Buletinul Ştiinţific al Universităţii "Politehnica" din Timişoara

Seria ELECTRONICĂ şi TELECOMUNICAŢII
TRANSACTIONS on ELECTRONICS and COMMUNICATIONS

Tom 53(67), Fascicola 2, 2008

On the Selection Combining Over Correlated *α-μ* Fading Channels

Mihajlo Stefanović[1], Dragana Krstić[2], Stefan Panić[3], Ilija Temelkovski[4]

Abstract-In this paper, system performances of selection combining and correlated *α-μ* (Generalized Gamma) fading channels are analyzed. Fading between the diversity branches is correlated and distributed with *α-μ* distribution.Very useful closed-form expressions are obtained for the output signal's probability density function (PDF) and cumulative distribution function (CDF). The main contribution of this analysis for dualbranch signal combiner, is that it has been done for general case of *α-μ* (Generalized Gamma) distribution, which includes as special cases important other distributions such as Weibull and Nakagami-*m* (therefore, the One-Sided Gaussian and Rayleigh are also special cases of it), so our analysis has high level of generality.

Keywords: selection combining, *α-μ* distribution

I. INTRODUCTION

Various techniques for reducing fading effect and influence of cochannel interference are used in wireless communication systems [1]. Diversity reception is an effective remedy that exploits the principle of providing the receiver with multiple faded replicas of the same information-bearing signal. The goal of diversity techniques is to upgrade transmission reliability without increasing transmission power and bandwidth and to increase channel capacity. When multiple receiver antennas are used space diversity is an efficient method for amelioration system's quality of service (QoS) [2].

There are several principal types of combining techniques and division can be generally performed by their dependence on complexity restriction put on the communication system and amount of channel state information available at the receiver. One of the least complicated combining methods is selection combining (SC). Combining techniques like equal gain combining (EGC) and maximal ratio combining (MRC) require all or some of the amount of the channel state information of received signal. MRC and EGC combining techniques require separate receiver chain for each branch of the diversity system, which increase its complexity.

SC receiver process only one of the diversity branches, and is much simpler for practical realization, in opposition to these combining techniques.

In general, selection combining, assuming that noise power is equally distributed over branches, selects the branch with the highest signal-to-noise ratio (SNR), that is the branch with the strongest signal [1-3]. There is also a type of selection combining that chooses the branch with highest signal and noise sum [4]. In fading environments as in cellular systems where the level of the cochannel interference is sufficiently high as compared to the thermal noise, SC selects the branch with the highest signal-to-interference ratio (SIR-based selection diversity) [5]. When the correlation is considered, most of the works deal with the case of dual branch because the analysis becomes complicated as the diversity order grows [6], [7].

The multipath fading in wireless communications, is modeled by several distributions including Weibull, Nakagami-*m* Hoyt, Rayleigh, and Rice. By considering two important phenomena inherent to radio propagation, namely non-linearity and clustering, the α-μ fading model was recently proposed in [8], [9]. The model provide a very good fit to measured data over a wide range of fading conditions. This distribution has the same functional form as the Generalized Gamma or Stacy distribution [10].The α-μ distribution is written in terms of physically-based fading parameters, namely α and μ. Roughly speaking, α is related to the non-linearity of the environment whereas μ is associated with the number of multipath clusters [11]. The effect of correlated fading has been extensively analyzed on the performance metrics of wireless communication system. In papers [12, 13] selction diversity over Wielbull

[1]Department of Telecommunications, Faculty of Electronic Engineering, University of Nis, Aleksandra Medvedeva 14, 18000 Nis, Serbia), misa@elfak.co.yu
[2]Department of Telecommunications, Faculty of Electronic Engineering, University of Nis, Aleksandra Medvedeva 14, 18000 Nis, Serbia), dragana@elfak.co.yu
[3]Department of Telecommunications, Faculty of Electronic Engineering, University of Nis, Aleksandra Medvedeva 14, 18000 Nis, Serbia stefanpnc@yahoo.com
[4]Department of Telecommunications, Faculty of Electronic Engineering, University of Nis, Aleksandra Medvedeva 14, 18000 Nis, Serbia

fading channels has been analyzed. In recent works [14-15] the joint PDF and CDF of the multivariate Nakagami-*m* and Rayleigh distributions. Moreover to the best author's knowledge, no analytical study of selection combining involving assumed correlated α-µ fading for desired signal, has been reported in the literature.

In this paper, we consider diversity system with multiple correlated α-µ fading channels model in the presence of mutually correlated interferences. We model fading and interference by α-µ fading distribution with constant correlation model, which is an adequate for multipath waves propagating in a nonhomogenous environment.

In order to study the effectiveness of any modulation scheme and the type of diversity used, it is required to evaluate the system's performance over the channel conditions [3]. In this paper, for proposed system model, expressions for cumulative distribution function (CDF) and probability distribution function (PDF) of the output signal for selection combining diversity are derived. Numerical results for PDF and CDF are graphically presented.

Outage probability is shown graphically for different system parameters. The main contribution of this analysis is that α-µ fading channels model includes as special cases important other distributions such as Weibull and Nakagami-*m* (therefore, the One-Sided Gaussian and Rayleigh are also special cases of it), so our analysis has high level of generality.

II. STATISTICS OF THE SC OUTPUT

The α-µ distribution is a general fading distribution that can be used to better represent the small-scale variation of the fading signal in a non line-of-sight fading condition. In provides good fits to other distributions such as Weibull and Nakagami-*m* which are used in many wireless communications applications. In this paper, wireless communication system with dual SIR-based SC diversity is considered. The desired signal received by the *i*-th antenna can be written as [13]:

$$D_i(t) = R_i e^{j\phi_i(t)} e^{j[2\pi f_c t + \Phi(t)]} \qquad i = 1,2 \qquad (1)$$

where f_c is carrier frequency, $\Phi(t)$ desired information signal, $\phi_i(t)$ the random phase uniformly distributed in [0.2π], and $Ri(t)$, a α-µ distributed random amplitude process given by [9]:

$$f_{R_i}(t) = \frac{\alpha_i \mu_i^{\mu_i} t^{\alpha_i \mu_i - 1}}{\hat{t}^{\alpha_i \mu_i} \Gamma(\mu_i)} \exp\left(-\mu_i \frac{t^{\alpha_i}}{\hat{t}^{\alpha_i}}\right) \qquad t \geq 0, \qquad (2)$$

where Γ(•) is the Gamma function, $\Omega = t^2/m$, with $\hat{t} = \sqrt[\alpha_i]{E(R_i^{\alpha_i})}$ being the α -root mean value, µ is the inverse of normalized variance of R^{α} i.e.

$$\mu = \frac{E^2(R^{\alpha})}{V(R^{\alpha})}, \qquad \mu > 0 \qquad (3)$$

and *E()* and *V ()* are, respectively, the expectation and variance operators.
The performance of the dualbranch SC can be carried out by considering, as in [13], the effect of insufficient antennae spacing, so desired, envelopes experience correlative α -µ fading, with joint distributions [11]:

$$f_{R_1,R_2}(R_1, R_2) = f_{R_1}(R_1) f_{R_2}(R_2) \sum_{l=0}^{\infty} \frac{l!\,\Gamma(\mu_1)}{\Gamma(\mu_1 + l)} \rho_{12}^{l} L_l^{\mu_1 - 1}\left(\frac{\mu_1 R_1^{\alpha_1}}{\hat{R}_1^{\alpha_1}}\right) L_l^{\mu_1 - 1}\left(\frac{\mu_1 R_2^{\alpha_2}}{\hat{R}_2^{\alpha_2}}\right), \qquad (4)$$

substituting (2) into (4):

$$f_{R_1,R_2}(R_1,R_2) = \frac{\alpha_1 \mu_1^{\mu_1} R_1^{\alpha_1\mu_1 - 1}}{\hat{R}_1^{\alpha_1\mu_1} \Gamma(\mu_1)} \exp\left(-\frac{\mu_1 R_1^{\alpha_1}}{\hat{R}_1^{\alpha_1}}\right) \frac{\alpha_2 \mu_1^{\mu_1} R_2^{\alpha_2\mu_1 - 1}}{\hat{R}_2^{\alpha_2\mu_1} \Gamma(\mu_1)} \exp\left(-\frac{\mu_1 R_2^{\alpha_2}}{\hat{R}_2^{\alpha_2}}\right)$$
$$\times \sum_{l=0}^{\infty} \frac{l!\,\Gamma(\mu_1)}{\Gamma(\mu_1 + l)} \rho_{12}^{l} L_l^{\mu_1 - 1}\left(\frac{\mu_1 R_1^{\alpha_1}}{\hat{R}_1^{\alpha_1}}\right) L_l^{\mu_1 - 1}\left(\frac{\mu_1 R_2^{\alpha_2}}{\hat{R}_2^{\alpha_2}}\right) \qquad (5)$$

It is important to quote that *ρ* denotes the power correlation coefficient defined as $cov(R_i^2, R_j^2)/(var(R_i^2)var(R_j^2))^{1/2}$. $L_n^k(x)$ is generalized Laguerre polynomial given by [16]:

$$L_n^k(x) = \sum_{m=0}^{n} (-1)^m \frac{(n+k)!}{(n-m)!(k+m)!m!} x^m \qquad (6)$$

The selection combiner chooses and outputs the branch with the largest signal level.

$$R = R_{out} = \max(R_1, R_2). \qquad (7)$$

For this case joint cumulative distribution function can be written as [3]:

$$F_{R_1 R_2}(R_1, R_2) = \int_0^{R_1}\int_0^{R_2} f_{R,R_2}(x_1, x_2) dx_1 dx_2 \qquad (8)$$

Cumulative distribution function of output SIR, could be derived from (6) by equating the arguments $R_1 = R_2 = R$ as in [13]:

$$F_R(R) = \sum_{l=0}^{\infty} \sum_{m=0}^{l} \sum_{j=0}^{l} G_1 \gamma\left(\mu_1 + j, \frac{\mu_1 R^{\frac{\alpha_1}{2}}}{\hat{R}_1^{\frac{\alpha_1}{2}}}\right) \gamma\left(\mu_1 + m, \frac{\mu_1 R^{\frac{\beta_1}{2}}}{\hat{R}_2^{\frac{\beta_1}{2}}}\right), \qquad (9)$$

with:

$$G_1 = \frac{(-1)^{j+m}\, l!\, \rho_{12}^{l}}{j!\, m!\, (l-j)!\, (l-m)!} \frac{((\mu_1 - 1 + l)!)^2}{(\mu_1 - 1 + j)!\, (\mu_1 - 1 + m)!\, \Gamma(\mu_1)\, \Gamma(\mu_1 + l)} \tag{10}$$

and γ (a, x), being the lower incomplete Gamma function [17].

The nested infinite sum in (9), ,for two branches diversity case, converges for any value of the parameters ρ, $\hat{R}_1$, $\hat{R}_2$, μ_1, α_1 and α_2. The number of the terms need to be summed to achieve a desired accuracy, depend strongly on the correlation coefficient ρ. The number of the terms increases as correlation coefficient increases. For the special case of μ_1=1 we can evaluate expression for cdf for Weibull - desired signal and co-channel interference, and For the special case of α_1=2 and α_2, =2 we can evaluate expression for cdf for Nakagami-*m* - desired signal and co-channel interference. This generality of cumulative distribution function of output signal for a number of fading distributions is the main contribution of our work.

Probability density function (PDF) of the output SIR can be obtained easily from previous expression:

$$p_R(R) = \frac{d}{dR} F_R(R) = \sum_{l=0}^{\infty} \sum_{m=0}^{l} \sum_{j=0}^{l} G_1 \left(\frac{\alpha_1 \mu_1^{\mu_1 + j} R^{\frac{\alpha_1(\mu_1+j)}{2} - 1} e^{-\frac{\mu_1 R^{\frac{\alpha_1}{2}}}{\hat{R}_1^{\frac{\alpha_1}{2}}}}}{2 \hat{R}_1^{\frac{\alpha_1(\mu_1+j)}{2} - 1}} \gamma\left(\mu_1 + m, \frac{\mu_1 R^{\frac{\alpha_2}{2}}}{\hat{R}_2^{\frac{\alpha_2}{2}}}\right) + \frac{\alpha_2 \mu_1^{\mu_1 + m} R^{\frac{\alpha_2(\mu_1+m)}{2} - 1} e^{-\frac{\mu_1 R^{\frac{\alpha_2}{2}}}{\hat{R}_2^{\frac{\alpha_2}{2}}}}}{2 \hat{R}_2^{\frac{\alpha_2(\mu_1+m)}{2} - 1}} \gamma\left(\mu_1 + j, \frac{\mu_1 R^{\frac{\alpha_1}{2}}}{\hat{R}_1^{\frac{\alpha_1}{2}}}\right) \right) \tag{10}$$

Fig. 1 shows probability density function of output signal for balanced and unbalanced ratio of signal at the input of the branches and various values of correlation coefficient and various fading types (various values of α and μ, including special cases of Nakagami-m and Weibull distributions)

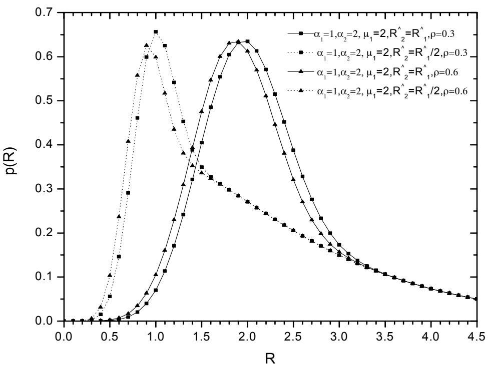

Fig. 1. Probability density function of output signal for various values of fading types and corelation coefficient

Outage probability P_{out} is one of the accepted performance measure for diversity systems operating in fading environments. Outage probability is a measure of the system's performance, used to control the cochannel interference level, helping the desingers of wireless communications system's in order to meet the QoS and grade of service (GoS) demands. P_{out} is defined as the probability that the output signal of the SC falls below a given outage threshold *q* also known as a protection ratio.

$$P_{out} = P_R(\xi < q) = \int_0^q f_R(R)\, dt = F_R(q) \quad . \tag{11}$$

Protection ratio depends on modulation technique and expected QoS. Outage probability versus normalized parameter $\hat{R}_1/q$ for balanced and unbalanced ratio of signal at the input of the branches and various values of corelation coefficient and fading severity parameters is shown on Fig. 2.

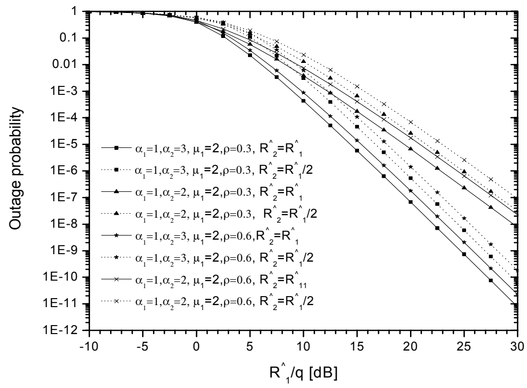

Fig. 2. Outage probability versus $\hat{R}_1$ /q for various values of fading types and corelation coefficient

III. CONCLUSION

In this paper, system performances of selection combining over correlated α-μ channels are analyzed. Fading between the diversity branches is correlated and α-μ distributed. The complete statistics for the SC output signal is given in the closed form, i.e., PDF, CDF. As an illustration of the mathematical formalism, numerical results of these performance criteria are presented, describing their dependence on correlation coefficient and fading severity. The proposed approximations turns out to be simple, effective, and highly accurate tool for evaluating the outage probability of selection diversity systems. The main contribution of this analysis for dualbranch signal combiner, is that it has been done for general case of *α-μ* (Generalized Gamma) distribution, which includes as special cases important other distributions such as Weibull and Nakagami-*m* (therefore, the One-Sided

Gaussian and Rayleigh are also special cases of it), so our analysis has high level of generality.

REFERENCES

[1] Stuber, G.L.: ‘Mobile communication’ (Kluwer, USA, 2003, 2nd edn.)

[2] Lee, W.C.Y.: 'Mobile communications engineering' (New York, Mc-Graw-Hill, 0-7803-7005-8/01, IEEE, 2001, 1992.)

[3] Simon, M.K., and Alouini, M.S.: ‘Digital communication over fading channels’ (Wiley, New York, 2005, 2nd edn.)

[4] Neasmith, E.A., Beaulie, N.C.: 'New Results in selection diversity', IEEE Trans Commun., 1998, 46, pp. 695–704.

[5] Okui, S.: 'Effects of SIR selection diversity with two correlated branches in the m-fading channel',IEEE Trans. Commun., 2000;48, pp.1631–3.

[6] Schwartz, M., Bennett, W. R., and Stein, S.: 'Communication Systems and Techniques' (New York, 1966.)

[7] Izzo, L., Fedele G., and Tanda M.: 'Dual Diversity Reception of M-ary DPSK Signals over Nakagami Fading Channels', IEEE, 1995

[8] Yacoub, M. D.: 'The α-μ Distribution: A Physical Fading Model for the Stacy Distribution', IEEE Trans. Veh. Technol., Jan. 2007,. 56 (1), pp. 27–34.

[9] Yacoub, M. D.: 'A general fading distribution', IEEE Inter. Symp. on Personal, Indoor and Mobile Radio Communications, PIMRC2002, 2, pp. 629-633, 2002

[10] Stacey, E. W.: 'A generalization of the Gamma distribution', Annal. Math. Stat. , 1962., 33, pp. 1187–1192

[11] De Souza Rausley, A. A., Fraidenraich, G., Yacoub, M. D. :'On the Multivariate α-μ Distribution With Arbitrary Correlation',VI International Telecommunications Symposium (ITS2006), September 3-6, 2006, Fortaleza-CE, Brasil

[12] Stefanović, M., et al.: 'Performance analysis of system with selection combining over correlated Weibull fading channels in the presence of cochannel interference', Int J Electron Commun (AEU) (2007), Digital Object Identifier: 10.1016/j.aeue.2007.09.006.

[13] Sagias, N.C., Karagiannidis, G.K., Zogas, D.A., Mathiopulous P.T., Tombras G.S.: 'Performance analysis of dual selection diversity over correlated Weibull fading channels', IEEE TransCommun 2004; 52(7), pp.1063–7.

[14] Mihajlo Stefanović, Aleksandar Mosić, Stefan Panić, Srđan Jovković, "SIR analiza SC diverziti sistema sa tri grane za Nakagami-*m* model sa konstantnom korelacijom signala i interferencije", *ETRAN 2008*, Palić 2008

[15] M. Stefanović, S.Panić, A. Mosić, " Analyses of Triple SC over Constant Correlated Rayleigh Signal and Interference Based on Signal to Interference Ratio", *ICEST 2008*, Niš 2008
the multivariate Nakagami-*m* distribution with exponential correlation',IEEE Trans Commun 2003, 51, pp .1240–4.

[16] http://mathworld.wolfram.com/LaguerrePolynomial.html , accessed May 2008

[17] Gradshteyn I. and Ryzhik. I.: 'Tables of Integrals, Series, and products', (Academic Press, 1980 New Yourk)