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A GENERIC CONDUCTIVITY NON HOMOGENEITY MODEL FOR THE LINEARIZED EIT PROBLEM

Ioan Jivet[1]

Abstract – The paper introduces a new dipole like model for a perturbation induced by a circular slab of incremental conductivity in a uniform current density field. The analytical solution for the forward problem is derived. The dipole model analysis reveals that the uniform parallel field is not sensitive to spatial frequencies of the conductivity function. A parallel impressed field was found to be very accurate in localizing areas of non homogeneity by their 'center of gravity'. A couple of useful properties of the field projection on curves at distance from non homogeneity is also described.

***Keywords*— dipole model, linearized EIT problem, dipole co-ordinates.**

I. INTRODUCTION

One of the difficulties encountered in devising high accuracy solution for Electrical Impedance Tomography (EIT) practical use is the non local property of the conductivity when probed with current distributions [1]. The intense research conducted by research groups interested in EIT problem solutions in the past years benefited from an abundance of new theoretical results obtained by abstract mathematical investigations [2] [3] [4].

The present paper analyses a conductivity non homogeneity effect on an impressed parallel current density field emphasising local geometry perspective. The best and appropriate field patterns and data collection arrangement is a problem that still has not received a complete answer. Several particular patterns have been explored only and some have been studied to be associated with appropriate practical tactics of implementation [5] [1].

The recent advances in the EIT problem formulated from a global functional perspective of boundary injected current fields, seeking a inverse solution to the boundary measured data constituted valuable inspiration [4] [2].

The basic problem of imaging a generic perturbation of conductivity is central to the linearised EIT formulation. A generic circle of changed conductivity from background is a good starting point for the study of the problem . The homogeneous parallel field pattern was chosen for its simplicity and ease of practical implementation.

A derivation of the dipole analytical model is original and it was obtained based on the charge distribution at the boundary of the non homogeneity generated by the boundary constraints.

For the case of electric current flow in domains with general conductivity distributions the literature is not very generous with examples of solutions. The case is overshadowed by electrostatics and circuit representations [2].

The positive answer reported in the present paper is that the dipole model is valid representation for a incremental non homogeneity as a basic perturbation in the linearized EIT formulation.

The inverse imaging based on the domain boundary measurement is shown not to be sensitive al all to conductivity spatial frequencies.

Finally a local dipole co-ordinate system was found to be a useful tool in area partitioning and its metrics analysis use for devising solutions to the inverse problem.

II. THE DIPOLE MODEL FOR THE LINEARIZED EIT PROBLEM

The dipole model is an appropriate model for incremental conductivity change from the background as required by the linearised formulation of the EIT problem. The paper does not address the high contrast conductivity case used in application areas like geophysics or particle inclusion determination [2] [6].

In a impressed uniform and parallel current field the change due to the presence of a small generic non homogeneity in conductivity results in a dipole like field added to the original one. The derivation of the equations of the added field is similar to the formulation of the problem in terms of electrostatic field. [7] [8].

The electrostatic formulation of the problem for a *infinite conductivity* generic unit non homogeneity in a parallel electrostatic field is the case most often presented. This case of extreme change in the region property is useful in the study of high contrast conductivity EIT not addressed in this paper [6].

[1] Facultatea de Electronică şi Telecomunicaţii, Departamentul Electronica Aplicata, Bd. V. Pârvan Nr. 2, 300223 Timişoara, e-mail ioan.jivet@etc.upt.ro

The case of a *small conductivity* change with respect to the background as it is defined in the linearised EIT problem is analysed in the following. The set of equations is formally equivalent to the electrostatic case but it relates now current density normal to the non homogeneity boundary and conductivity.
The current density continuity at the boundary results in a electric field discontinuity hence a charge distribution $q\,|_{r\,=\,r0}$ as presented in Fig 1.and equations (1 - 4).

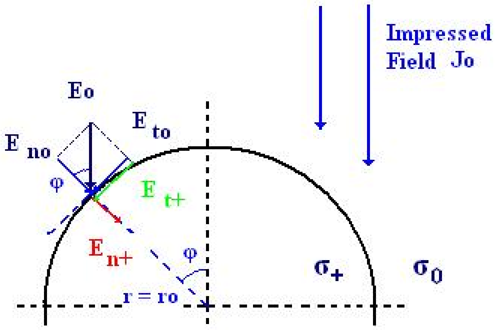

Fig 1.Boundary charge as resulting from electric field and conductivity constraint at the boundary.

For a very small perturbation the normal component of the impressed current density is proportional to the cosine of the central angle.

$$E_{n\,0}\,\sigma_0 = E_{n\,+}\,\sigma_+ \qquad (1)$$

(1)

$$E_{t\,0} = E_{t\,+} \qquad (2)$$

$$q\,|_{r\,=\,r0} = \varepsilon_0\,E_{n\,0}\,(\sigma_0/\,\sigma_+ - 1) \qquad (3)$$

$$q\,|_{r\,=\,r0} = \varepsilon_0\,J_0\,/\,\sigma_0\,(\sigma_0/\,\sigma_+ - 1\,)\,(\cos(\varphi)) \qquad (4)$$

where, E_n and E_t are the normal and tangential electric field, σ_0 and σ_+ are the initial and perturbed conductivity, ε_0 is the electric permitivity, J_0 is the initial current density and the φ central angle of a characteristic vector to the circular boundary.
It is important to note that the charge density *q* due to the conductivity change is a valid relation even when the conductivity perturbation tends to zero in the limit.
For a charge distribution, the edge of the circular slab, in the form of a cosine function an analytical solution is known and the result is a dipole like field perturbation [7].
The potential *V* of the induced electric field due to the perturbation can be summarized in the following formula:

$$V = A\cos(\varphi)\,/\,r^{dim-1} \qquad (5)$$

where A is a constant, r, φ are the polar co-ordinates and *dim* takes value 2 for a cylindrical case in two dimensions and 3 for the three dimensional case [7].
Even if the formula is not the same for two and three dimensions the filed lines and equipotential lines topology is the same as presented in fig 2.

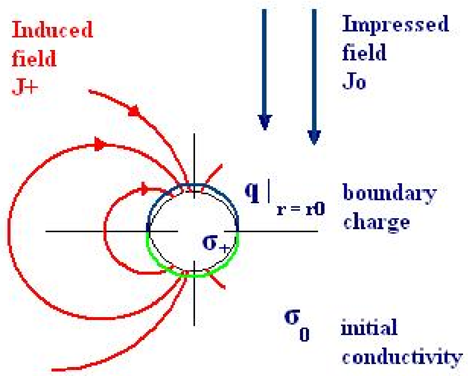

Fig 2. Dipole model, boundary charge and induced field resulting from a conductivity perturbation.

Analysis of the model reveals a number of interesting properties of interest to the EIT problem.
The projection of the perturbation component of the field on lines and circles at a distance from the centre of the non homogeneity has a 'mexican hat' form as presented in Fig 3.

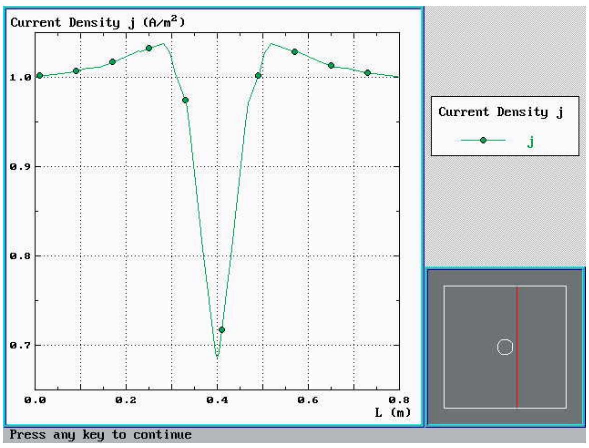

Fig 3. Induced field projection on a parallel line to the original impressed field.

Our analysis of the field projection reviled the following interesting properties:

P1. The width of the positive peak of the field projected on a line parallel to the original field is equal to twice the distance to the non homogeneity centre. A similar proportional property can be deduced for any curve of projection.

P2. The integral of the field projection on each half space determined by the non homogeneity is zero.

A simple proof can be immediately obtained using Gauss theorem given the fact that the total charge distribution at the boundary of the non homogeneity is of zero sum due to symmetry.

The found properties are of interest in devising sparcity promoting inverse problem methods of reconstruction. A multiresolution analysis remains as a future objective that needs more work to reach results valuable for practical use [9].

III. DISSCUSSION

Analysing the determination of an inverse image of the non homogeneity from measurements at a conductivity object boundary it is easy to observe that it admits as solution a Dirac like function. The magnitude is equal to the conductivity variation scaled by the non homogeneity area. The absolute precision in the' centre of gravity ' position is in evident contrast with the possible total undetermined conductivity boundary localisation.

The proof is immediate given the fact that the form of the perturbed field lines as determined for the dipole model are independent of the non homogeneity radius. The filed lines follow outside the non homogeneity area the form of a ideal dipole located at the 'centre of gravity' of the perturbation.

The filed lines map at the non homogeneity boundary, therefore are independent from non homogeneity radius. Any circle of with the same centre is a possible solutions to the inverse problem.

The result above leads to the important practical conclusion that the homogeneous parallel impressed field is not suitable for imaging high frequency components of the conductivity.

A measurement strategy using uniform filed will permit the location of the centre of gravity of the simply connected regions of conductivity.

Other inverse imaging methods can be used in follow up to determine the location of the boundaries of the regions [3].

The described projections of the field on curves at distance from the non homogeneity do provide valuable information for the development of a general EIT multi resolution approach [9].

The field change due to the perturbation is a result obtained for a particular initially impressed field. The results are of value for other types of initial fields that can be decomposed in terms of parallel fields as well.

One interesting subject of future work is the study of the perturbed field geometry.

The dipole co-ordinates and variations as used in other fields constitute an important starting point. [10]. The hyperbolic function change of co-ordinates that 'flatten' the field suggest a similar approach for conductivity non homogeneity problem in EIT.

IV. CONCLUSIONS

The paper introduces an original dipole like model for the perturbation induced by a circular slab of incremental conductivity in a uniform and parallel current field. The charge distribution on the boundary is found to be arranged according to a cosine profile. This finding leads to a simple derivation of the analytic solution for the perturbing potential. Probing for an inverse solution the absolute precision in position was found in evident contrast with the undetermined conductivity boundary localisation. This finding proves that a parallel impressed field is a not suitable for imaging conductivity distributions with high spatial frequency content. It is very accurate as far as the localization of areas of non homogeneity by its' center of gravity'. The projections of the field change due to the perturbation was found to have useful properties for its use in developing a multi - resolution algorithm for EIT. The proportional scaling with depth is a important finding but it needs more work to turn it into a practically usable method. Field geometry and co-ordinates promises to be a interesting and idea reach subject for future work.

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