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The estimation of the instantaneous frequency using time-frequency methods

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Abstract – Instantaneous frequency (IF) is a very important parameter in a large number of applications. Generally, the IF is a non-linear function of time. For such cases the analysis of time-frequency content provides an efficient solution. In this paper is analyzed the performance in IF estimation of the two time-frequency based methods. The first estimation method uses the complex argument distribution (CTD) and the second one uses the ridges extraction method from the time-frequency distribution based on mathematical morphology operators (TF-MO). Monocomponent signals with non-linear and highly non-linear IF corrupted by Gaussian white noise are considered as numerical examples.

Keywords: Instantaneous frequency, time-frequency distribution, complex argument, mathematical morphology, signal analysis, image analysis.

I. INTRODUCTION

In signal processing the decision (detection, denoising, estimation, recognition or classification) is a basic problem. Knowing that the real environments are generally highly non-stationary, it is necessary to use a method able to provide suggestive information about the signal structure. A potential solution is based on time-frequency representations that provide a good concentration around the law of the IF and realize a diffusion of the perturbation noise in the time-frequency plane.

The CTD has been introduced in [1] as an efficient way to produce almost completely concentrated representations along the IF, it considerably reduces the artifacts due to the complexity of the analyzed signal.

The TF-MO estimation method [2] is based on the conjoint use of two very modern theories, that of time-frequency distributions and that of mathematical morphology. This strategy permits the enhancement of the set of signal processing methods with the aid of some methods developed in the context of image processing.

The paper is organized as follow. In section II is presented the CTD. The TF-MO method is illustrated in section III. In section IV some simulation results are depicted. Section V will close this communication.

II. COMPLEX TIME DISTRIBUTION

The complex time distribution as an IF estimator have been proposed and analyzed in [1]. It provides a highly concentrated distribution along the IF law with reduced interferences (noise and cross-terms).

Mathematically, the complex time distribution (CTD) of signal is defined as:

$$CTD(t,\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} s\left(t+\frac{\tau}{4}\right)s^{*}\left(t-\frac{\tau}{4}\right) \times s^{-j}\left(t+j\frac{\tau}{4}\right)s^{j}\left(t-j\frac{\tau}{4}\right)\, e^{-j\omega\tau}\,d\tau \tag{1}$$

where the continuous form of the “complex-time signal” is:

$$s(t+j\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty} S(j\omega)\,e^{-\omega\tau}\,e^{j\omega t}\,d\omega \tag{2}$$

where $S(j\omega)$ is the Fourier transform of signal s.

The main property of the CTD consists in the capability to attenuate the high-order terms of the polynomial decomposition of the IF, for signals of the form $s(t)=r\,e^{j\phi(t)}$. The spread factor $Q(t,\tau)$ around the IF for this distribution is:

$$Q(t,\tau)=\phi^{(5)}(t)\frac{\tau^{5}}{4^{4}5!}+\phi^{(9)}(t)\frac{\tau^{9}}{4^{8}9!}+\dots \tag{3}$$

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As proved in [4], in the case of the CTD, this function has a fifth order dominant term (for comparison, the spectrogram and the Wigner-Ville distribution (WVD) have a second and a third order dominant term, respectively), which corresponds to a drastic reduction of the higher terms of $Q(t,\tau)$.

The CTD satisfies some important properties:

1) the CTD is real for the frequency modulated signals $s(t) = re^{j\phi(t)}$;

2) the time-marginal property $\frac{1}{2\pi}\int_{-\infty}^{\infty} CTD(t,\omega)d\omega = |s(t)|^2$;

3) the unbiased energy condition $\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} CTD(t,\omega)\,dt\,d\omega = \int_{-\infty}^{\infty}|s(t)|^2\,dt = E_s$;

4) the frequency-marginal property;

5) time-frequency shift-invariance properties;

6) the CTD of the scaled signal $\sqrt{|a|}\,s(at)$ is $CTD(at,\omega/a)$.

III. TIME-FREQUENCY MORPHOLOGICAL OPERATORS METHOD

The estimation method based on time-frequency and image processing techniques has been introduced in [2]. The quality of estimating the IF depends on the time-frequency distribution and on its ridges projection mechanism. The TF-MO method proposes a time-frequency representation based on cooperation of linear and bilinear distributions: the Gabor and the Wigner-Ville distributions. It is known [3] that the Gabor representation has a good localization and free interference terms properties. Unfortunately, the linear distributions, except the Discrete Wavelet transform, correlate the zero mean white input noise, as shown in [4]. The WVD is a spectral-temporal density of energy that does not correlates the input noise, thus having a spreading effect of the noise power in the time-frequency plane [2]. The WVD has also a good time-frequency concentration.

To combine these useful advantages, the time-frequency distribution is calculated according to the following algorithm [2]:

1) Calculate the Gabor transform for the signal *s*, $G(t,\omega)$.

2) Filter the image obtained with a hard-thresholding filter:

$$Y(t,\omega) = \begin{cases} 1, & if\ |G(t,\omega)| \geq tr \\ 0, & if\ |G(t,\omega)| < tr \end{cases} \tag{4}$$

where *tr* is the threshold used.

3) Calculate the WVD for the signal *s*, $WV(t,\omega)$.

4) Multiply the modulus of the $Y(t,\omega)$ distribution with the $WV(t,\omega)$ distribution.

In step 2) the proposed threshold value is:

$$tr = \frac{\max\limits_{(t,\omega)}\{|G(t,\omega)|\}}{5} \tag{5}$$

This operation decreases the amount of noise that perturbs the ridges of $G(t,\omega)$ and brings to zero the values in the rest of the time-frequency plane. The effect of the multiplication in step 4) is the reduction of the interference terms of the WVD and the very good localization of the ridges of the resulting ditribution.

To estimate the ridges of the obtained distribution, some mathematical morphology operators are used, the above resulting distribution being regarded as an image. This mechanism is applied through the following steps [2]:

1) Convert the image obtained in step 4) in the procedure described earlier in binary form.

2) Apply the dilation operator on the image in 1).

3) Skeletonization of the last image, an estimation of the IF of the signal being obtained. This image represents the result of the TF-MO method. The conversion in binary form realizes a denoising of the time-frequency distribution. The role of the dilation operator is to compensate the connectivity loss, produced by the preceding conversion. The skeleton produces the ridges estimation.

IV. RESULTS

Example 1: Consider a noisy monocomponent signal with non-linear IF:

$$s(t) = \exp\{j(5\pi t^3 - 9.5\pi t)\} + n(t) \tag{6}$$

within the interval [-1, 1] where *n(t)* is a Gaussian white noise.

Based on the CTD and TF-MO method, the IF is estimated for various values of the SNR. In Fig. 1.a and Fig. 1.b, are presented the estimated IF with the two methods, for SNR=30dB (left image) and SNR=3dB (right image) along with the real IF law.

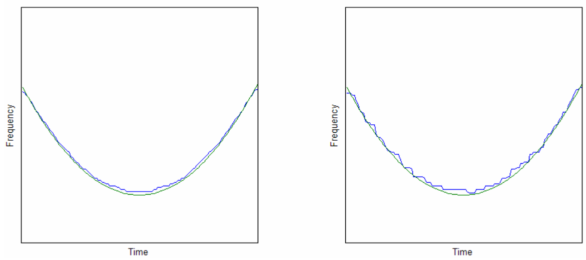

Fig. 1.a. IF estimation based on TF-MO method for SNR=30dB (left image) and SNR=3dB (right image)

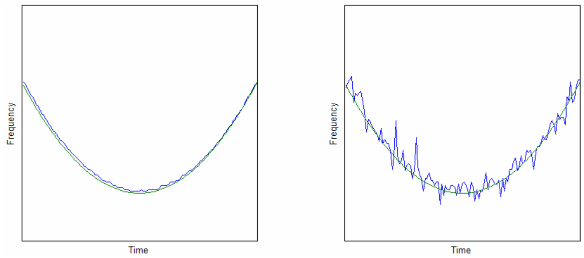

Fig. 1.b. IF estimation based on CTD method for SNR=30dB (left image) and SNR=3dB (right image)

From Fig. 1.b can be also observed that for low SNR (right image) the variance in the CTD is higher thus degrading the performance of the estimation.
Mean squared errors of the IF estimation calculated in 128 realizations for SNR values within the interval [3dB, 30dB], based on CTD, TF-MO method and WVD are showed in Fig. 2.a and 2.b (zoomed image).

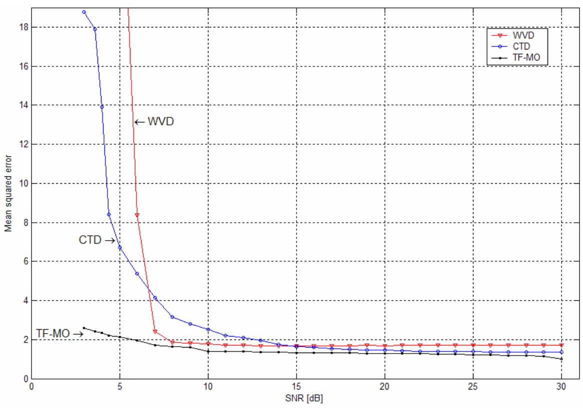

Fig. 2.a. Mean squared error of the IF estimation for SNR between [3dB, 30dB] based on CTD , TF-MO method and WVD

It can be noticed that the performances for the three distributions in the SNR range [10dB, 30dB], are slightly similars, nevertheless the TF-MO method providing a better results. As the SNR decreases, the estimation error in the CTD and WVD increases more rapidly than in the TF-MO method.

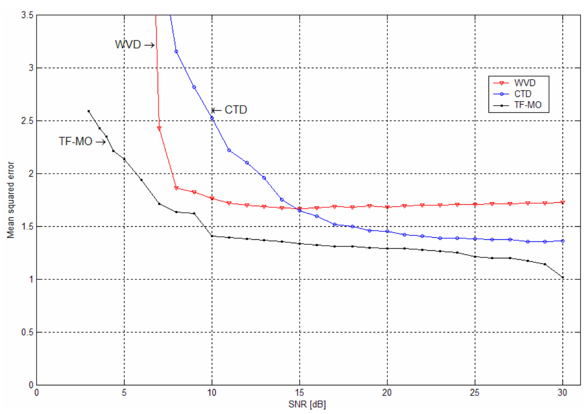

Fig. 2. b. Mean squared error of the IF estimation based on CTD , TF-MO method and WVD (zoomed image)

Example 2: Consider now a noisy monocomponent signal with highly non-linear IF:

$$s(t) = \exp\{j(3\cos(\pi t) - \cos(3\pi t)/2 + \\ + \cos(5\pi t)/1.5))\} + n(t) \quad (7)$$

within the interval [-1, 1] where *n(t)* is a Gaussian white noise.
The IF is estimated for various values of the SNR, based on the CTD and TF-MO method. Fig. 3.a and Fig. 3.b, represent the estimated IF with the two methods, for SNR=30dB (left image) and SNR=3dB (right image) along with the real IF law.

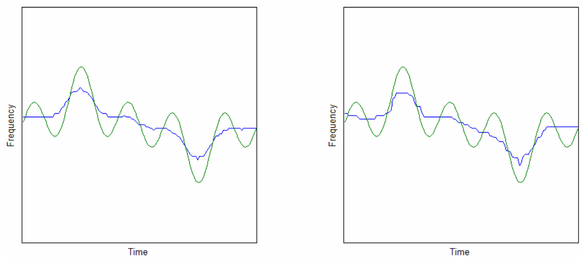

Fig. 3.a. IF estimation based on TF-MO method for SNR=30dB (left image) and SNR=3dB (right image)

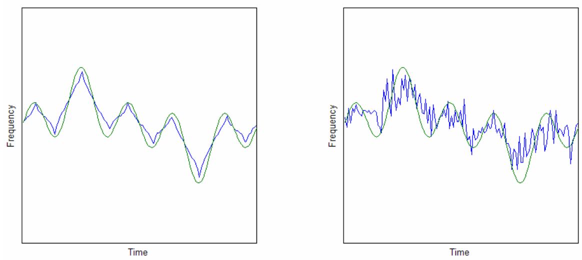

Fig. 3.b. IF estimation based on CTD method for SNR=30dB (left image) and SNR=3dB (right image)

Mean squared errors of the IF estimation calculated in 128 realizations for SNR values within the interval [3dB, 30dB], based on CTD, TF-MO method and WVD are showed in Fig. 4.

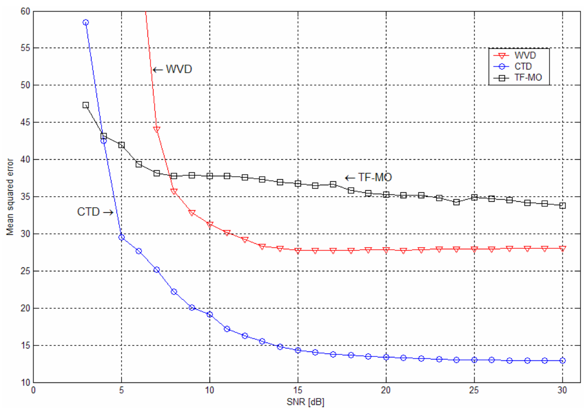

Fig. 4. Mean squared error of the IF estimation for SNR between [3dB, 30dB] based on CTD , TF-MO method and WVD

Fig. 3.a illustrates the fact that the bias in the TF-MO method is significant and dominates in the estimation error. The IF estimator in this case cannot accurately follow the rapid transitions in the IF. When the noise is increased, the variance in the CTD is higher (Fig. 3.b) thus reducing the quality of the estimation, whereas the variance and the bias in the TF-MO method remains slightly unchanged, which assures almost the same estimation error. Fig. 4 proves that behavior.

From these numerical examples, it can be noticed that the TF-MO method is very dependent on the choice of the threshold *tr*. A high value can preserve the noise peaks in the time-frequency plane outside the region where the signal component is located. This side effect is very significant for relatively high noise, thus degrading the estimation process. The performance can be improved by applying the morphological operators only in the region around the signal component. This can be done using a detection technique. Moreover, the parameters of the morphological operators have an important role for the ridges extraction precision and it has been observed that for high SNR skipping the application of the dilation operator in the TF-MO method provides an improvement of the IF estimation.

A low value for the *tr* can cause connectivity breaks of the time-frequency distribution used, this inconvenient being compensated by the reconstruction capacity of the morphological operators. This can be seen from Fig. 3 and Fig. 4, where for relatively high noise the performances of the TF-MO method are remaining acceptable. However, a greater connectivity breaks can induce false IF estimation.

From the analysis of the two examples considered it could be concluded that for signal with swift transitions over a short duration of time and SNR high enough, the accuracy of the IF estimation in the TF-MO method is poorer than in the CTD case. Still, for low SNR the performances become relatively equals, the TF-MO method being robust at the noise influence.

For signals not so complicated, the performances of the methods analyzed are similar, but the TF-MO method provides the better results.

V. CONCLUSIONS

In this paper, it has been analyzed the performances of the IF estimation for the CTD distribution and TF-MO method, in two illustrative numerical examples.

For monocomponent signals with highly non-linear IF and high SNR, the CTD distribution is more adapted than the TF-MO method. It is possible to improve the IF estimation results combining the qualities of the two methods. An immediate solution is to apply the mechanism of the ridges projection of the TF-MO method on the image obtained in the CTD distribution. Another possibility is to analyze the benefit of the reallocation principle in time-frequency domain, [5].

For signals with a time-frequency structure not so complicated, the performances of the distributions studied in this paper are relatively the same.

Further research could be directed toward the estimation of the polinomyal order of the estimated IF.

Other possible further research directions are the following:

- the comparison of the two IF estimation methods for multicomponent signals;
- the conception of a new IF estimation method combining the qualities of the two methods analyzed in this paper;
- the simulation of a system for time-frequency signal analysis, starting from the model proposed in [6].

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