

Performance Analysis of STBCs for Mobile Wireless Systems

Ancuta Moldovan¹ Tudor Palade¹ Emanuel Puschita¹

Abstract - The main goals in developing new wireless communication systems are increasing the transmission capacity and improving the spectrum efficiency. Space-time block coding (STBC) exploit the richly scattered wireless channel by transmitting redundant data streams from multiple antennas in order to improve the reliability of data transfer and greatly increase data transmission rates without additional radio resource requirements. Significant performance gain is achieved by varying some transmit parameters at almost no processing expense, while a simple, linear maximum decoding algorithm operates at the receiver.

Keywords: space-time block codes, diversity order, bit error rate

I. INTRODUCTION: MIMO-OFDM

MIMO (Multiple Input Multiple Output) is one of the enabling physical layer technologies that can provide high-capacity wireless links. With multiple antennas plus adaptive signal processing at the transmitter and receiver, a MIMO system takes advantage of the spatial diversity obtained by spatially separated antennas in a dense multipath scattering environment. When combined with adaptive modulation, a MIMO system can achieve a much higher transmission rate than that of a single-input single-output system without additional radio resource requirements.

The spectral efficiency of a MIMO system grows linearly with the minimum number of transmit and receive antennas. However, the complexity of a MIMO receiver increases in a broadband transmission environment in the presence of delay spread (inter-symbol interference). The solution to this problem is to use OFDM (orthogonal frequency-division multiplexing) based modulation, which avoids inter-symbol interference by modulating narrow orthogonal carriers. Combining OFDM and MIMO results in narrowband MIMO transmission, and thereby, simplifies the implementation of MIMO without loss of capacity.

The use of MIMO can provide array gain, since multiple antennas can coherently combine signals to increase the signal to noise ratio value and hence improve coverage, diversity gain, which combats fading and significantly improve reliability and

multiplexing gain, which leads to an increase in data rate[1]. The achievable capacity and performance depend on the channel conditions and on the structure of the transmit signal. The signal design directly influences the complexity of the transmitter and, particularly the receiver.

The MIMO coding techniques can be split into three groups according to [4]: space-time coding (STC), space division multiplexing (SDM) (if channel state information –CSI- is not available at the transmission) and beamforming (if CSI is available at the transmitter). STC increases the performance of the communication system by coding over the different transmitter branches, while SDM achieves a higher data by transmitting independent data stream on the different transmitter branches simultaneously and at the same carrier frequency. The last type of codes exploits the knowledge of channel at the transmitter. It decomposes the channel coefficient matrix using singular value decomposition (SVD) and uses these decomposed unitary matrices to achieve near capacity.

The purpose of this paper is to evaluate the performance of space-time block codes and to see which are the transmit parameters that most influence the quality of the transmission. Also the influence of the number of receive antennas is analyzed while performing a linear processing at the receiver.

The rest of the paper is organized as follows. In Section II we describe the coding principle of the space-time codes and we provide a transmission model based on space-time block codes. Section III analyzes the performances of the space-time block codes and show the performance that are achieved by varying different parameters on the transmission side. The simulations were done using Matlab 7.0.1. Finally, Section IV presents the conclusions and the final comments.

II. SPACE-TIME CODING

The technique is characterized by joint encoding, which means that the original bit stream is first encoded and then demultiplexed into coded

¹ Faculty of Electronics, Telecommunications and Information Technology, Communications Dept.
26 G.Baritiu Street, 400027 Cluj-Napoca, Romania, e-mail Ancuta.Moldovan@com.utcluj.ro

substreams of which each is modulated and mapped onto the corresponding transmit antenna[6]. Coding is performed in both spatial and temporal domains to introduce correlation between signals transmitted from various antennas at various time periods. The spatial-temporal correlation is used to exploit the MIMO channel fading and minimize transmission errors at the receiver. Space-time coding can achieve transmit diversity and power gain over spatially uncoded systems without sacrificing the bandwidth and without the need to increase the transmit power. STC includes space- time block codes (STBC), space-time trellis codes (STTC), space-time turbo codes and linear dispersive codes.

For the evaluation of the STBC encoding algorithm a wireless communication system is considered, with N_t transmit antennas and N_r receive antennas. The channel is assumed to be quasi-static, flat fading channel and the path gain from transmit antenna i to receive antenna j is defined to be h_{ij} , modeled as sample of independent complex Gaussian random variables with variance 0.5 per real dimension. The path gains are constant over one encoded frame and may vary from one frame to another, as the channel is assumed to be quasi-static.

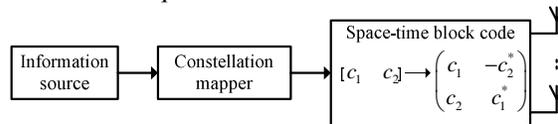


Fig.1. Transmission model using STBC

The incoming bit stream is mapped into a number of modulated symbols from a real or complex constellation[7]. Then a block of Y data symbols are encoded into a codeword matrix C of size $[N_t \times N_s]$, which will then be sent simultaneously through N_t antennas in N_s OFDM blocks. The codeword matrix C can be expressed as:

$$C = \begin{pmatrix} c_{11} & \dots & c_{1N_s} \\ \vdots & \ddots & \vdots \\ c_{N_t 1} & \dots & c_{N_t N_s} \end{pmatrix} \quad (1)$$

where rows correspond to transmit antennas and columns to symbol time instants[6]. The rate of STBCs is defined as $R = Y / N_s$. If the channel state information (CSI) is available at the receiver, the optimal maximum likelihood detection can be performed[6]. The received signal at each receive antenna is a linear superposition of the n transmitted signals perturbed by noise and can be computed as:

$$r_{jt} = \sum_{i=1}^{N_t} h_{ij} c_{it} + n_{jt} \quad (2)$$

where r_{jt} is the received signal at antenna j , at the time t and n represents the AWGN and are modeled as i.i.d. complex Gaussian random variables with zero mean and with variance $1/(2SNR)$ per complex dimension.

When the number of transmitted antennas grows, R improves related to the fact that number of transmitted symbols per unity of time increases. From an efficiency point of view, the codes should be designed such that they achieve a rate as high as possible. On the other hand redundancy is added to obtain a robust communication link. Both goals cannot be achieved always at the same time.

STBC achieve the maximum possible transmission rate for any number of transmit antennas using any arbitrary real constellations such as PAM. For an arbitrary complex constellation such as PSK and QAM, space-time block codes are designed to achieve $1/2$ of the maximum possible transmission rate for any number of transmit antennas. For the specific cases of two, three and four transmit antennas, STBC can achieve $3/4$ of maximum possible transmission rate[3]. STBCs which attain full code rate and full diversity do not exist for more than two transmit antennas for complex valued constellations. Orthogonal-STBCs designed for more than two antennas can achieve full diversity order but they have a code rate less than unity.

Regarding the efficiency, the orthogonal STBCs do not always fully exploit the available MIMO channel capacity. A STBC is optimal with respect of capacity when it is rate one and it is used over a channel of rank one[5]. So, only rate one STBCs used over any channel with one receive antenna are optimal with respect to capacity. The rate of complex orthogonal STBCs with more than two transmit antennas drops below one, the result is a capacity penalty. Non-orthogonal STBCs are able to achieve rate one, but at the expense of performance.

III. PERFORMANCE ANALYSIS OF STBC

There are a number of parameters that affect the performances of the transmission based on space-time block codes. These parameters are: the number of transmit/receive antennas, the modulation type and constellation size, the type of encoder, the coding rate. The influence of these parameters will be analyzed in the simulations bellow.

The data used for the simulations are based on the orthogonal designs presented in [2].

In the first simulation the input data are modulated with 4-PSK modulation and encoded with a rate one code. The number of received antennas is varied between 1 and 5 in order to analyze the effect on the performance of the system. The number of transmit antennas is equal to two. As it can be seen in Fig. 2, receive diversity is an efficient and simple possibility to increase the link reliability.

With the increase in the number of received antennas the bit error rate is reduced. Since each element receives an independently copy of the same signal, the

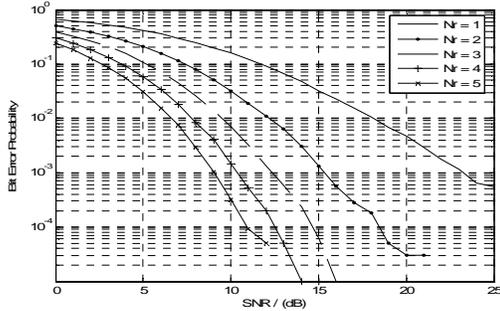


Fig.2. Number of receive antennas influence

output SNR not only increases, but also the fluctuations in the output SNR are reduced, so the bit error rate is smaller. The increase of the diversity order determines the exponential decay of the error-rate with the SNR; as it can be noticed the slope of the BER curves also increases with the number of received antennas.

The table below contains the value needed for the SNR in order to obtain a $BER = 10^{-4}$, when varying the number of receive antennas.

Table 1. Simulation results: different number of receive antennas

N_r	1	2	3	4	5
$SNR[dB]$	>25	18.5	14.5	12.5	11

In order to test the influence of the constellation size on the transmission, a simulation was done in which a rate one orthogonal data design G_2 was used for the transmission of the encoded data and it can be compared with the transmission without the encoder.

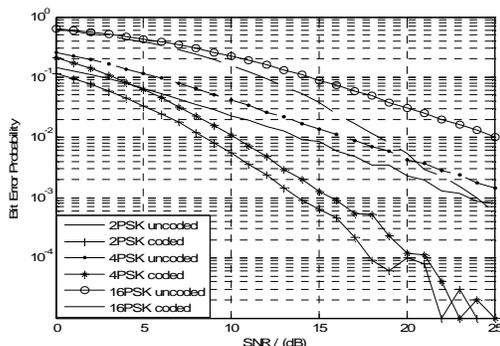


Fig 3. Coded and uncoded transmission for different PSK constellation size

It can be observed in Fig.3. that the transmission based on encoded data and modulated with 2-PSK is better by about 10dB that the transmission with no encoded data, for a BER of 10^{-3} . The same difference is achieved for the 4-PSK constellation. For the 16-PSK constellation, at low SNRs, there is a small difference between the encoded and not coded data, but the performers are poor compared to the

other constellations. At high SNRs the encoded 16-PSK is comparable and even better that the not coded 2-PSK and 4-PSK modulations, because it exploits the signal space more efficiently. It is noticeable that higher-order modulation exhibit higher error-rates but in exchange they deliver a higher raw data rate. In Fig. 3 can be observed that the data transmitted and encoded have the same slope, the same diversity order equal to two, while the magnitude of the slope of the not coded data is smaller as it does not benefit from the spatial diversity. The following table presents the results of the simulation.

Table 2. Simulation results: coded and uncoded transmission for $BER = 10^{-4}$

M-PSK	2-PSK	4-PSK	16-PSK
Coded	G_2	-	G_2
uncoded	-	G_2	-
$SNR[dB]$	18	35	21
		35.5	28.5
			>40

Fig.4. shows the performance of a system with the space-time encoder based on the rate one complex orthogonal design G_2 , while the type of modulation and the constellation size are varied.

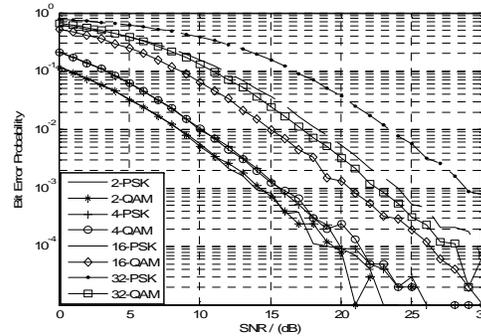


Fig 4. M-PSK vs. M-QAM

For the small size constellation (2-4 PSK/QAM) the performances are almost the same. For the higher size constellations the QAM modulation exploits the signal space more efficiently than the PSK modulation. For a $BER = 10^{-3}$, the 32-QAM modulation is 7dB better than the 32-PSK modulation, and it performs even better than the 16-PSK by about 2 dB.

Table 3. SNR values for different modulation needed to obtain a $BER = 10^{-4}$

	PSK	QAM
Mary=2	18.5dB	19.5dB
Mary=4	20dB	21dB
Mary=16	29.5dB	26dB
Mary=32	>30dB	28dB

From Table 2. and Table 3. we can conclude that the same performances can be obtained by using the 2-PSK modulation with one receive antenna or by using 4-PSK modulation with two receive antennas, while keeping the same rate one code ($SNR = 18dB, BER = 10^{-4}$).

In the next simulations are examined the performances of STC schemes for identical spectral efficiencies. This can be achieved by using an appropriate modulation scheme for each STC.

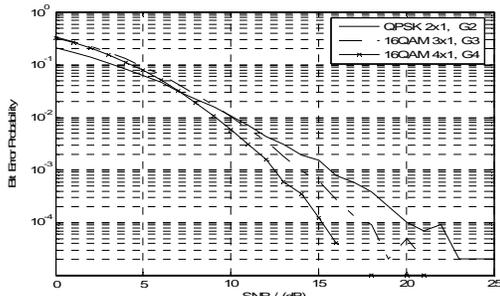


Fig. 5. Transmission performance for the same spectral efficiency and different orthogonal STCs

For a spectral efficiency of 2 bits/s/Hz the data are modulated using QPSK and encoded with the complex orthogonal design G_2 with the code rate $R_c = 1$, then the modulation used is 16-QAM with the encoded schemes based on G_3 and G_4 orthogonal designs, codes with a lower rate $R_c = 1/2$. At low SNRs the code with two transmit antennas is has the best results. QPSK is more robust than 16-QAM against the influence of noise. At high SNRs, the code with four transmit antennas gains about 1dB and 2dB relative to the codes with two and three antennas. So, the higher diversity degree becomes obvious only for high SNRs.

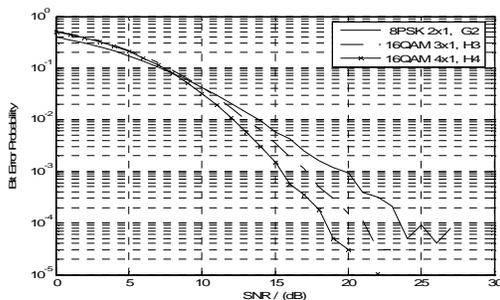


Fig. 6. Transmission performance for the same spectral efficiency and different orthogonal STCs

The same spectral can be obtained by using the 16-QAM modulation combined with two higher rate codes. Three and four antennas systems employ rate $3/4$ codes using the H_3 and H_4 codes, with 16-QAM modulation, which yields to a spectral efficiency of 3bits/s/Hz. The same efficiency is obtained with a 8-PSK modulation with G_2 complex orthogonal design. From the figure above, it can be seen that at the BER of 10^{-4} , the code H_4 is better by about 3dB and 5dB than the code H_3 , respectively G_2 . At lower BER the difference is even higher. At low SNRs the PSK modulation performs better, but the difference is even smaller than in the first case.

The last two simulation show that the transmit diversity improves the performance. For the both cases the number of receive antennas is one.

Table 4. SNR needed to obtain a $BER = 10^{-4}$

$\eta = 2 \text{ bits / s / Hz}$		$\eta = 3 \text{ bits / s / Hz}$	
QPSK - G_2	20dB	8-PSK - G_2	23dB
16-QAM - G_3	18dB	16-QAM - H_3	21dB
16-QAM - G_4	15dB	16-QAM - H_4	18.5dB

For a fixed spectral efficiency, in the high SNR regime, diversity is most important and overcompensates the larger sensitivity of high-order modulation schemes. At low SNRs, robust modulation schemes such as QPSK should be preferred because the diversity gain is smaller than the loss associated with the change of the modulation scheme.

IV. CONCLUSIONS

The simulation results confirm that with space-time block coding and multiple transmit antennas a significant performance gain can be achieved at almost no processing expense. Although STBC do not provide a coding gain, they have the great advantage that decoding requires some simple combinations of the symbols received at each antenna. Moreover, STBCs provide the full diversity degree achievable with a certain number of transmit and receive antennas. Redundancy in STBCs is utilized to achieve diversity gain.

From the simulations performed, it can be concluded that a simple way to improve the quality of the transmission is to increase the number of receive antennas. For small devices, where the number of antennas is limited, a solution is to increase the number of transmit chains with very little decoding complexity. When the SNR is high diversity is important and overcompensates the larger sensitivity of high-order modulation schemes. At low SNR robust modulation schemes should be used because the diversity gain is smaller.

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