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Blocking Probabilities in GSM/(E)GPRS Cells with Different Radio Resources Allocation Strategies

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Abstract – Resources allocation in GSM/(E)GPRS networks is very important because they are designed to transmit mixed traffic (voice and data) with varying characteristics. Based on a bi-dimensional model developed for voice and data traffic we make a study regarding the blocking situations that occur due to the lack of resources or voice priority. Complete partitioning (CP) and Partial partitioning (PP) schemes for cells allocation are considered. Original formulas for data blocking probability and voice preemption probability are established in both cases based on the Erlang-B law.
Keywords: GSM, (E)GPRS, modeling, blocking probability, Erlang-B law.

I. INTRODUCTION

The GSM/GPRS network is designed for supporting voice traffic as well as several types of data traffic such as: Wap, Web, E-mail etc. Unlike GSM that was designed for voice service and requires a circuit switching transmission mode, GPRS provides a packet switching transmission mode in GSM network.
The main challenge in engineering of GSM/GPRS networks is the integration and support of a wide variety of applications such as voice and data services. Both require quality of service but especially for data services the traffic corresponding to each service is highly bursty and can be characterized by a typical ON/OFF process [1].
Moreover the possibility of sharing a time slot between different services complicates the analysis.
The main purpose of radio resource dimensioning is to define the necessary number of physical channels (number of TRX's) to guarantee an appropriate quality of service in the covered area.
The Radio Resources Manager (RRM) is in charge of optimizing the usage of radio resources, based on a specific resource sharing algorithm. In the literature we can distinguish three main static resources sharing schemes [2].

- In the first one, called Complete Sharing (CS) all radio channels are accessible for both data and voice traffic, with preemptive priority of voice calls.
- In the second scheme, known as Complete Partitioning (CP), time-slots are divided into two sets and each type of traffic is allowed to use only its dedicated set.
- These two strategies can be mixed to form a third hybrid scheme containing the following channel sets: one set shared between voice and data traffic and two sets each one being reserved for strict usage of its dedicated traffic: voice or data. This scheme, referred as Partial Portioning (PP), is a good compromise as it combines the advantages of both CS and CP schemes. First, reserving a set of time-slots for each type of traffic allows to guarantee, as in CP a minimum QoS for each type of traffic. Second, PP scheme provide a better efficiency than CP which is not suitable for maximizing radio utilization, especially in highly varying demand. Due to these advantages PP is widely implemented in a number of actually operating GSM/GPRS networks.

A major problem of GSM/GPRS operators is the choice of strategy to partition the available cell capacity between traditional GSM and new GPRS services.
A GSM carrier consists of 8 TDMA channels also called time-slots (TS). In a cell the total number of TS is dedicated to voice and data traffic in a PP scheme according to the traffic characteristics. In very low traffic cells, like rural area, the number of time-slots dedicate to data is very small in comparison with the busiest data traffic areas. Data time-slots have a cost, in particular for transmission links and connectivity, hence operators can't just decide for simplicity to have all voice time-slots shared with data and preemtable by voice.
A modeling tool to appropriately define the sharing strategy is essential.
GSM operators have been dimensioning their networks for voice service in terms of offered voice traffic and blocking probability. The Erlang-B formula [3] is the reference model for this system. This formula gives the proportion of calls that are blocked as a simple function of system capacity and traffic intensity. It provides the relationship between

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the offered traffic in Erlangs and the number of resources (time-slots) for a typical blocking probability (1% or 2% being the typical blocking values).

Our goal in this paper is to derive a reference model for data traffic in GPRS networks, based on formulas that have the simplicity of Erlang laws that means to find an expression for the blocking probability depending on the data traffic intensity and the number of available resources. We focus on the downlink assumed to be the limited direction because of traffic asymmetry. The CP scheme as well as the PP scheme are studied but.

The rest of the paper is organized as follow: in Section II we make a discussion on related work. Section III gives the main characteristics of GSM/(E)GPRS systems that have an influence on the analytical models and presents the voice and data traffic models used in each case CP and PP schemes. In Section IV we present an approximation of the bi-dimensional model, called the conditional product-form model, that takes into account PP strategy. We propose, based on Erlang-B law, original formulas for the blocking probabilities in both cases; CP and PP strategies and for the preemption probability in PP strategy.

The conditional product-form model is implemented and evaluated under different conditions using the proposed formulas.

II. RELATED WORKS

Several research works have addressed traffic studies in GSM/(E)GPRS networks. The major works in this field are based on analytical models using continuous-time Markov chains [4] - [7]. All these models assume an infinite number of users in the cell and thus model the arrival of traffic by a Poisson distribution. Recently, in [8]-[10] analytical models based on discrete-time Markov chains have been proposed and a single type of traffic (data traffic) is considered. It is assumed to be generated by a finite number of users. In [11] and [12] an Erlang-like law is presented with a specific emphasis on its validation on the real field data. It is based on markovian modeling and permit to compute network performances in a cell where voice and data traffic share a common resource.

III. SYSTEM DESCRIPTION AND MODELLING

Our study consider two types of traffic: GSM voice calls and GPRS data flows. It is based on the analysis of the bottleneck i.e. the radio downlink, studied in a particular cell.

A. *Main characteristics of GPRS system*

The GSM/GPRS system is characterized by the following parameters:

- t_B: the radio block duration which is equal to $20ms$.
- x_B: the number of data bytes that are transferred over one time-slot. $\frac{x_B}{t_B}$ is the throughput offered by the RLC/MAC layer to the LLC layer. The value of x_B depends on the radio coding scheme, according to Table 1 for the coding schemes CS1-CS4 used for GPRS and according to Table 2 for coding schemes MCS1-MCS9 for EGPRS. The used coding schemes depend on the radio condition.

Table 1

GPRS coding scheme	CS1	CS2	CS3	CS4
x_B (bytes)	20	30	36	50

Table 2

EGPRS coding scheme	MCS1	MCS2	MCS3
x_B (bytes)	22	28	37

MCS4	MCS5	MCS6	MCS7	MCS8	MCS9
44	56	74	112	136	148

- TS: the number of time-slots of the TDMA partitioned into a contiguous set of TS_V time-slots dedicated to voice calls, TS_{VD} time-slots shared between voice and data and TS_D time-slots dedicated to GPRS; time-slots used by data $TS_D + TS_{VD}$ are on a single TDMA which has a total of 8 time-slots.
- d (resp. u): is the number of time-slots that can be used simultaneously for downlink (resp. uplink) traffic. All GPRS mobiles have the same radio capability, denoted $d+u$.
- $tbf_{\max}$: the maximum number of mobiles that can simultaneously have an active downlink (Temporary Block Flow i.e. ON period). If there is a single TDMA, $tbf_{\max} = \min(32, 7TS, mTS)$ because of the GPRS system limitations on signaling capabilities (no more than 32 TFI's per TDMA, no more than 7USF's per uplink time-slot) and m is an additional settable parameter which describes a minimum throughput per mobile if an admission control scheme is used (no more than m mobiles per time-slot).
- We assume that the RRM continuously performs a time-slot rearrangement so as to maintain a perfect repacking. Time-slots used by phone calls and by GPRS flows are always contiguous.
- We suppose that on the lack of an available channel, a voice call will be lost on arrival according to a number of actual implementations.
- Voice calls have a preemptive priority over data flows on the shared part of the TDMA due to the fact that they generate the largest amount of

revenue in most actual operating systems. As a consequence, if all TS_V time-slots dedicated to voice are occupied and all TS_{VD} time-slots are in use with at least one of them allocated to data, then one time-slot assigned to GPRS traffic in the shared part of the TDMA will be reallocated to voice on the arrival of a GSM request.

B. *Voice traffic model*

Obviously, the classical Erlang-B model applies for voice. The steady-state voice probabilities given by relation (1) are generated by the birth-death structure of this model shown in Fig.1 .

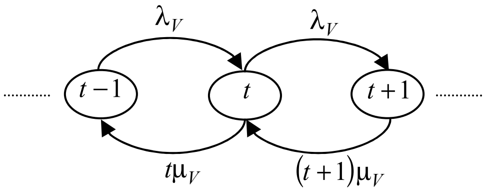

Fig.1 The birth-death model

$$p_V(t) = \frac{\rho_V^t}{t!} p_V(0), \quad t \in [0, TS_V] \tag{1}$$

$\rho_V = \dfrac{\lambda_V}{\mu_V}$ and $p_V(0)$ is obtained by normalization.

- The voice calls arrival process is characterized by a Poisson model with rate λ_V.
- Call durations are exponentially distributed with mean $\dfrac{1}{\mu_V}$.

The transition from a state *t* to a state *t*+1 is performed with the rate λ_V. The transition from a state *t* to a state *t*-1 is performed with a rate $t\mu_V$.

As voice has a preemptive priority over data, the blocking probability for voice can be obtained by Erlang-B formulas (because data traffic is transparent to voice), with a number of resources equal to $TS_V + TS_{VD}$.

$$P_{BV} = \frac{\rho_V^{(TS_V + TS_{VD})}}{(TS_V + TS_{VD})!} p_V(0) \tag{2}$$

C. *Data traffic model*

Data traffic is modeled as in [11] and [12] assuming that there is a fixed number *N* of data mobiles in the cell. Each mobile is doing an ON/OFF traffic with an infinite number of pages:

- ON periods correspond to the download of an element like a WAP, a WEB page, an email, a file, etc. Its size is characterized by a discrete random variable X_{on}, with an average value x_{on}.
- OFF periods correspond to the reading time of the last downloaded element, which is modeled as a random variable T_{off} with an average value of t_{off} seconds.

For constructing our model based on the ON/OFF traffic we define the average data traffic parameters as follows:

- The average data rate of data arrival process:

$$\lambda_D = \frac{1}{t_{off}} \tag{3}$$

- The average data rate per time-slot:

$$\mu_D = \frac{x_B}{x_{on} t_B} \tag{4}$$

Based on these two parameters we define, as shown in relation (5) a parameter ρ_D that characterized data traffic, similar to ρ_V.

$$\rho_D = \frac{\lambda_D}{\mu_D} = \frac{t_B x_{on}}{x_B t_{off}} \tag{5}$$

For the GPRS system associated to a single cell we consider a continuous-time Markov model.

System with complete partitioning (CP)

If we consider the system with complete partitioning, the data model applies on the TS_D time-slots reserved to data traffic.

There is a limitation $n_{\max}$ of the number of data mobiles that can be in active transfer in the cell due to the system constrain $tbf_{\max}$ and the total mobile population:

$$n_{\max} = \min(tbf_{\max}, N) \tag{6}$$

Based on the memoryless assumptions for the ON and OFF distributions discussed and validated against the most commonly used data traffic models [11] –[15] data traffic model is represented by the continuous-time Markov-chain shown in Fig.2.

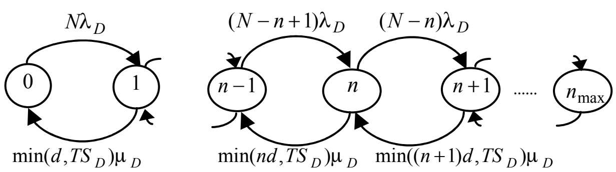

Fig.2 Continuous-time Markov-chain model

As indicated in Fig.2, the state n of the Markov chain corresponds to the number of the data mobiles that are simultaneously in active transfer. The maximum bandwidth capacity they can use is TS_D.

Because of the maximum downloading capacity d of each GPRS mobile, two situations can be distinguished:

(1) If $nd < TS_D$, the available bandwidth is not fully utilized by data mobiles. As a consequence the transition rate from state n to state $n-1$, given by the accomplished transfer of one mobile, is $nd\mu_D$.

(2) If $nd \geq TS_D$ the alocator has to share the TS_D time-slots among the n data mobiles and the transition rate from state n to state $n-1$ is $TS_D\mu_D$.

The stationary probabilities of having n data mobile in active transfer, derived from the birth-death structure of the Markov chain are:

(1) for $n \in (0, n_0]$

$$p_D[n] = \frac{N!}{n!d^n(N-n)!}\rho_D^n p_D(0) \qquad (7)$$

(2) for $n \in (n_0, n_{\max}]$

$$p_D[n] = \frac{N!}{n_0!d^{n_0}TS_D^{n-n_0}(N-n)!}\rho_D^n p_D(0) \qquad (8)$$

Where n_0 is the maximum value of n such that $nd < TS_D$:

$$n_0 = \left\lfloor \frac{TS_D}{d} \right\rfloor \qquad (9)$$

For the blocking probability we propose the formula given in relation (10), similar to the Erlang-B law:

$$P_{BD} = \frac{N!}{n_0!d^{n_0}TS_D^{n_{\max}-n_0}(N-n_{\max})!}\rho_D^{n_{\max}} p_D(0) \qquad (10)$$

System with partial partitioning (PP)

We define $TS_{\max}(t)$ as the number of time-slots that can be used by data when there are t voice calls in the system:

$$TS_{\max}(t) = TS_D + TS_{VD} - \max(0, t - TS_V) \qquad (11)$$

with t taking the values: $t \leq TS_V + TS_{VD}$.

Also we expressed $N_{\max}(t)$ the maximum number of data mobiles that can simultaneous be in active transfer, when there are t pending voice calls. Due to the GPRS system limitations on signaling capabilities, $N_{\max}(t)$ can be derived, for $t \leq TS_V + TS_{VD}$, as follows:

$$N_{\max}(t) = \min(32, 7TS_{\max}(t), mTS_{\max}(t), N) \qquad (12)$$

The model applied to this system is the bi-dimensional Markov chain proposed in [11]. A state of the bi-dimensional Markov chain is represented by a couple (t,n) of the number t of voice calls pending and the number n of data mobiles in active transfer. For better understanding the model we have represented in Fig.3 the transitions out of a generic state (t,n) with $0 < t < TS_V + TS_{VD}$ and $n = 0, \cdots, N_{\max}(t+1)$. The vertical transitions in Fig. 3 correspond to the classical Erlang-B model transitions for voice traffic described in Fig.1. Horizontal transitions correspond to the continuous-time model derived in Fig.2, where we have replaced the value TS_D by the adequate value $TS_{\max}(t)$. The transition rate from the state (t,n) to the state $(t,n-1)$ is given by: $\min(nd, TS_{\max}(t))\mu_D$. On the other hand the transition rate from state (t,n) to state $(t,n+1)$ is equal to $(N-n)\lambda_V$ because of the ON/OFF assumptions regarding the data traffic.

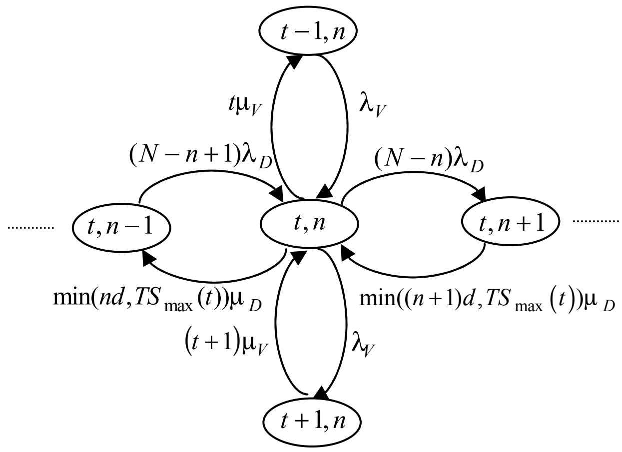

Fig. 3 Transactions out of a generic state in the bi-dimensional model

Another important part of the bi-dimensional model is represented by the transitions that occurs if a new voice call starts when the number of existing voice communications is greater then TS_V (and lower than $TS_V + TS_{VD}$). This new call preempts a time-slot that could be in use by GPRS mobile and can results in GPRS rejections, when the data mobile capacity is reached. These transactions out of a limiting state $(t, N_{\max}(t))$ are represented in Fig.4.

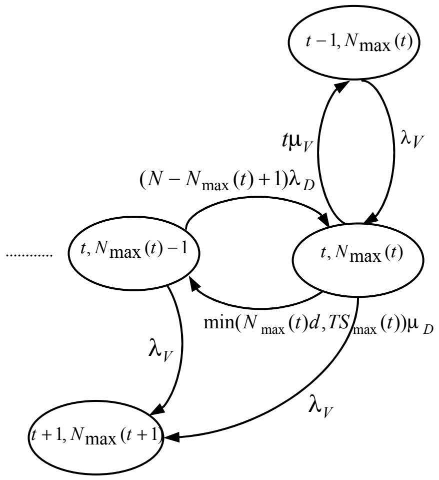

Fig.4 Transactions out of a limiting state in the bi-dimensional model

As indicate in Fig.4, the rejections are represented in the Markov chain by transitions going from any state (t,n) where $t \in [TS_V, TS_V + TS_{VD}]$ and $n = N_{\max}(t+1)+1,\cdots,N_{\max}(t)$ to state $(t+1, N_{\max}(t+1))$. At most $\min(m,7)$ rejections may occur.

IV. PERFORMANCE EVALUATION

In order to introduce the blocking probability formula for the system with partial partitioning we apply the approximation of the bi-dimensional model with the conditional product-form model developed in [11] and represented in Fig.5.

The model shown in Fig.5 represents the decomposition of the original bi-dimensional model into several one-dimensional Markov chains. As a consequence the probability of a generic state (t,n) becomes:

$$p(t,n) = p_V(t)\, p_D(n|t) \qquad (13)$$

As mentioned before, voice traffic can be modeled with the Erlang model. Due to the fact that there are currently t voice mobiles in communication, the data mobile traffic is modeled by the Erlang-like model developed in [12] with a number of time-slots equal to $N_{\max}(t)$. We define $n_0(t)$ the maximum value of n such that $nd < TS_{\max}(t)$:

$$n_0(t) = \left\lfloor \frac{TS_{\max}(t)}{d} \right\rfloor \qquad (14)$$

The stationary probabilities of having n data mobiles in active transfer conditioned by the state t of voice calls can be derived as follows:

(1) for $(0, n_0(t)]$:

$$p_D(n|t) = \frac{N!\rho_D^n p_D(0|t)}{n!d^n(N-n)!} \qquad (15)$$

(2) for $(n_0(t), N_{\max}(t)]$:

$$p_D(n|t) = \frac{N!\rho_D^n p_D(0|t)}{n_o(t)!d^{n_0(t)} TS_{\max}^{n-n_o(t)}(t)(N-n)!} \qquad (16)$$

where ρ_D is the same as in relation (5) and $p_D(0|t)$ is obtained by normalization for any value of t.

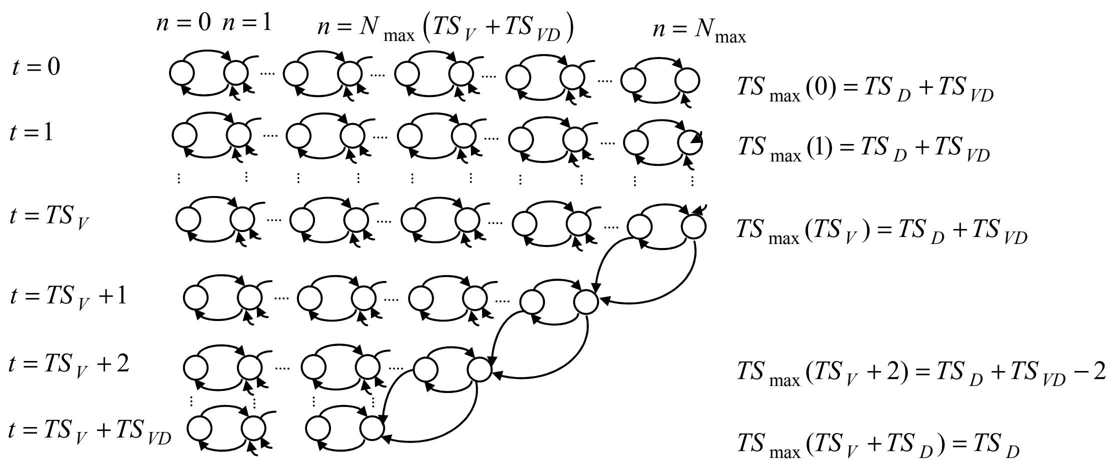

Fig.5 Product-form model

The data blocking probability and the probability that a data transfer ends prematurely because of voice call preemption can be derived easily from the detailed stationary probabilities, $p_D(n|t)$.

We propose to express the data blocking probability, i.e. the probability that a new data transfer request is rejected because of the lack of available resources, as:

$$P_{BD} = \sum_{t=0}^{TS_V+TS_{VD}} p\left(t, N_{\max}(t)\right) = \sum_{t=0}^{TS_V+TS_{VD}} p(t) p_D(N_{\max}(t)|t) \quad (17)$$

This formula is inspired by the Erlang-B formula and can be applied because of the structure of the model used based on "an Erlang like-law".

For the preemption probability we propose the formula:

$$P_P = \sum_{t=TS_V}^{TS_V+TS_{VD}} \sum_{n=N_{\max}(t+1)+1}^{N_{\max}(t)} p(t,n) = \sum_{t=TS_V}^{TS_V+TS_{VD}} \sum_{n=N_{\max}(t+1)+1}^{N_{\max}(t)} p(t) p_D(n|t) \quad (18)$$

The probabilities shown in (17) and (18) can be expressed as functions of data traffic ρ_D as follows:

$$P_{BD} = \sum_{t=0}^{TS_V+TS_{VD}} \frac{\rho_V^t}{t!} \frac{N! \rho_D^{N_{\max}(t)}}{n_0! d^{n_0(t)} TS_{\max}^{n-n_0(t)}(t)(N - N_{\max}(t))} p_V(0) p_D(0|t) \quad (19)$$

$$P_P = \sum_{t=TS_V}^{TS_V+TS_{VD}} \sum_{n=N_{\max}(t+1)+1}^{N_{\max}(t)} \frac{\rho_V^t \rho_D^n N! p_V(0) p_D(0|t)}{t! n_o(t)! d^{n_0(t)} TS_{\max}^{n-n_o(t)}(t)(N-n)!} \quad (20)$$

The product-form model has been implemented by a simple program written in Matlab and various scenarios were experimented.

In the first scenario we have considered a cell equipped with a single TRX that provides $TS_V = 3$, $TS_{VD} = 3$, $TS_D = 1$. One time slot is dedicated to broadcast and signaling purposes in the cell. The GPRS coding scheme was CS2, that provides $x_B = 30$ bytes transferred during t_B over one time slot and for the reading time we consider a typical average value $t_{off} = 7$.

Fig.6 and Fig.7 show respectively the blocking probability and the preemption probability obtained according to relations (19) respectively (20) for a large number of N values.

For each value N, we have considered four loading situations that correspond to different occupancies of the time-slots that can be used by voice as indicated in Table 3.

In each case a 2% for the lost voice traffic was considered according to Erlang-B formula.

Table 3

ρ_V	$TS_V + TS_{VD}$	TS_V (occupied/ available)	TS_{VD} (occupied/ available)
0.6	3	3/3	0/3
1.0	4	3/3	1/3
1.6	5	3/3	2/3
2.2	6	3/3	3/3

The value of $x_{on} = 4000$ correspond to a moderate GPRS data traffic load.

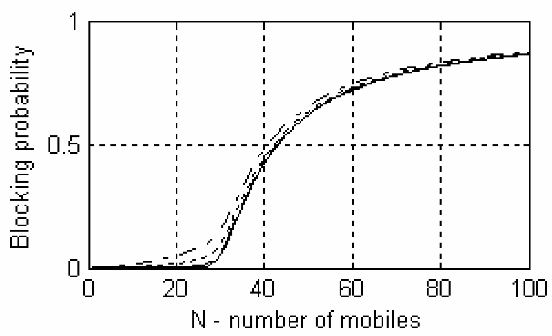

Fig.6 The blocking probability - 1TRX and x_{on}=4000 bytes

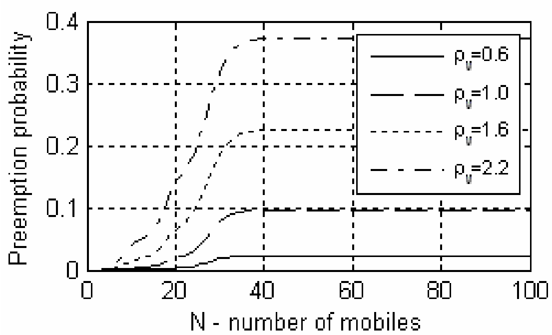

Fig.7 The preemption probability - 1TRX and x_{on}=4000 bytes

Fig. 8 and Fig.9 illustrate respectively the blocking probability and the preemption probability calculated using the previous system and the voice loading conditions indicated in Table 3. Only the value of data traffic was changed according to a heavy data traffic load, $x_{on} = 8000$.

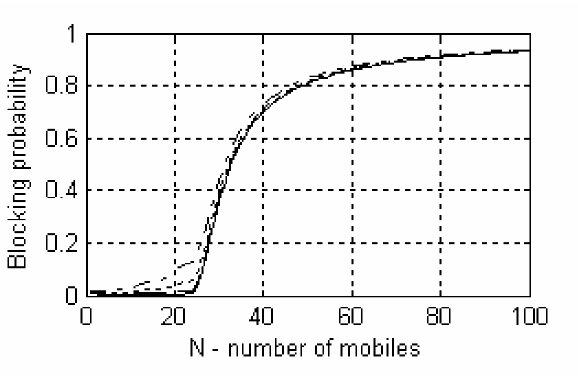

Fig.8 The blocking probability - 1TRX and x_{on}=8000 bytes

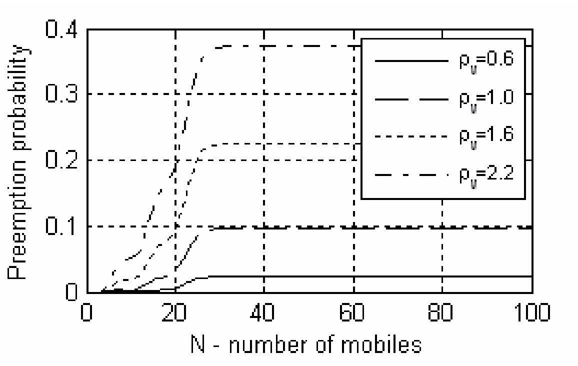

Fig.9 The preemption probability - 1TRX and x_{on}=8000 bytes

The curves of Fig.6 and Fig.8 show that the blocking probability for data (P_{BD}) depends on the number of data mobiles N and does not depend on the voice traffic load ρ_V.

The preemption probability (P_P) strongly depends on voice traffic loads and is weakly influenced by the number of data mobiles N as shown in Fig.7 and Fig.9.

The results show clearly that the limit values of the preemption probability (P_P) does not depend on data traffic load x_{on} values.

In the second scenario we have considered a cell equipped with two TRX that provides $TS_V = 6$, $TS_{VD} = 7$, $TS_D = 1$. In that case two time slots are dedicated to broadcast and signaling purposes in the cell. The system parameters are the same as in the previous case ($x_B = 30$, $t_{off} = 7$).

Again, for each value of N, we have considered four loading situations as indicated in Table 4, that correspond to different occupancies of the time-slots that can be used by voice calls.

In our first experiment we have considered the data traffic load according to: $x_{on} = 8000$.

The results for the blocking probability and the preemption probability are shown in Fig.10 respectively in Fig.11.

Table 4

ρ_V	$TS_V + TS_{VD}$	TS_V (occupied/ available)	TS_{VD} (occupied/ available)
2.2	6	6/6	0/7
4.3	9	6/6	3/7
5.8	11	6/6	5/7
7.4	13	6/6	7/7

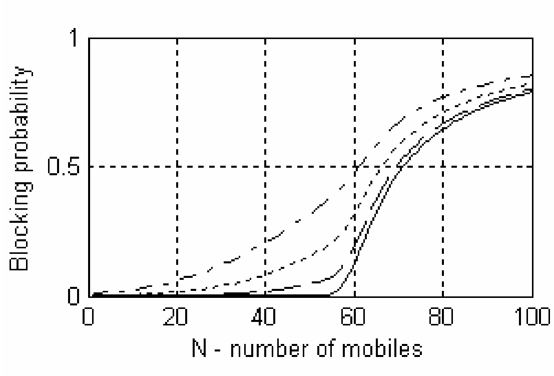

Fig.10 The blocking probability - 2TRX and x_{on}=8000 bytes

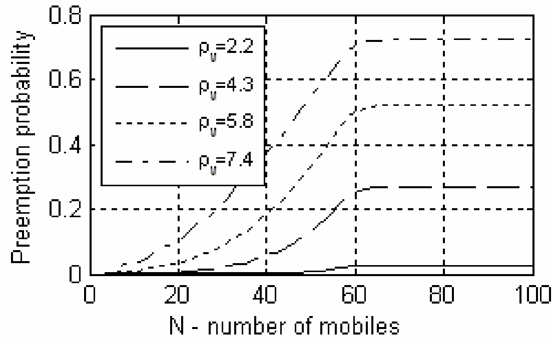

Fig.11 The preemption probability - 2TRX and x_{on}=8000 bytes

Comparing Fig.8 and Fig.10 the conclusion is that the blocking probabilities decrease according to the increased number of resources, for the same value of the data traffic x_{on}.

We have represented the same probabilities as before in Fig.12 and Fig.13 for a heavy data traffic load considering $x_{on} = 16000$.

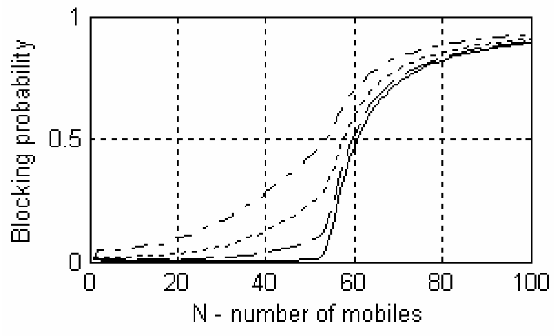

Fig.12 The blocking probability - 2TRX and x_{on}=16000 bytes

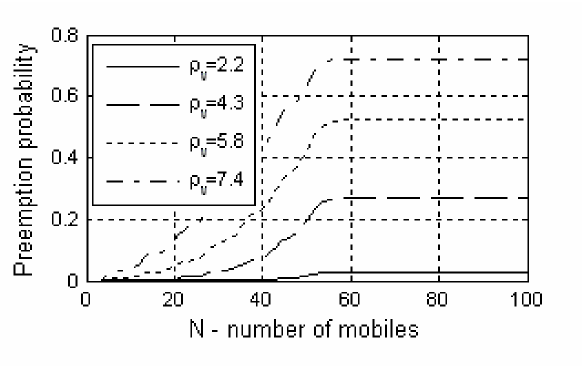

Fig.13 The preemption probability - 2TRX and x_{on}=16000 bytes

The preemption probabilities (P_P) shown in Fig.11 and Fig.13 have the same behavior as in our first scenario.

V. CONCLUSIONS

In this paper we have made a study regarding the blocking situations that occur due to the lack of resources and voice traffic priority over data traffic in GSM/(E)GPRS cells. Two radio resources allocation strategies have been considered: systems with complete partitioning as well as systems with partial partitioning.

In the first case (CP) the model on which we have based our study consists of two parts: one part dedicated to voice and the other one dedicated to data.

For the voice traffic we have used the classical birth-death model and the Erlang-B formula to measure the blocking probability.

For the data traffic we have developed a model based on the continuous-time Markov chain and for the blocking probability we have proposed a formula similar to the Erlang-B law.

We have started considering in the second case (PP) a model represented by the bi-dimensional Markov chain, where voice and data are treated together. In order to introduce the blocking probably formulas we have approximated the bi-dimensional model with the conditional product-form model. This model has allowed us to introduce the blocking probabilities formulas according to the PP strategy.

For the voice traffic the model is exactly the same as mentioned before. Only the number of resources was changed to correspond to the PP strategy.

For the data traffic the conditional product-form model has allowed us to define two original formulas to better measure the performance of the system according to the traffic load of each service.

The first formula, called the data blocking probability was inspired by the Erlang-B law and it expresses the data rejections probability, because of the lack of available resources, as function of data traffic.

The second one, called the voice preemption probability, measures the probability for a data transfer to end prematurely because of voice calls preemption.

The evaluations of the model were obtained using a Matlab program for various scenarios.

The results show that the blocking probability for data (P_{BD}) depends on the number of data mobiles N and is not depending on the voice traffic load .

The preemption probability (P_P) strongly depends on the voice traffic loads and is less influenced by the number of data mobiles N .

For the same value of the data traffic the blocking probabilities decrease according to the increased number of resources.

The proposed formulas can be used in the future as dimensioning tools for radio resources allocation.

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