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The Mealy in Moore conversion of an automaton

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Abstract – In this article , it presents a new demonstration of the Mealy in Moore conversion theorem and an new matrix algoritm witch, just manually, can be easy realised for automatons with a great number of internal states. The Mealy in Moore conversion is a stage in the logical synthesis of a Mealy automaton using, for example, programmable logic arrays or delays.

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I. INTRODUCTION

An automaton is noted $A=(X, Z, Y, f, g)$ and has the next significances for notations: X=lot of input combinations , Z=lot of internal states, Y=lot of output combinations, f=transition logical function defined by

$$f : X\times Z\rightarrow Z \ , \ \ f(x_n, z_n) = z_{n+1} \ ,$$

g=output logical function witch for an Mealy automaton is defined by

$$g : X\times Z\rightarrow Y \ , \ \ g(x_n, z_n) = y_{n+1}$$

and for an Moore automaton by

$$g : X\times Z\rightarrow Y \ , \ \ g(z_n) = y_{n+1} \ .$$

An automaton is represented by truth table or connections matrix. In the connections matrix, for every internal state it allocats one row and one column, the column containing the input transitions to the internal state and the row containing the output transitions from the internal state. In a automaton synthesis, it must identifier, in the Z lot, the equivalence classes for to generate the reduced form of the automaton by keeping only one single state from every equivalence class. Because the truth table and the connections matrix are easier handled for an Moore automaton, in a Mealy automaton synthesis is necessary a Mealy in Moore conversion ; [1], [2].

II. THEORETICAL SUPPORT

The possibility of a Mealy in Moore conversion of an automaton is justifyed by the next

<u>Theorem</u>
Any Mealy automaton has an equivalent Moore one.
<u>Demonstration</u>
Let it be $A = (X, Z, Y, f(x_n, z_n), g(x_n, z_n))$ an Mealy automaton, whose output function can be writed

$$g(x_n, z_n) = g(\overset{-1}{f}(z_{n+1})) \overset{not}{=} G(z_{n+1}) \qquad (1)$$

only if the transitions to a internal state of the Mealy automaton have the same output for automaton, so that the function

$$\overset{-1}{f} : Z \rightarrow X\times Z \qquad \overset{-1}{f}(z_{n+1}) = (x_n, z_n) \qquad (2)$$

don't introduce output undeterminations. The last relation shows that it's an Moore automaton, noted $A' = (X, Z, Y, f(x_n, z_n), G(z_n))$, having the same outputs like the Mealy automaton, but the outputs being delayed by one single transition. So, for to pass from the Mealy automaton at the Moore automaton it uses the functions group $h=(\alpha, \beta, \delta)$, in witch :

$$\begin{cases} \alpha : X\rightarrow X \ , & \alpha(x_n) = x_n \\ \beta : Z\rightarrow Z \ , & \beta(z_n) = z_n \\ \delta : Y\rightarrow Y \ , & \delta(g(x_n, z_n)) = g(x_{n-1}, z_{n-1}) \end{cases} \qquad (3)$$

Because :

$$g(x_{n-1}, z_{n-1}) = g(f(z_n)) = G(z_n) = G(\beta(z_n)) \qquad (4)$$

and, by building manner of the Moore automaton, $h=(\alpha, \beta, \delta)$ is both injective and surjective, the Moore and Mealy automatons are in a equivalence relation. Any Mealy automaton can be transformed, so that the transitions to a internal state have the same output for automaton, dividing any internal state in so many new internal states how many different outputs are in the transitions to the internal state. Thus, a new internal state keeps, from the origin internal state, the same output transition, but only the input transitions with the same output for automaton. An transformation example of a internal state z_k in two new internal

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states z_{k0} and z_{k1} is presented in Fig.1, by fluency graph.

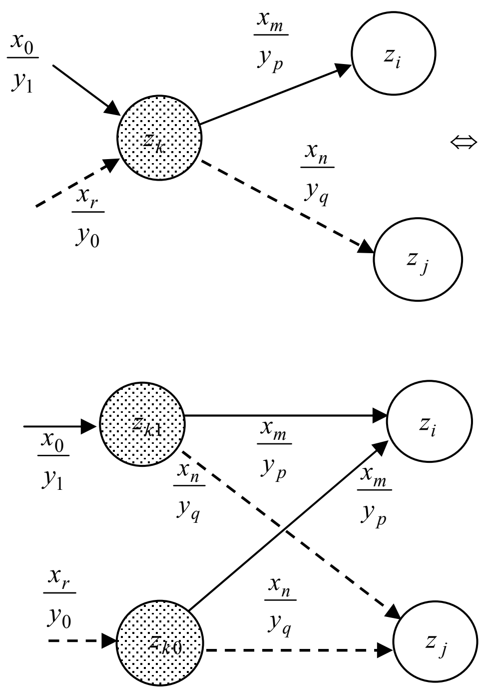

Fig.1

III. THE CONVERSION ALGORITHM

The Mealy in Moore conversion algorithm of an automaton uses the previous theorem, and has the next two phases:

a) In the initial Mealy automaton, it forms an intermediary Mealy automaton having in the input transitions to a internal state the same output for automaton. So, it obtains the connections matrix of the intermediary Mealy automaton by dividing every internal state in so many new internal state, how many different outputs are in the input transitions from the column of the divided internal state. Every new internal state keeps from origin internal state, in its row all output transitions and in its column only input transitions with the same output for automaton, so that the new internal states with same origin have identical rows, but different columns.

b) It forms the equivalent Moore automaton delaying the outputs of the intermediary Mealy automaton by one single transition. For that, in the connections matrix of the intermediary Mealy automaton, it moves the outputs from input transitions of a internal state column, to the output transitions of the same internal state row ; [3].

In the example presented below, the matrix noted M and the incomplete matrix noted M_i show witch internal states are divided and how it does this. In the intermediary matrix noted M_{INT} it's shown the forming mode of the rows for the new internal states z_{11} and z_{21}.

<u>Example</u>

$$A \overset{def}{=} \begin{array}{c|ccc|} & \uparrow z_1 & \uparrow z_2 & \uparrow z_3 \\ z_1 \rightarrow & - & \frac{x_0}{y_0} & \frac{x_2}{y_0} \\ z_2 \rightarrow & \frac{x_2}{y_0} & \frac{x_1}{y_0} & \frac{x_3}{y_0} \\ z_3 \rightarrow & \frac{x_0}{y_1} & \frac{x_1}{y_1} & \frac{x_3}{y_0} \end{array} Mealy =$$

$$= \begin{array}{c|ccc|} & z_1 \rightarrow \begin{cases} z_{10} \\ z_{11} \end{cases} \uparrow & z_2 \rightarrow \begin{cases} z_{20} \\ z_{21} \end{cases} \uparrow & z_3 \uparrow \\ z_1 \rightarrow & - & \frac{x_0}{y_0} & \frac{x_2}{y_0} \\ z_2 \rightarrow & \frac{x_2}{y_0} & \frac{x_1}{y_0} & \frac{x_3}{y_0} \\ z_3 \rightarrow & \frac{x_0}{y_1} & \frac{x_1}{y_1} & \frac{x_3}{y_0} \end{array} = M =$$

$$= \begin{array}{c|ccccc|} & \uparrow z_{10} & \uparrow z_{20} & \uparrow z_3 & \uparrow z_{11} & \uparrow z_{21} \\ z_{10} \rightarrow & & & & - & \frac{x_0}{y_0} \\ z_{20} \rightarrow & & & & \frac{x_2}{y_0} & \frac{x_1}{y_0} \\ z_3 \rightarrow & & & & - & - \\ z_{11} \rightarrow & & & & & \\ z_{21} \rightarrow & & & & & \end{array} = M_i =$$

$$= \begin{array}{c|ccccc|} & \uparrow z_{10} & \uparrow z_{20} & \uparrow z_3 & \uparrow z_{11} & \uparrow z_{21} \\ z_{10} \rightarrow & - & - & \frac{x_2}{y_0} & - & \frac{x_0}{y_0} \\ z_{20} \rightarrow & - & - & \frac{x_3}{y_0} & \frac{x_2}{y_0} & \frac{x_1}{y_0} \\ z_3 \rightarrow & \frac{x_0}{y_1} & \frac{x_1}{y_1} & \frac{x_3}{y_0} & - & - \\ z_{11} \rightarrow & - & - & \frac{x_2}{y_0} & - & \frac{x_0}{y_0} \\ z_{21} \rightarrow & - & - & \frac{x_3}{y_0} & \frac{x_2}{y_0} & \frac{x_1}{y_0} \end{array} = M_{INT} =$$

$$
\begin{array}{cc}
 & \begin{array}{ccccc} z_{10} & z_{20} & z_3 & z_{11} & z_{21} \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \end{array} \\
\begin{array}{c} z_{10} \rightarrow \\ z_{20} \rightarrow \\ = z_3 \rightarrow \\ z_{11} \rightarrow \\ z_{21} \rightarrow \end{array} &
\left| \begin{array}{ccccc}
- & - & \frac{x_2}{y_1} & - & \frac{x_0}{y_1} \\
- & - & \frac{x_3}{y_1} & \frac{x_2}{y_1} & \frac{x_1}{y_1} \\
\frac{x_0}{y_0} & \frac{x_1}{y_0} & \frac{x_3}{y_0} & - & - \\
- & - & \frac{x_2}{y_0} & - & \frac{x_0}{y_0} \\
- & - & \frac{x_3}{y_0} & \frac{x_2}{y_0} & \frac{x_1}{y_0}
\end{array} \right|_{Moore} = A'
\end{array}
$$

IV. CONCLUSIONS

The Mealy in Moore conversion algorithm, proposed in this article, is, just manually, easier handled for automatons with a great number of internal states and is based on a siple mathematical demonstration. This algorithm simplifies logical synthesis of an sequantial automaton witch can be realised with programmable logic arrays or relays.

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