

Wavelet OFDM Performance in Flat Fading Channels

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Abstract – This paper represents an investigation of the wavelet based multi-carrier modulation performance in flat fading channels. The fading envelope is distributed according to a Rayleigh probability density function. BER performance of the multicarrier wavelet method is computed and analyzed against the classical Orthogonal Frequency Division Multiplexing (OFDM) case, in various scenarios with respect to the Doppler shift influence and to the noise level.

Keywords: OFDM, wavelet-based OFDM, fading, Doppler shift.

I. INTRODUCTION

Multi-carrier modulation techniques were widely used in the last decade in various standards for wireline and wireless communications. Amongst others, versions of Fourier-based OFDM are employed at the physical level to provide good performance over the air interface in systems like Digital Audio & Video Broadcasting (DVB), WiMAX (described by IEEE 802.16), WiFi (802.11) or Qualcomm's Flash OFDM. This proves the reliability and the efficiency of the multi-carrier modulation concept, which is very well suited to radio transmissions. Besides its incontestable advantages, OFDM presents some well known drawbacks as: diminished spectral efficiency because of the cyclic prefix (CP) overhead, slow decay of the out-of-band side-lobes, high sensitivity to time and frequency synchronization, increased peak-to-average-power ratio [1,2].

Recent research focused on the multi-carrier transmission techniques [3,4], highlighted that some of these disadvantages can be steadily counteracted using wavelet carriers instead of OFDM's complex exponential waveforms. Due to the fact that these wavelet carriers form an orthogonal family, they can be separated at receiver's side by correlation techniques. The authors in [5] have shown that wavelet-based OFDM (WOFDM) has better spectral efficiency, is simpler and at least as rapid as OFDM in practical implementations. Furthermore, the performance of the two systems is similar in AWGN channels. Note however that, from this point of view, the real gain of multi-carrier techniques can be highlighted in conditions specific to the radio channels, which are both frequency-selective and

time-variant. With respect to these conditions, the author will investigate the OFDM and WOFDM performance in different flat, Rayleigh fading scenarios. A deeper analysis of the wavelet based method is performed, taking into account the influence of the chosen wavelets mother, as well as of the number of decomposition levels used in Inverse Discrete Wavelet Transform (IDWT) computation.

In the next section, an overview of the multi-carrier modulation concept is provided, focusing on the WOFDM principles. The third section will describe the simulation scenarios, whose results will be shown and discussed in section 4. The last section is dedicated to concluding remarks and to possible future directions for the continuation of the present work.

II. MULTI-CARRIER TRANSMISSIONS AND WAVELETS

Largely used in the modern communication systems, Orthogonal Frequency Division Multiplexing (OFDM) relies on a multicarrier approach, where data is transmitted using several parallel substreams. Every stream modulates a different complex exponential subcarrier, the subcarriers involved being orthogonal to each other. The orthogonality is the key point that allows subcarrier separation at receiver. The multicarrier approach has the advantage of a long symbol duration, issued from the simultaneous transmission of several low-rate parallel streams. OFDM implementation is based on the Fast Fourier Transform (FFT) algorithm, which allows reduced complexity and low implementation cost. The idea which gathers OFDM and wavelets is that in the same manner that the complex exponentials define an orthonormal basis for any periodic signal, a wavelet family forms a complete orthonormal basis for $L^2(\mathfrak{R})$. The orthogonality condition for wavelet family members is illustrated in (1).

$$\langle \Psi_{j,k}(t), \Psi_{m,n}(t) \rangle = \begin{cases} 1, & \text{if } j = m \text{ and } k = n \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Wavelet family members from (1) can be obtained by translating and scaling a unique function called

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wavelets mother and denoted by $\psi(t)$, according to (2):

$$\psi_{j,k}(t) = s_0^{-j/2} \psi(s_0^{-j} \cdot t - k\tau_0) \quad (2)$$

Equation 2 corresponds to a sampled version of a wavelet family, the discrete variables being s_0 (the scale) and k (the position within the scale).

The relation (1) indicates that all the members of the wavelet family $\{\Psi_{j,k}(t) = 2^{-j/2} \Psi(2^{-j}t - k)\}_{k \in \mathbb{Z}}$ (we considered $s_0=2$ and $\tau_0=1$) are orthogonal to each other. Consequently, if instead of complex exponential waveforms we use wavelet carriers, we will still be able to separate these subcarriers at receiver, due to their orthogonality. This is the main idea that lies behind the wavelet-based OFDM techniques [3,6]. As for the classical OFDM, the WOFDM symbol can be generated by digital signal processing techniques, such as IDWT. In this case, the transmitted signal is "synthesized" from the wavelet coefficients $w_{j,k} = \langle s(t), \psi_{j,k}(t) \rangle$ located at the k -th position from scale j ($j=1, \dots, J$), and from the approximation coefficients $a_{J,k} = \langle s(t), \phi_{J,k}(t) \rangle$, located at the k -th position from the coarsest scale J . Taking into account the constraints of a practical implementation, we can reformulate equation (2). Thus, the computation of the IDWT using Mallat's algorithm [7] requires finite-length dyadic data sequences at system input. If we denote by N the length of our input data sequence (which must be a power of 2), then the maximum number of decomposition levels for the DWT equals $L = \log_2(N)$, and the formula which will be employed for the WOFDM symbol computation is:

$$s(t) = \sum_{j=1}^J \sum_{k=1}^{2^{L-j}} w_{j,k} \psi_{j,k}(t) + \sum_{k=1}^{2^{L-J}} a_{J,k} \phi_{J,k}(t) \quad (3)$$

where J stands for the number of decomposition levels used, (whose the maximum value is L). $\phi_{j,k}(t)$ in the equation above is the scaling function associated to the wavelet mother. This formula corresponds to IDWT computation, which translates into a time domain signal some wavelet and approximation coefficients. Note that, in practice, a sampled version of the output signal, $s[n]$ is generated. The total number of samples composing this signal (referred to as WOFDM symbol in wavelet modulation terms) is equal to the number of samples of the input data sequence.

III. SIMULATION SCENARIO

The BER performance of WOFDM and OFDM will be compared in flat Rayleigh fading channel. The transmission chain used for simulations is shown in figure 1.

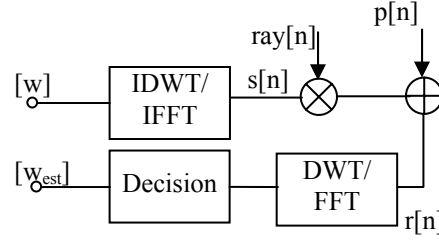


Fig.1: Baseband implementation of a WOFDM system.

A. The transmitter

For the case of a classical OFDM system, the input data vector $[w]$ can be interpreted as being composed of frequency-domain coefficients. These coefficients are randomly generated from bipolar symbols $+1$ and -1 , which are combined into some complex numbers such a way that the output of IFFT block to generate a real sequence [5].

If a WOFDM transmission is implemented instead, then the input data vector $[data]$ represents a sequence of wavelet-domain detail and approximation coefficients, as shown below:

$$data = [\{a_{J,k}\}, \{w_{J,k}\}, \{w_{J-1,k}\}, \dots, \{w_{1,k}\}] \quad (4)$$

This data sequence is modulated onto a contiguous finite set of dyadic frequency bands and onto a finite number of time positions k within each scale.

The composition of the time-domain signal (the "WOFDM symbol") is explicitly illustrated in figure 2, with respect to equations 3 and 4.

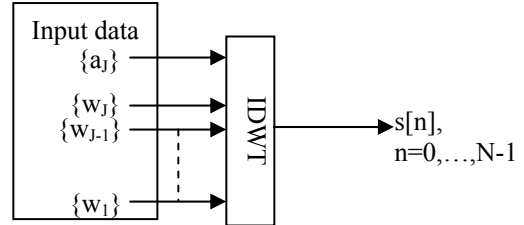


Fig. 2: Composition of WOFDM symbol using IDWT: practical implementation.

Note that in the figure above, J represents the coarsest scale used for IDWT computation. The choice of the approximation and detail coefficients which compose the input data vector (see fig. 2) can be done in different manners. The authors in [8] consider the data at scale $J-1$ as being a repetition of the useful stream from the previous coarser scale J . Since at scale $J-1$ we have two times more wavelet coefficients, one can state that at this scale we transmit the same data as at scale J but with a two times higher rate.

The author's approach in this paper is to transmit independent data streams at each scale. This data corresponds to a vector of N equally likely bipolar symbols ± 1 . The meaning of each symbol composing the vector is given by equation 4: first we have the approximation coefficients, then the coarsest scale

wavelet coefficients, next finer scale wavelet coefficients etc.

B. The channel

The radio channels exhibit small scale fading, which confers to this transmission environment two independent characteristics: time variance and frequency selectivity [9]. The variance in time of the radio channel's behavior can be expressed by the mean of the Doppler shift parameter, which depends on the relative motion between transmitter and receiver (v) and on the transmission wavelength (λ). The maximum value of this parameter is:

$$f_d = v/\lambda \quad (5)$$

The author uses in this paper a normalized version of Doppler shift:

$$f_m = f_d \cdot T_S \quad (6)$$

where T_S is the duration of a transmission symbol (from the data vector identified by (4)). The values taken into account for f_m in our simulations belong to the set $\{0.001, 0.005, 0.01, 0.05\}$. In slow fading scenarios, T_S must be much smaller than the coherence time of the channel expressed as:

$$T_C = \frac{0.423}{f_d} \quad (7)$$

Taking into account (6,7), our worst case scenario ($f_m=0.05$) leads to a coherence time T_C which is approximately 8 times higher than T_S . In the best case (the lowest Doppler shift), the coherence time is 400 times longer than the symbol duration.

These values seem to fit to the slow fading model, where the channel remains unchanged for the duration of a symbol. Though, when evaluating the channel behavior, one should take into account that in multi-carrier communications the transmitted symbol is longer. Usually, since the whole data vector is required at demodulator to identify the transmitted symbols, we can consider that the multicarrier symbol duration (an OFDM or a WOFDM block) is N times longer than the serial symbols brought at modulator's input. Note that in these conditions, the channel response changes during the transmission of one symbol, or block.

From the frequency selectivity point of view, the scenario taken into account refers to flat fading model, where the frequency response of the channel is considered approximately constant in the transmission band. This means flat frequency response of the channel, which can be implemented as a one-tap filter.

Small scale fading envelope can be modeled with a Rayleigh distribution, generated using the method described in [10]. The impact of the Rayleigh flat

fading is given by the multiplicative ray[n]. Rayleigh pdf is given in equation 8:

$$\text{pdf}(x|\sigma) = \frac{x \cdot e^{-x^2/2\sigma^2}}{\sigma^2} \quad (8)$$

In our simulations we consider unitary variance ($\sigma^2=1$). This is a simplifying hypothesis, because the variance of the signal obtained after multiplication is equal to the variance of the useful signal, s . A white noise $p[n]$ is then added to the signal above, obtaining the sequence $r[n]$ to be processed by the demodulator:

$$r[n] = s[n] \cdot \text{ray}[n] + p[n] \quad (9)$$

C. The receiver

The receiver is composed of a demodulator (the FFT or DWT block respectively) and a simple detector using a threshold comparison. Neither synchronization, nor equalization issues are taken into consideration. For OFDM case, a real symbol is generated, using the method described in [5].

IV. RESULTS AND DISCUSSIONS

Simulations were made under Matlab 7. For both investigated methods, the author considers the transmission of 10000 data blocks of 1024 symbols each. For the OFDM simulations, 512 complex symbols were composed from 1024 randomly generated bipolar values (+1 and -1), in order to obtain real values at the output of IFFT block. Neither synchronization, nor equalization issues were taken into account.

All the following simulated scenarios refer to flat Rayleigh fading channels. Two different wavelet mothers were used for DWT computation: Haar and Daubechies-10. The channel exhibit flatness (no frequency selectivity) and a variant behavior over time. The first set of simulations aims to investigate the BER performance of the multicarrier methods in slow fading (for low Doppler shifts). The results are shown in figure 3.

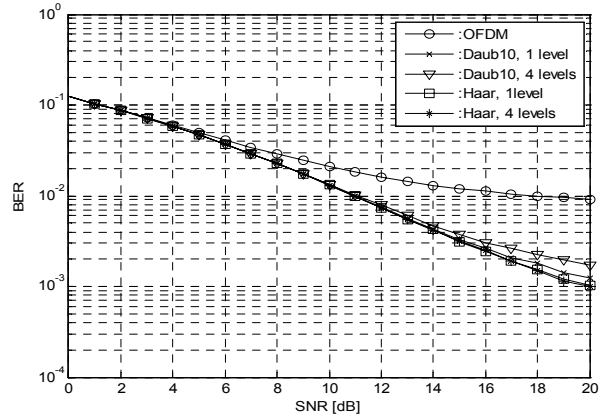


Fig.3: BER performance in slow fading channels ($f_m=0.001$).

Figure 3 shows that OFDM performance degrades significantly even in the case of a slow fading channel. Thus, wavelet based methods provide a gain of almost 10 dB compared to OFDM.

The same conclusion is strengthened by figure 4, where Doppler value corresponds to a fast fading scenario. In the worst case, with $f_m=0.05$, the coherence time of the channel would be only 1.62% from the duration of a multicarrier modulated symbol. An important gain is brought again by the use of wavelets as orthogonal carriers instead of OFDM's complex exponentials.

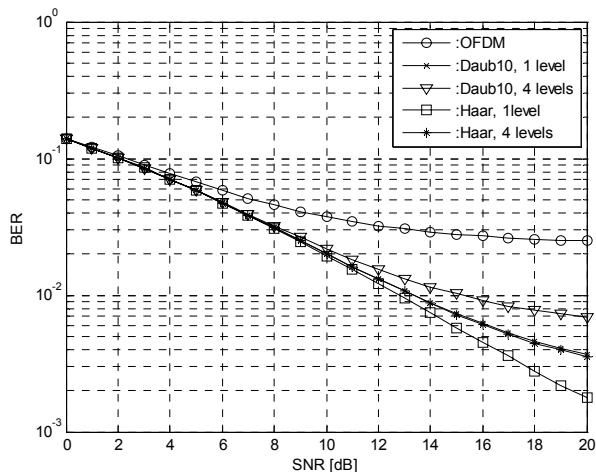


Fig.4: BER performance in fast fading channels ($f_m=0.05$).

This time, the wavelet-based system provides an even higher gain. Thus, OFDM system needs 20 dB of SNR to reach a BER of 0.02, approximately 9dB more than required by all the wavelet-based methods. These results can be explained intuitively by the well known OFDM sensitivity to the time variant character of the radio channel. Indeed, the orthogonality of the sine carriers in OFDM is very fragile: a small Doppler displacement of one carrier will move its position and will transform this carrier into an interfering source for the other ones. Furthermore, for an OFDM system, the following remark can be made: higher the number of subcarriers, lower their frequency separation and higher the probability of interference. Nevertheless, wavelet carriers provide a better compromise between their time and frequency localization: they are less concentrated in frequency than the complex exponentials. This feature seems to enhance their resilience to Doppler. Furthermore, Haar wavelet, which has the best time and the poorest frequency localization amongst all wavelets achieves the lowest BER, remark that supports the conclusions above.

A deeper analysis of the WOFDM system is carried out in the following. This analysis is conducted with respect to some WOFDM related parameters. The influence of the wavelet mother choice is first investigated. For this purpose, the other parameter is kept constant (one DWT iteration in our simulations). Figure 5 shows the results for the slow fading case. No noticeable difference can be

highlighted between the two wavelet mothers from figure 5. The results are almost identical even in the case of the higher Doppler shift taken into account ($f_m=0.005$). These results change significantly for the fast fading case. Whereas there is still no difference between the two tested wavelets at $f_m=0.01$, the use of Haar wavelet mother leads to a 2 dB gain over Daubechies-10 at $f_m=0.05$.

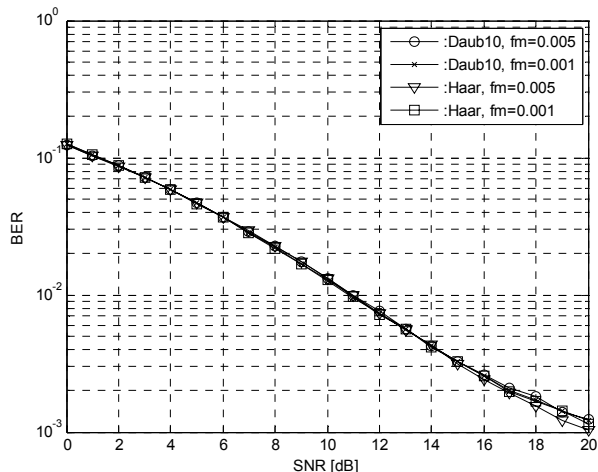


Fig.5: BER performance in slow fading channels with various wavelets mother.

This supports to a certain extent the remark made above, about a higher resilience to Doppler of wavelets with a better time localization. Another argument, which remains only at a stage of intuitive remark, is that Haar's wavelet matches "the best" to the bipolar data transmission (a wavelet's mother looks like a sequence of two symbols -1 and +1 transmitted sequentially).

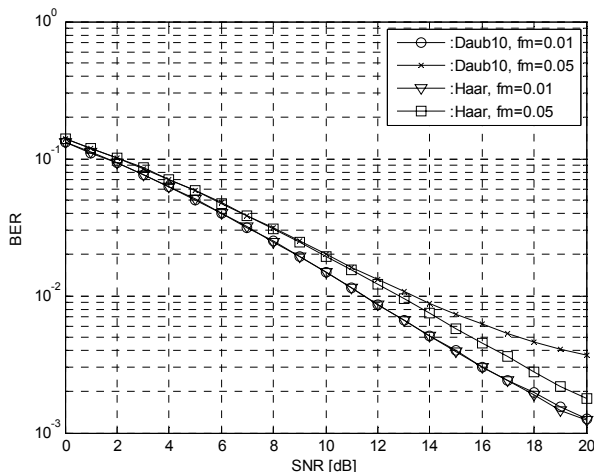


Fig.6: BER performance in fast fading channels with various wavelets mother.

Thus, the variability in time of the radio channel seems to cause more problems when carrier waves with good frequency localization are used (this conclusion could also refer to OFDM's sine carriers, as an extreme case). Note yet that this conclusion should be enforced by well structured theoretical

computations and by more extensive simulations, including several types of wavelets.

The next goal of our study is to investigate the influence of another parameter of the WOFDM system, namely the number of iterations used in the DWT computation. Simulations were made for the two considered wavelets mother, with one and four iterations. The results are displayed in figures 7 and 8, for slow and fast fading respectively.

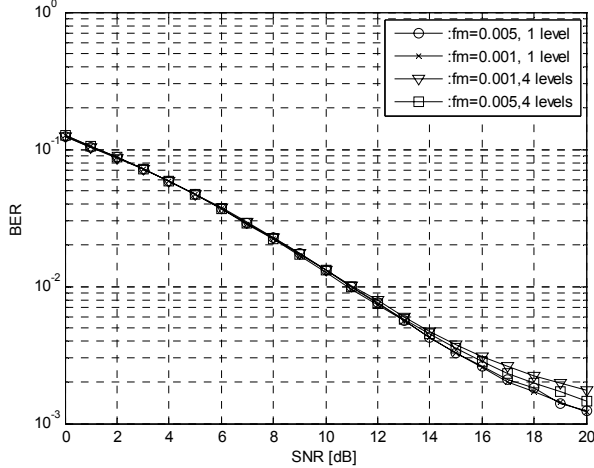


Fig. 7: BER performance in fast fading channels with various numbers of DWT iterations and Daubechies 10 wavelet mother.

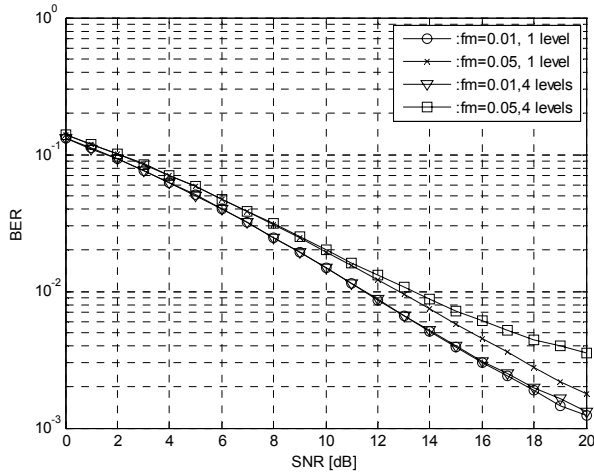


Fig. 8: BER performance in fast fading channels with various numbers of DWT iterations and Haar wavelets mother.

Both figures above support the following hypothesis: higher the number of iterations used in DWT/IDWT computation, lower the system protection against Doppler shift and poorer the system performance. The degradation brought by an increased number of iterations is smaller in the flat fading case, but can be clearly highlighted in the fast fading scenario. Hence, the transmission with 1 DWT iteration provides a 5dB gain over the case with 4 iterations

These observations are interesting and their interpretation is not straightforward. One possible explanation is given in the following. When a single

iteration is used for IDWT computation, the wavelet carrier employed has the best time localization and the poorest frequency localization amongst all other wavelets from the same family. With respect to figure 2, the data at the IDWT synthesizer input will be composed of a_j and w_j only. If we refer to equation 3, then w_j coefficients will modulate a wavelet $\Psi_1(t) = \Psi(t/2)$. When more iterations are used (4 iterations in our simulations), then the wavelet carriers will be not only those from the finest scale but wavelets from coarser scales too ($\Psi_2(t) = \Psi(t/4)$, $\Psi_3(t) = \Psi(t/8)$ etc). Previous studies made [6, 8] have shown that higher Doppler shifts (short coherence time) will mainly affect the symbols transmitted at coarser scales, where symbol (or equivalently "sample") duration is longer, becoming comparable with channel's coherence time. This remark must be strengthened too by carrying out the evaluation of errors distribution "per scale", which will be the subject of a future paper.

V. CONCLUSIONS AND FURTHER WORK

The performance of wavelet-based OFDM in flat Rayleigh fading conditions is investigated in this paper. The author carries out a comparison between this technique and the classical OFDM, based on complex exponential carriers. It is proven by simulation means that the two techniques perform differently in flat fading channels. Showing less sensitivity to the Doppler shift caused by the time-variant character of the radio channel, WOFDM has better BER performance, mainly at high Doppler shift values (a gain of more than 10 dB under certain circumstances).

A second goal of this paper was to study what is the influence of certain parameters used for WOFDM implementation: the type of wavelets mother and the number of decomposition levels used in DWT/IDWT computation. Some interesting conclusions can be drawn. Thus, Haar-based WOFDM works better than Daubechies 10 –based WOFDM. Intuitively, this can be explained by the poor frequency localization of the first wavelet, which is less affected by the frequency offset caused by the Doppler effect. On the other hand, the number of iterations used for DWT/IDWT computation proves to be important too. Noticeable differences were observed mainly for the fast fading case, when the system with one iteration provides a significant gain compared to the 4 iterations case. The explanation resides in the inherent structure of WOFDM, which acts "across the scales": less iterations means finer scales, shorter duration wavelet carriers (and implicitly shorter transmitted samples). Next, short-duration data symbols transmitted on a scale means less sensitivity to Doppler (the coherence time of the channel being significantly higher than the symbol time).

These interesting conclusions open some new research directions on this topic. Thus, the effect of wavelets mother choice can be clearly identified only

by a more comprehensive theoretical and practical study, which should be carried out on more wavelets families (e.g. Symmlet, Coiflet, Daubechies etc). On the other hand, the relevance of the number of decomposition levels can be investigated in a more detailed fashion only by computing "number of errors per scale" statistics. Intuitively, these "BER across scales" statistics could be used to adaptively select the appropriated error correction codes which would lead to an optimized performance.

Finally, the next logical step in this direction will be to take into consideration the second critical feature of the radio channel, besides its time-variant behavior, namely its frequency selectivity. Indeed, the influence of all parameters considered in this study could be redefined in a frequency-selective context, where equalizations issues become critical.

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