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Failure Analysis of Logical Network

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Abstract - **Measures for logical network failure are considered in paper. They are Dynamic Reliability Indices (DRIs) and Reliability Function (RF). The DRIs allow investigating the influence of one gate breakdown to failure of logical network. Methods of Logical Differential Calculus and structure function of a logical network are used for calculation of these indices. The RF is a probability of logical network failure and it is calculated by special form of structure function. Algorithms for DRIs and RF computation are proposed in paper.**

Keywords: Reliability Analysis, Dynamic Reliability Indices, Reliability Function

I. Introduction

Reliability has been considered as an important design measure in many technical systems [1 – 4]. A logical network is one of them [1, 2, 4]. One of principal problems in reliability analysis of logical network is investigation of breakdown of each individual gate influence upon the logical network failure [1, 2]. Generally, the system reliability model and its indices are required for solving the problem.

Discrete probability models are typically employed in design of a technical system for reliability analysis [1, 3, 4]. Markov processes and structure function are tools for reliability analysis in system design. Markov processes analyze the system state transition process [3, 5, 6], and the structure function investigates the system topology [3, 7, 8].

In this paper we propose new method on the basis of structure function to estimate the system reliability evolving results proposed in [8, 9]. Two type indices are used in this method for measure of logical network reliability. Firstly, it is *Reliability Function* (RF) that is characterized probability of system failure. RF is well-know measure in reliability analysis [1 – 4, 9, 10]. Secondly, we have been proposed *Dynamic Reliability Indices* (DRIs) for evaluation of dynamic properties of system (logical network) reliability.

These indices are computed based on structure function and Logical Differential Calculus [11, 12]. DRIs characterize the change of a system reliability that is caused by the change of a component state and include two groups [13 – 15]: *Component Dynamic Reliability Indices* (CDRIs) and *Dynamic Integrated Reliability Indices* (DIRIs). CDRIs allow measuring an influence of each individual gate to the logical network reliability. DIRIs characterize a probability of impact of one gate to the system reliability.

In this paper DRIs are considered for special class of system, it is logical network. Basic conceptions for reliability analysis of a logical network by DRIs are defined in this paper (structure function of logical network, gate probability, failure of logical network etc.). Presented results are evolution of investigations that have been obtained in papers [13 – 15].

II. Basic Conception

A. Structure Function of Logical Network

A logical network of n logical gates realize a logical function $F(y_1, \ldots, y_k) = F(\mathbf{y})$ (y_i is the i-th variable of logical function and i-th input of logical network; $i = 1, \ldots, k$). With relation to reliability analysis a logical network and its every gate have two states of efficiency s: "zero" designates network or gate failure (is not working) and state "one" declares of working of network or its gate.

A structure function declares a logical network reliability according to its gate states x_i ($i = 1, \ldots, n$):

$$\phi(x_1, \ldots, x_n) = \phi(\mathbf{x}): \{0, 1\}^n \to \{0, 1\}. \quad (1)$$

The structure function (1) is Boolean function but it is different form logical function $F(\mathbf{y})$ that is realized by logical network. The structure function describes the topology of a logical network and can be equal to logical function $F(\mathbf{y})$ in some time. Note that different logical networks with different logical function may have equal structure functions.

For example, there are two different logical networks in Fig.1. These logical networks are system of three components (n = 3) and have similar connections of gates. The graphical presentation by Block Diagram [1 – 3] is illustrates it. Therefore

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considered logical networks have equal structure function (1):

$$\phi(x) = (x_1 \vee x_2)\, x_3. \qquad (2)$$

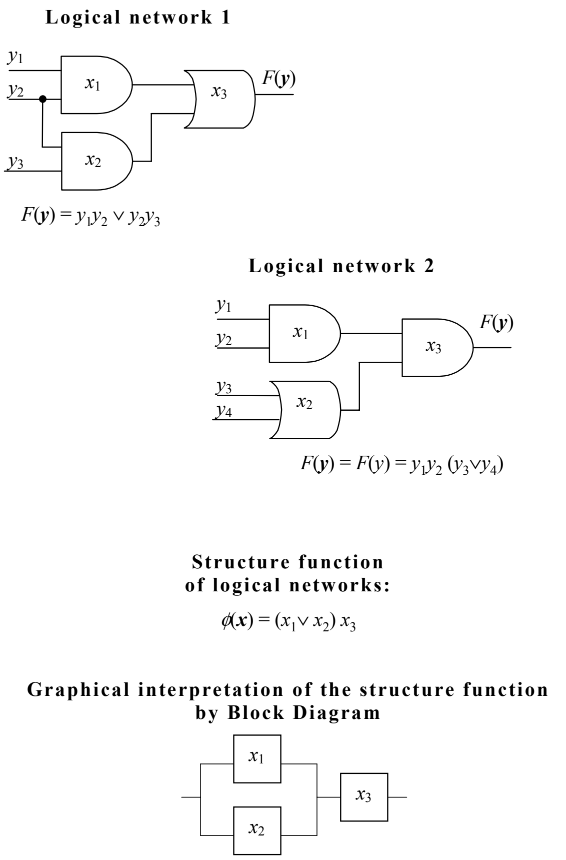

Fig. 1. Logical networks and they structure function

Every gate of a logical network is characterized by the probability of its working state:

$$p_i = \Pr\{x_i = 1\} \quad \text{and} \quad r_i = (1 - p_i) = \Pr\{x_i = 0\}. \qquad (3)$$

There are assumptions for structure function (1) that are peculiar to Reliability Analysis:

(*a*) the structure function $\phi(\mathbf{x})$ is monotone and $\phi(\mathbf{s}) = s$ ($s \in \{0, 1\}$ and s is the vector whose all components are equal to s) [1 – 3];

(*b*) all components are s-independent and are relevant to the system [1 – 3].

B. Structure Function of Logical Network

We propose to use Direct Partial Logic Derivatives (it is a part of Logical Differential Calculus) for investigation of logical network reliability by structure function [11 – 12]. The possibility to use Logical Differential Calculus of Boolean Function (the Logical Derivatives or Logical Differences) in Reliability Analysis is mentioned in [10] too. But the author of that paper considered the tool of the Logical Derivatives (the Logical Differences is another name of this tool) that is very simple and does not realize correct analysis of system reliability change that is caused by changes of system component states [16, 17]. There is another tool of Logical Differential Calculus that improves dynamic analysis of Boolean Function and is named Direct Partial Logic Derivatives [17]. These derivatives reflect the change in the value of the underlying function when the values of variables change.

Direct Partial Logic Derivative $\partial\phi(j \to h)/\partial x_i(a \to b)$ of a function $\phi(x)$ of n variables with respect to variable x_i reflects the fact of changing the function from j to $\bar{j}$ when the value of variable x_i changes from a to $\bar{a}$:

$$\partial\phi(j \to \bar{j})/\partial x_i(a \to \bar{a}) = \{\phi(a_i, x) \sim j\} \wedge \{\phi(\bar{a}, x) \sim \bar{j}\}, \qquad (4)$$

where $\phi(a_i, x) = \phi(x_1,\ldots, x_{i-1}, a, x_{i+1},\ldots, x_n)$ $a, j \in \{0, 1\}$ and $\sim$ is the symbol of equivalence operation.

The logical network failure measures play significant role in reliability analysis and are defined depending on a change of one system component states with relation to Direct Partial Logic Derivatives of a structure function $\phi(\mathbf{x})$ of n variables with respect to variable x_i (4).

In Direct Partial Logic Derivative terminology *the system failure* is represented as the changing of logical network running order $\phi(\mathbf{x})$ from 1 into 0 ($\phi(\mathbf{x})$: $1 \to 0$) and breakdown of a gate (Fig.3):

$$\partial\phi(\mathbf{x})/\partial x_i = \partial\phi(1 \to 0)/\partial x_i(1 \to 0)$$

and is declared as

$$\partial\phi(\mathbf{x})/\partial x_i = \phi(1_i, \mathbf{x}) \wedge \overline{\phi(0_i, \mathbf{x})}. \qquad (5)$$

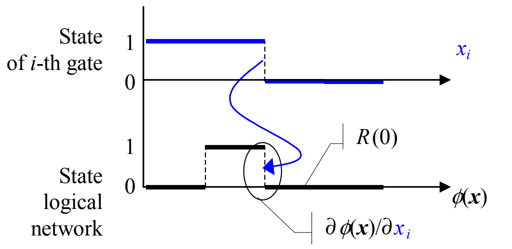

Fig. 2. Logic Derivatives and changes of logical network running order

Direct Partial Logic Derivative (5) permits to determine boundary logical states. Nonzero elements of the derivative indicate logical network states (x_1 x_2 $\ldots x_n$) for witch breakdown of i-th gate caused logical network failure.

For example, boundary logical states of the system with structure function (3) are calculated by Direct Partial Logic Derivatives $\partial\phi(\mathbf{x})/\partial x_1$, $\partial\phi(\mathbf{x})/\partial x_2$ and $\partial\phi(\mathbf{x})/\partial x_3$ (Table 1). Nonzero elements of these derivatives correspond to boundary system states. Therefore breakdown of i-th component causes system failure. So, boundary state $x_1x_2x_3$ = (010) for third component reveals that this component breakdown causes system failure if the first component isn't functioning (x_1=0) and the second component is working (x_2=1).

Table 1. Direct Partial Logic Derivatives

$x_1x_2x_3$	$\phi(\mathbf{x})$	$\partial\phi(\mathbf{x})/\partial x_1$	$\partial\phi(\mathbf{x})/\partial x_2$	$\partial\phi(\mathbf{x})/\partial x_3$
000	0	0	0	0
001	0	1	1	-
010	0	0	-	1
011	1	0	-	-
100	0	-	0	1
101	1	-	0	-
110	0	-	-	1
111	1	-	-	-

III. Reliability Measures of Logical Network

C. *Reliability Function (RF)*

The RF $R(j)$ is one of best known MSS reliability measures (Fig.3). It is probability of system work or failure [1 – 5]:

$$R(j) = \Pr\{\phi(\mathbf{x}) = j\},\ j \in (0, 1\}. \quad (6)$$

For estimation of logical network failure the RF (6) is declared as:

$$R(0) = \Pr\{\phi(\mathbf{x}) = 0\}. \quad (7)$$

There are some algorithms for a RF calculation [1, 3 – 7, 10]. Boundary states of investigated system are used for it computation.

In papers [4, 13] proposed algorithms for RF calculation on basis of system boundary states and structure function (that is presented as a *Disjunctive Normal Form* (DNF)) orthogonalization [13]. An orthogonal DNF transform in form for determination of RF by replacement [1, 4]: gate states x_i (or $\overline{x_i}$) to probability gate state p_i (or $(1 - p_i) = r_i$); operations AND and OR to product and sum probabilities. As it is usually accepted, we base our calculations on the following two theorems from probability theory [1, 4, 7, 13]:

(*a*) the probability of product *ab* of two *independent* events (happening simultaneously), *a* and *b*, is equal to the product of their probabilities, $p(ab) = p(a)p(b)$;

(*b*) the probability of sum $a + b$ of two *incompatible* events (at least one of them happening), *a* and *b*, is equal to the product of their probabilities, $p(a + b) = p(a) + p(b)$.

Consider the algorithm for calculation of the logical network RF in paper [13]. In first step the the logical network structure function (1) is presented by matrix *T* whose columns correspond to the basic events (gate states x_i, $i = 1, \ldots, n$) and rows define the boundary states of logical network.

In the second step the logical network structure function declared as matrix *T* is orthogonalized. It is based on the idea of disjunctive expansion of an elementary conjunction into a number of other conjunctions, every of which being orthogonal to the conjunctions from a certain family or absorbed by one of them [4]. In the latter case, the absorbed conjunction is removed from the result and the number of the remaining conjunctions is minimized possibly.

The operation of expanding k_i as to k_j where k_i and k_j are non-orthogonal elementary conjunctions is the key operation for the algorithm:

(*a*) the set of all variables that are in k_i and are not in k_j is selected (assume that the number of them is *t*).

(*b*) k_i is expanded disjunctively as to the first of these variables; one of the resultants of this expansion is orthogonal to k_j, and the other is expanded as to the second variable from the extracted set, etc.

As a result, conjunction k_i will be replaced by *t* conjunctions orthogonal to k_j.

For example, compute RF for logical network structure function (2), if probability state of every gate is equal and is 0.3 ($p = 0.3$).

Boundary states of this structure function are determined by Direct Partial Logic Derivative (5) and are $(x_1x_2x_3)$ = {(001), (110)}. The matrix *T* for this case is:

$$T = \begin{bmatrix} 1 & 1 & - \\ - & - & 1 \end{bmatrix}.$$

The Fig.3 illustrate the orthogonalization of this matrix. As a result of this procedure we obtain orthogonal matrix T^+ and describe equation for RF calculation:

$$R(0) = p_1p_2 + (1\text{-}p_1)\,p_3 + (1\text{-}p_1)p_2(1\text{-}p_3) = 0.357$$

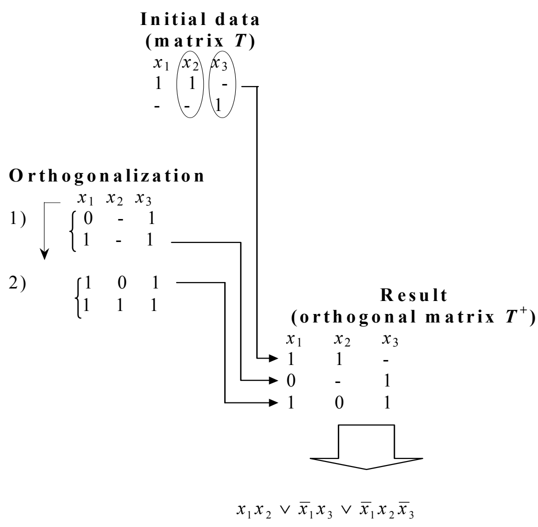

Fig. 3. The orthogonalization for calculation of RF

Therefore, probability of logical network (that is has structure function (2)) failure is 0.357.

The algorithm for RF calculation is in Fig.4.

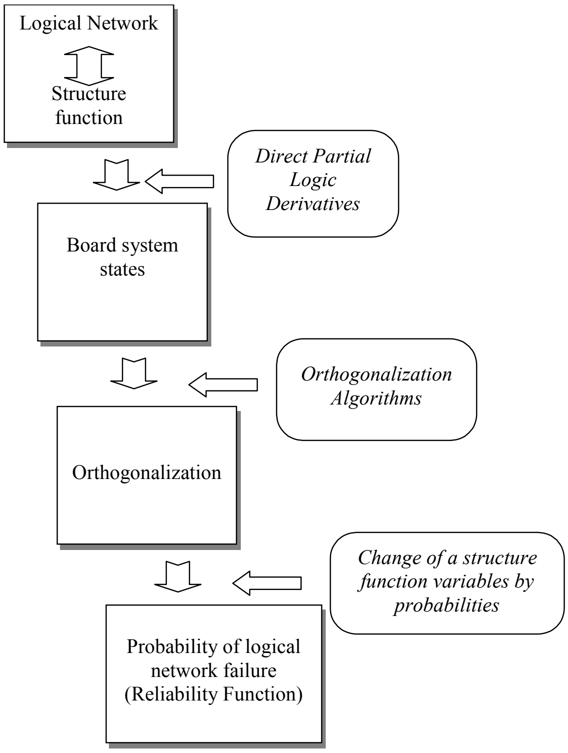

Fig. 4. Reliability Function calculation algorithm

Note, the measure (7) does not permit the analysis of the logical network failure that is caused by a gate breakdown. At the same time, in papers [13 –15] indices for the estimation of the influence of component states changes into the MSS reliability have been proposed. These indices are named as Dynamic Reliability Indices (DRIs).

D. Dynamic Reliability Indices (DRIs)

There are two kinds of DRIs [13 – 15]: *Component Dynamic Reliability Indices* (CDRIs) and *Dynamic Integrated Reliability Indices* (DIRIs). CDRIs are declared as the probability of the system failure and repair if the state of the *i*-th system component changes [6, 7]. DIRIs are another kind of DRIs and represent the probability of the system reliability changes with a modification of one or fixed system component states [6, 7, 10]. Now we consider these measures for logical network reliability analysis and in particular of logical network failure.

Definition 1. CDRIs of a logical network failure is the probability of the network failure that is caused by breakdown of the *i*-th gate:

$$P_f(x_i) = (\rho_f/\rho_1)\cdot r_i, \tag{8}$$

where ρ_f is the number of system states when the breakdown of the *i*-th gate forces the system failure (is the number of nonzero values of Direct Partial Logic Derivative with respect to a corresponding variable (5)); ρ_1 is the number of system states when $\phi(1_i, \boldsymbol{x}) = 1$ and is computed by structure function; r_i is the probability that is determined by (3).

Definition 2. DIRIs of a logical network failure is the probability of its failure that is caused by breakdown of any gate:

$$P_f = \sum_{i=1}^{n} P_f(x_i) \prod_{\substack{q=1 \\ q\neq i}}^{n} (1 - P_f(x_q)), \tag{9}$$

where $P_f(x_i)$ is CDRIs of the logical network failure (8) at the *i*-th gate breakdown.

The assumption (*b*) for structure function that all components are independent and relevant to the system is taken into account in DIRIs definitions.

For example, continue previous example and determine CDRIs and DIRIs for logical network structure function (2), if probability state of every gate is equal and is 0.3:

$$p_i = p = 0.3.$$

CDRIs for this logical network failure are calculated by (8) and DIRIs are determined according to (9). CDRIs for this system are in Table 2. The numbers ρ_f are computed as the numbers of nonzero elements of derivatives $\partial\phi(\boldsymbol{x})/\partial x_i$ and the number ρ_1 is computed from the structure function of this network.

Table 2. CDRIs of Structure Function (2)

Gate	Number ρ_f	Number ρ_1	CDRIs, $P_f(x_i)$
x_1	1	2	0.150
x_2	1	2	0.150
x_3	3	3	0.300

The logical network failure will be most possible if the third gate breaks down, because CDRIs $P_f(x_3)$ have the maximum value $P_f(x_8) = 0.3$.

DIRIs for failure of this logical network are P_f = 0.395 and is calculated by (9). It is probability of network with structure function failure if one of its gates fails.

IV. Application of DRIs for Reliability Analysis of a Logical Network

Consider an example of a logical network (Fig.5) analysis by RF, CDRIs and DIRIs, that realises the logical function:

$$F(\boldsymbol{y}) = ((y_1 y_2 \vee y_1 y_3 y_5) \vee (y_2 y_3 y_4 \vee y_4 y_5)).$$

The network in Fig. 5 includes nine gates (n = 9) and let the probabilities of their states be in Table 3. The structure function of this network in accord of (1) is:

$$\phi(\boldsymbol{x}) = ((x_1 \vee x_2\, x_5)\, x_7 \vee (x_4 \vee x_3\, x_6)\, x_8)\, x_9. \quad (10)$$

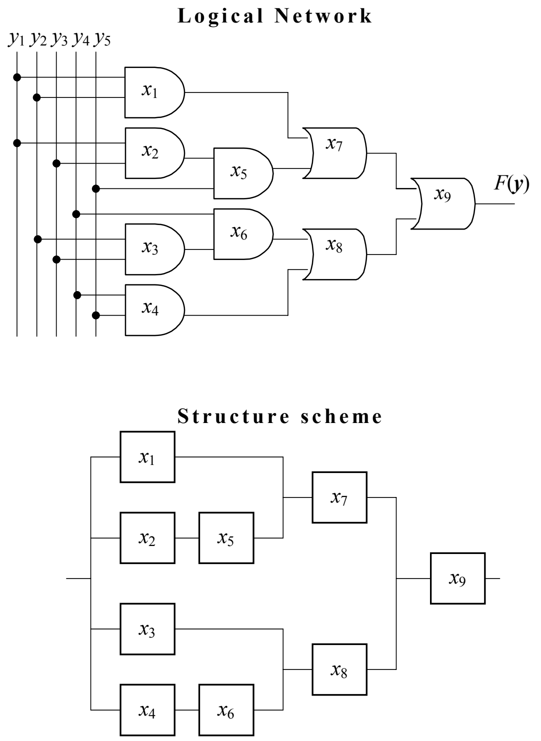

Fig. 5. Example of logical network

Table 3. The State Probabilities of Gates for the Logical Network in Fig.4

	Components		
	x_1	x_2	x_3
r_i	0.03	0.12	0.03
p_i	0.97	0.88	0.97
	x_4	x_5	x_6
r_i	0.08	0.02	0.07
p_i	0.92	0.98	0.93
	x_7	x_8	x_9
r_i	0.01	0.23	0.02
p_i	0.99	0.77	0.98

The RF of this network characters possibility of system to be failed. This probability is calculated by the algorithms from [13] and is $R(0) = 0.023$.

CDRIs for this logical network failure are calculated by (8) and DIRIs are determined according to (9). CDRIs for this system are in Table 4. The numbers ρ_f are computed by derivatives $\partial \phi(\boldsymbol{x})/\partial x_i$ and the number ρ_1 is computed from the structure function of this network.

The logical network failure will be most possible if the eighth gate breaks down, because CDRIs $P_f(x_8)$ have the maximum value $P_f(x_8) = 0.1332$. Therefore, replacing this gate by another gate with the larger probability of perfect working makes for decrease of possibility of the network failure if the eighth gate fails. For example, CDRIs $P_f(x_8)$ would be equal to 0.0289 if the probability for this gate working is $p_8 = 0.95$ and $r_8 = 0.05$.

Table 4. CDRIs for the Logical Network Failure

Gate	Number ρ_f	Number ρ_1	CDRIs, $P_f(x_i)$
x_1	33	84	0.0118
x_2	11	73	0.0181
x_3	11	73	0.0045
x_4	33	84	0.0314
x_5	11	73	0.0030
x_6	11	73	0.0105
x_7	55	95	0.0058
x_8	55	95	0.1332
x_9	135	135	0.0200

DIRIs for failure of this logical network are $P_f = 0.2033$. It is probability of network in Fig.4 failure if one of its gates fails. This probability is different of RF of this network that is $R(0) = 0.023$. Therefore we have three types of measures for the logical network failure: RF ($R(0) = 0.023$), CDRIs (Table 3), DIRIs ($P_f = 0.2033$).

V. Conclusion

In this paper, we suggest a new method for estimation of logical network failure, which is calculated from structure function of the network only. We consider two types of reliability measures: RF and DIRs. The measure, which is named CDRIs, involves the probabilities of logical network failure depending on breakdown of *i*-th gate. DIRIs are another measures of logical network reliability that is calculated from structure function. We include the RF in method for general estimation of logical network failure and elaborate the algorithms for it calculation. Presented measures permit to estimate reliability (failure) of logical network with relation to a structure of this network mainly.

Note proposed methods for reliability analyses of logical network can be used in another application. For example, DRIs may be employed for reliability analyses of computer and transport networks, power system [2, 3, 15].

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