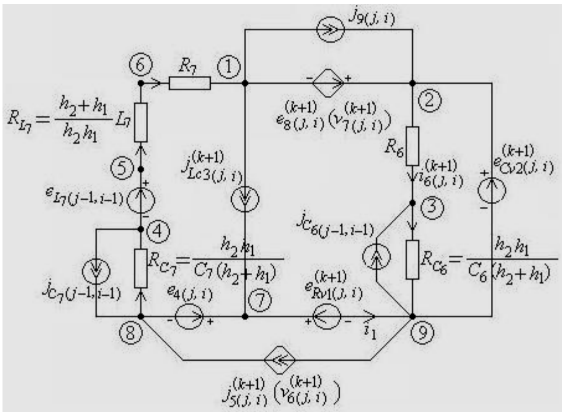
We observe that since the hybrid submatrices $\boldsymbol{G}_{v,v}$, $\boldsymbol{B}_{v,c}$, $\boldsymbol{A}_{c,v}$, and $\boldsymbol{R}_{c,c}$ and the two source vectors $\boldsymbol{S}_{(j,i)}$, $\boldsymbol{S}_{LcCv(j-1,i-1)}$ and $\boldsymbol{S}_{LC(j-1,i-1)}$ (relations (2, b)) are fixed, only $\boldsymbol{G}_{dv}\left(s^{(k)}_{(j,i)}\right)$, $\boldsymbol{R}_{dc}\left(s^{(k)}_{(j,i)}\right)$, $\hat{\boldsymbol{j}}_{v}\left(s^{(k)}_{(j,i)}\right)$, and $\hat{\boldsymbol{e}}_{c}\left(s^{(k+1)}_{(j,i)}\right)$ need be changed in each iteration. Therefore, the *Jacobian matrix* in Eq. (8) can be obtained simple from the "slope" of the appropriate segment of the nonlinear curves. Keeping as symbols only the parameters associated to the nonlinear circuit elements the Newton-Raphson algorithm becomes very efficient. Structure of the hybrid equations (8) is adequate also to solve the nonlinear circuits by the electrical machine heating and/or cooling system is modelled.

III. EXAMPLE

Let be the nonlinear circuit shown in Fig. 6, a.



(a)



(b)

Fig. 6. a) Diagram circuit; b) Linear *m*-port $\hat{C}$.

The algorithm of the partial symbolic hybrid analysis, for the nonlinear circuit in Fig. 6, a, consists in the following steps:

1. According to the assumptions from Section II, we generate the normal tree. Tree branches is represented in Fig 6, a by dashed lines.

2. We substitute, at the time moment $\bar{t}(i,j)$, and the $(k+1)^{\text{th}}$ iteration, all c.v. nonlinear elements by ideal voltage sources and all c.c. nonlinear elements by ideal current sources, and replacing the linear capacitors and inductors by their resistive discrete circuit models associated with a given integration algorithm (for example, the backward Euler algorithm), we obtain the linear and time-invariant circuit in Fig. 6, b.

3. For the numerical values of the linear element parameters: $L_7 = 1\mu$H, $C_6 = 2$pF, $R_6 = 2$kΩ, $C_7 = 4$pF, $R_7 = 1$kΩ, $G_{5_6} = 0.001$S, $a_{8\_7} = 2$, $h_1 = 10^{-10}$s, $h_2 = 5.10^{-12}$s, $R_{C6} = 2.38\Omega$, $R_{L7} = 210$kΩ and $R_{C7} = 1.19\Omega$ it is analyzed, by partial symbolic hybrid method [10, 12-14], the linear m-port in Fig. 5, b. In this way we obtain the hybrid equations (1). Running SYMNA program [13] we obtain the following hybrid equations:

$$\begin{bmatrix} i_{1(j,i)}^{(k+1)} \\ i_{2(j,i)}^{(k+1)} \\ v_{3(j,i)}^{(k+1)} \end{bmatrix} = \begin{bmatrix} -0.4785\,10^{-5} & 0.001004 & 1.0 \\ 0.4785\,10^{-5} & -0.5044\,10^{-3} & -0.9998 \\ -1.01 & 1.01 & 0.0 \end{bmatrix} \begin{bmatrix} v_{1(j,i)}^{(k+1)} \\ v_{2(j,i)}^{(k+1)} \\ i_{3(j,i)}^{(j+1)} \end{bmatrix} +$$

$$+ \begin{bmatrix} -0.4785\,10^{-5} & -0.002377 & 0.5694\,10^{-5} \\ 0.4785\,10^{-5} & 0.001188 & -0.5694\,10^{-5} \\ -0.009572 & 0.0 & 0.01139 \end{bmatrix} \cdot \begin{bmatrix} e_{L7(j-1,i-1)} \\ j_{C6(j-1,i-1)} \\ i_{C7(j-1,i-1)} \end{bmatrix} +$$

$$+ \begin{bmatrix} 0.4785\,10^{-5} & 0.0 \\ -0.4785\,10^{-5} & 0.0 \\ 0.009572 & 0.0 \end{bmatrix} \begin{bmatrix} e_{4(j,i)} \\ j_{9(j,i)} \end{bmatrix}. \qquad (9)$$

4. Introducing into equations (9) the linear piecewise characteristic of the nonlinear circuit elements, we obtain the circuit equations from the time moment $\bar{t}(i,j)$, and the $(k+1)^{\text{th}}$ iteration:

$$\begin{bmatrix} G_{dv1}\left(s_{(j,i)}^{(k)}\right) + 0.4785\,10^{-5} & -0.001004 & -1.0 \\ -0.4785\,10^{-5} & G_{dCv2}\left(s_{(j,i)}^{(k)}\right) + 0.5044\,10^{-3} & 0.9998 \\ 1.01 & -1.01 & R_{dLc3}\left(s_{(j,i)}^{(k)}\right) \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} v_{1(j,i)}^{(k+1)} \\ v_{2(j,i)}^{(k+1)} \\ i_{3(j,i)}^{(k+1)} \end{bmatrix} = \begin{bmatrix} -0.4785\,10^{-5} & -0.002377 & 0.9117\,10^{-5} \\ 0.4785\,10^{-5} & 0.001188 & -0.9117\,10^{-5} \\ -0.009572 & 0.0 & 0.01824 \end{bmatrix} \begin{bmatrix} e_{L7(j-1,i-1)} \\ j_{C6(j-1,i-1)} \\ i_{C7(j-1,i-1)} \end{bmatrix} +$$

$$+ \begin{bmatrix} 0.4785\,10^{-5} & 0.0 \\ -0.4785\,10^{-5} & 0.0 \\ 0.009572 & 0.0 \end{bmatrix} \begin{bmatrix} e_{4(j,i)} \\ j_{9(j,i)} \end{bmatrix} + \begin{bmatrix} -j_{Rv1}\left(s_{(j,i)}^{k}\right) \\ j_{Cv2}\left(s_{(j,i)}^{k}\right) \\ e_{Lc3}\left(s_{(j,i)}^{k}\right) \end{bmatrix} + \begin{bmatrix} 0.0 \\ j_{Cv2(j-1,i-1)} \\ e_{Lc3(j-1,i-1))} \end{bmatrix}. \qquad (10)$$

5. The structure of the equations (10) allows to use an efficient iteration algorithm (Newton Raphson algorithm, Katzenelson algorithm [1, 3]). We remark that from an iteration to the other must be chanced only the parameters associated to the nonlinear circuit elements. The Jacobian matrix in Eqs. (10) can be obtained simple from the "slope" of the appropriate segment of the nonlinear curves. Considering $e_4(t) = 10\sin(2\pi 10^7 t)$ and $j_9(t) = 2\sin(2\pi 10^{10} t)$ mA, and using the ACAP program [14] we obtain the results represented in Figs. 7 and 8.

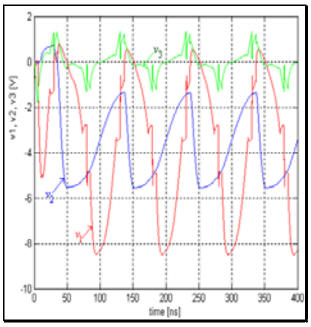


Fig. 7. Variations of v_1, v_2, and v_3.

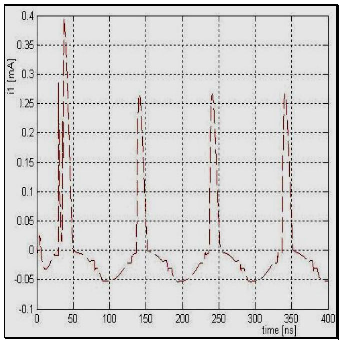


Fig. 8. Variation of i_1.

IV. CONCLUSION

The hybrid analysis method of the nonlinear analog circuits presents the advantage that it allows the computation only once at the beginning of the iteration process of those parts of the circuit equations that exclusively depend on the parameters of the linear elements.

The technique of the hybrid matrix generation is very useful for steady-state response computation and it may be successfully integrated in the frequency-domain approach. The method is remarkable by its great efficiency and generality. The procedure uses multiple time variables to describe multi-rate behavior, leading to multi-time partial differential equations. The hybrid equation formulation in a partially symbolic reduced form is used in order to obtain a MPDE form with a minimum number of independent variables. A new way to compute the appropriate BCs of the MPDE in order to accelerate the reaching of the periodic steady state is proposed.

Combining this hybrid procedure with a very efficient implicit integration algorithm, in which only the symbols of the parameters corresponding to the nonlinear circuit elements are considered, a significant efficiency in circuit analysis and an improvement of the accuracy in the numerical calculations are obtained.

ACKNOWLEDGMENT

The research is financed from CEEX 258 grant.

REFERENCES

[1] H. G. Brachtendorf, G. Welsch and R. Laur. A novel time-frequency method for the simulation of the steady state of circuits driven by multi-tone signals, *Proceedings of the ISCAS*, pp. 1508-1511, 1997.

[2] M. Iordache and Lucia Dumitriu. Efficient Decomposition Techniques for Symbolic Analysis of Large – Scale Analog Circuits by State Variable Method, *Analog Circuits and Signal Processing,* 40, 3, pp. 235-253, 2004.

[3] M. Iordache and Lucia Dumitriu. Circuit Diakoptic Analysis Based on Order Reduction of the State Equations. *Proceedings of the International Symposium on Signals, Circuits and Systems, ISSCS*, 1, pp. 187-189, 2005.

[4] R. Larchevêque, P. Bolcato and E. Ngoya. New and Efficient Method for the Multitone Steady-State Circuit Simulation, *Proceedings of the ISCAS,* pp. 330-333, 1998.

[5] T. Mei, *et al.* Robust, Stable Time-Domain Methods for Solving MPDEs of Fast/Slow Systems, *IEEE Trans on CAD of IC and Systems,* 24, 2, pp. 226-239, 2005.

[6] O. Narayan and J. Roychowdhury. Multitime simulation of voltage-controlled oscillators. *Proc. IEEE Design Automation Conf.*, pp. 629-634, 1999.
[7] O. Narayan and J. Roychowdhury. Analyzing Oscillators Using Multitime PDEs, *IEEE Trans.on CAS*-I, 50, 7, pp. 894-903, 2003.
[8] J. Roychowdhury. Efficient methods for simulating highly nonlinear multi-rate circuits, *Proc. of the Design Automation Conference*, pp. 269-274, 1997.
[9] J. Roychowdhury. Analyzing Circuits with Widely Separated Time Scales Using Numerical PDE Methods, *IEEE Trans. on Circuits and Systems* – I, 48, 5, pp. 578-594, 2001.
[10] A. E. Schwarz. Computer-aided design of microelectronic circuits and systems, Academic Press, London, 1987.
[11] A. Ushida and L. O. Chua.. Frequency-domain analysis of nonlinear circuits driven by multi-tone signals, *IEEE Trans. on Circuits and Systems* , 31, 9, pp. 766-779, 1984.
[12] M. Iordache, Lucia Dumitriu, "Multi-Time Method Based on State Equations for RF Circuit Analysis", Proceedings of 2006 IEEE International Symposium on Circuits and Systems, ISCAS'07, May 27-30, 2007, New Orleans, U.S.A., pp. 517-520.
[13] M. Iordache, Lucia Dumitriu, I.Matei, *SYMNA – Symbolic Modified Nodal Analysis*, Using Guide, Electrical Department Library, Bucharest, 2002.
[14] M. Iordache, Lucia Dumitriu, I. Matei, ACAP – Analog Circuit Analysis Program, User Guide, Library of Electrical Department, Politehnica University of Bucharest, 2001.