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Normalized Least Mean Squared Adaptive Algorithm with Fractional Tap (FT) Length

Sorin Zoican[1], Ph.D

Abstract – This paper illustrates a new fractional tap length (FT) algorithm that improves the performance of the normalized least mean squared (NLMS) algorithms. The proposed algorithm, named NLMS-FT, is compared with an adaptive fractional tap length algorithm named NLMS-FT (adaptive length). The computational effort is similar for the proposed and adaptive tap length algorithms but the proposed algorithm has better performance in terms of tracking capabilities, speed of convergence and miss-adjustment. The proposed algorithm has the advantage that it is a less sensitive to its parameters (that is, it has fewer control variables to set). Therefore, the proposed algorithm is more robust than the adaptive tap length algorithm.

Keywords: **NLMS, Fractional tap length, average squared error.**

I. INTRODUCTION

The least-mean-square (LMS) algorithm has been extensively used in many applications because of its simplicity and robustness [1], [2]. When applying the LMS algorithm, the tap length of the adaptive filter is generally fixed.

However, in certain applications, the tap length of the optimal filter is unknown or even variable. When the tap length is under-modeled, the mean-square output error is likely to increase, as shown in the analysis in [3] and [4]; thus, a variable tap length algorithm is needed in such environments.

Methods of searching for the optimal filter tap length have been proposed during recent years, and a summary of the work is given in [9] and [5]–[8]. The fractional tap length (FT) method is more robust and has lower computational complexity relative to other methods [9], but its performance can depend on the parameter choice, particularly when the channel coefficients are varying in time. Such sensitivity to parameter selection motivates a search for new approaches to variable tap length adaptive filtering.

Numerous adaptive algorithms can be found in the literature with a common point for most of them: they may not work very reliably since they depend on several parameters that are not simple to tune in practice. A less sensitive adaptive algorithm is proposed. It is easy to control and gives good performances.

On the other hand, a major goal of the method is to reduce the computational effort (e.g. the filter tap length) consider that most of channel coefficients are zero.

II. THE NLMS-FT ALGORITHM

The fractional tap NLMS algorithm (NLMS-FT) is designed to find the optimal filter tap length. As in most approaches to derive algorithms for adaptive filters, this problem is transferred to the optimization of some criteria related to the tap length.

For formulation convenience, we denote the steady state tap length of the FT algorithm as L; w_L and $x_L(n)$ are the corresponding steady-state filter vector and input vector, respectively, and n denotes the time index. In addition, we define the segmented steady state error as $e_M(n)$ [9]:

$$e_M(n) = d(n) - \mathbf{w}_M^T \mathbf{x}_M(n) \tag{1}$$

where $d(n)$ is the desired signal, $1 \leq M \leq L$, $\boldsymbol{w_M}$ and $\boldsymbol{x_M}(n)$ are vectors consisting of the first M coefficients of the coefficients vector $\boldsymbol{w_L}$ and the input vector $\boldsymbol{x_L}(n)$, respectively.

The mean square of this segmented steady-state error is $\varepsilon_M = E\{(e_M(n))^2\}$

The underlying basis of the FT method is to find the minimum value of the error of that complies with [9]

$$\varepsilon_{L-\Delta} - \varepsilon_L \leq \delta \tag{2}$$

where Δ is a positive integer, less than L, and δ is a small positive value determined by the system requirements. The minimum value of L that complies with (2) is then chosen as the optimum tap length. A detailed description of this criterion and another similar criterion can be found in [9].

[1] POLITEHNICA University of Bucharest, Electronic, Telecommunications and Information Technology Faculty, Telecommunications Department, e-mail sorin@elcom.pub.ro

Gradient-based methods can be used to find L from equation (2). However, the tap length that will be used in the adaptive filter structure must be an integer, and this constrains the adaptation of the tap length. Different approaches have been applied to solve this problem [5]–[9]. In [9], the concept of "pseudo fractional tap length" denoted by $l_f(n)$ is used to make instantaneous tap length adaptation possible. As explained in [9], $l_f(n)$ is no longer constrained to integer values, and the true tap length remains unchanged until the "change" of the fractional tap length accumulates to some extent.

Based on this approach, the FT algorithm can then be formulated as follows:

$$l_f(n+1) = (l_f(n) - \alpha) + \gamma(e^2_{L(n)} - e^2_{L(n)-\Delta}) \tag{3}$$

$$L(n+1) = \begin{cases} Q[l_f(n)], & \text{if} \quad |L(n) - l_f(n)| \geq \beta \\ L(n) & \text{otherwise} \end{cases} \tag{4}$$

where $Q[\ ]$ is the floor operator that rounds down the embraced value to the nearest integer and β is a given threshold.

Initially, the filter length L will be set to a maximum length L_{max}.

Although this FT method performs well under certain conditions (like white noise input or no coefficients changes of the FIR plant that generate the desired signal), its performance depends on the choice of the parameters.

For example, if the input is not white noise (e.g. speech input), fixed parameters that achieve both fast convergence rate and small steady state mean square error (MSE) will be difficult to obtain. If the plant FIR coefficients are varying in time, the adaptive tap length algorithm will not adjust very quickly the filter coefficients. This means that the residual error will decrease slowly and the steady state error will be relatively high.

III. THE PROPOSED ALGORITHM

A modified FT algorithm is proposed in this section. The underlying basis of the modified FT method is to find the minimum value of the filter length, L, i.e. the average squared error is less than a given threshold, instead of finding the minimum value of L that satisfied equation (2).

According with this idea, we consider a minimum filter length L_{min} and a maximum filter length L_{max} (the last one will be imposed by the computational time restrictions). The FT algorithm has the following steps performed at each input sample:

1. Compute the filter output for the minimum length:

$$y_{\min}(n) = \sum_{i=0}^{L_{\min}-1} x(n-i).w_n(i)$$
$$y(n) = y_{\min}(n) \tag{5}$$

2. Update the filter output by adding a new tap k, as follows:

$$y(n) = y(n) + x(n-k).w_n(k) \tag{6}$$

where $k = L_{min}, \dots, L_{max}$.

3. Compute the average squared error $e_{avg}(n)$:

$$e(n) = d(n) - y(n)$$
$$e_{avg}(n) = \frac{\sum_{k=0}^{M-1} e^2(n-k)}{M} \tag{7}$$

where M is chosen about 5 to 10.

4. Compare the average squared error with a given threshold and stop computing the filter output if the average error is less than the threshold e_T. If the condition $k \leq L_{max}$ is fulfilled then go to step two, or else the computing of the filter output will be stopped at $k=L_{max}$.

The next section will show that the performance of proposed algorithm has a better tracking capability than the adaptive tap length algorithm. Nevertheless, the computational time for the proposed algorithm is greater than the computational time for the adaptive tap length algorithm.

IV. THE MAIN RESULTS

Our simulations consider the case of an adaptive echo canceller that implies one of the above presented algorithms. We consider that the echo path consists of a single major coefficient and all others coefficients are very small comparing with this one. This case corresponds to the acoustic echo.

The following assumptions have been made (table 1):

Parameter	Value
L_{max}	100
L_{min}	20
e_T	10^{-5}
M	5
Δ	4
α	0.01
γ	0.1
β	5

Table 1 Values of parameters used in simulations

We consider that the echo path will change as illustrated in the figure 1.

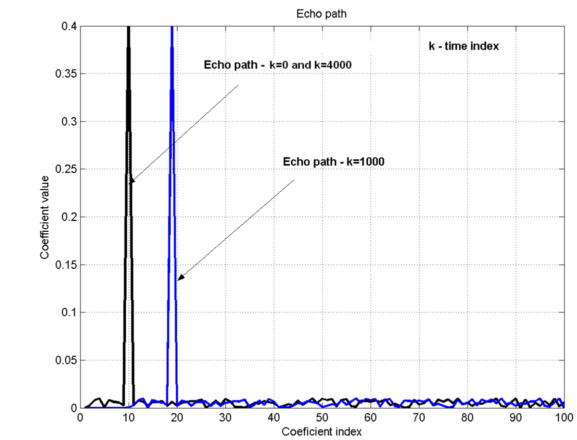

Figure 1. Echo path variation

The input signal was generated as the output of the plant FIR with the coefficients illustrated in figure 1. The plant FIR excitation has a normal distribution noise.

We computed the residual error for the classical NLMS adaptive algorithm, NLMS-FT adaptive algorithm, and NLMS-FT adaptive tap length algorithm.

These learning curves are shown in the figures 2, 3 and 4.

From these figures, one can observe that the NLMS-FT adaptive tap length algorithm exhibits a low tracking capability (after the second echo path change the residual error oscillates with a high value).

In the NLMS-FT algorithm, the number of taps will be decreased. The second term in equation (3) is negative but maintains the residual error small enough. When the echo path is changed, the residual error will increase. The second term in equation (3) is positive and the number of taps will be increased. The residual error will be reduced, but this process is slow.

The NLMS-FT algorithm tries to trade off between a relatively fast reducing of number of taps, when it converges, and the echo path tracking capability.

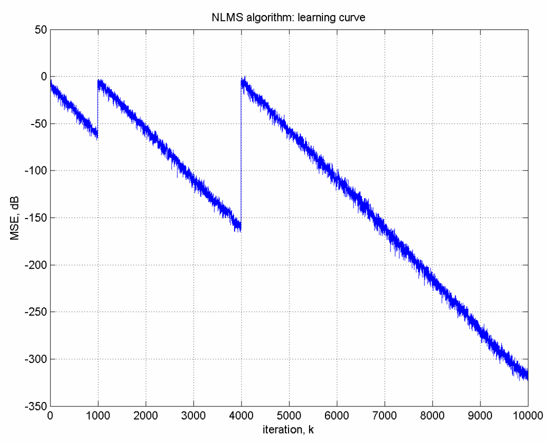

Figure 2. Learning curve – NLMS algorithm

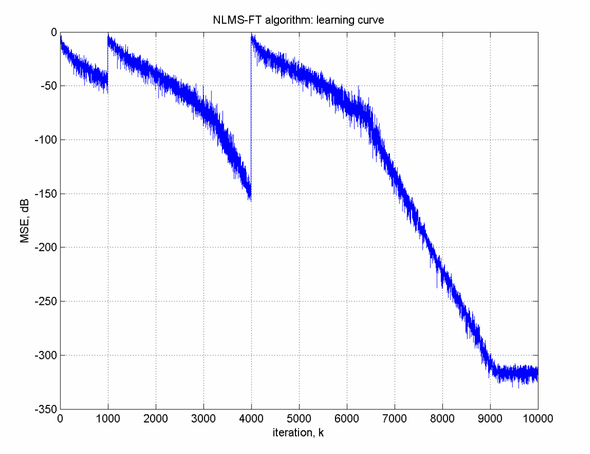

Figure 3. Fractional tap (FT) length learning curve

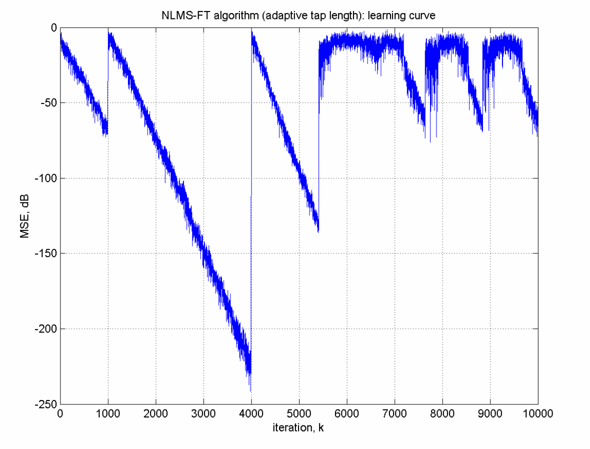

Figure 4. Fractional tap (FT) adaptive tap length learning curve

These two requirements are contradictory and are difficult to set all the parameters α, β and γ to fulfill both of them.

On contrast, the proposed algorithm (NLMS-FT adaptive tap) starts with a minimum value of the number of taps and increase it until the residual error is small enough. The number of taps is calculated at each sample interval so if the echo path is changed the algorithm can follow it quickly enough.

The computational effort will be increased comparing to NLMS-FT adaptive length tap algorithm but it remains lower than computational effort in NLMS algorithm.

Figure 5 illustrated the average number of taps for NLMS-FT and NLMS-FT adaptive tap algorithms. The number of filter tap for classical NLMS algorithm is set to 100.

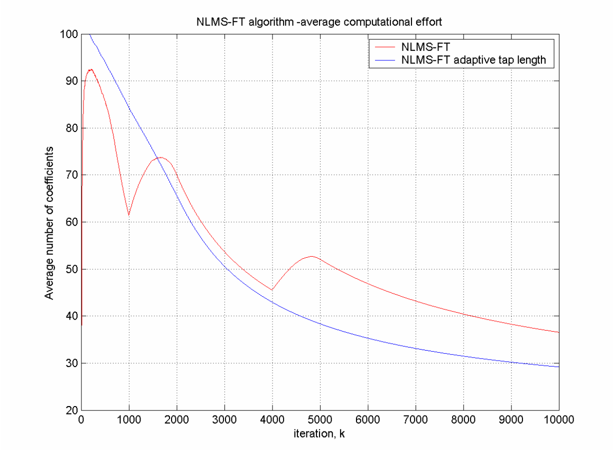

Figure 5. Fractional tap (FT) and fractional adaptive tap length computational effort

In the figure 2, the residual error was obtained for a number of 100 taps.

The table 2 indicates the computational effort for the above algorithms comparing with NLMS algorithm.

Average number of taps	NLMS-FT Algorithm	NLMS-FT Algorithm - adaptive tap length	NLMS Algorithm
100	53700	51065	50500
90	43750	41465	40950
80	34800	32865	32400
70	26850	25265	24850
60	19900	18665	18300
50	13950	13065	12750
40	9000	8465	8200
30	5050	4865	4650

Table 2. Computational effort (number of processor cycles)

The last column is only relevant to a comparison between all the three algorithms. The FT algorithms work with an average number of taps of about 30-35 and the computational effort must be compared with the computational effort for NLMS algorithm for 100 taps. One can observe that the computational effort decreases very much (about 20% of computational effort for NLMS) for a steady state of filter.

V. CONCLUSIONS

The paper presents a novel fractional tap length adaptive algorithm. This algorithm has better tracking capability with similar computational effort. On the other hand, the proposed algorithm is less sensitive to its parameters than other adaptive algorithms.

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