Buletinul Ştiinţific al Universităţii "Politehnica" din Timişoara

Seria ELECTRONICĂ şi TELECOMUNICAŢII
TRANSACTIONS on ELECTRONICS and COMMUNICATIONS

Tom 51(65), Fascicola 2, 2006

Evaluation of Information Capacity for a Class of MIMO Channels

Rodica Stoian, Lucian Andrei Perişoară[1]

Abstract - The Multiple Input Multiple Output (MIMO) Channels are usually used in wireless communications, by the use of spatial diversity at both sides of the link. The MIMO concept is more general and embraces many other scenarios such as wireline networks (LANs).
This paper summarizes the state of art in MIMO channels, presenting MIMO channel models, summarizing the computing of the information capacity for some particular MIMO channels and making a comparative analysis for different channel modeling parameters.
Keywords: Information Theory, Wireless Networks, LANs, MIMO systems, multipath propagation, information capacity.

I. INTRODUCTION

Based on generalization of Shannon's fundamental problem of communication, "that of reproducing at one point either exactly or approximately a message selected at another point", [1], the network information theory provides the theoretic basis to build up the best architecture for information transport.

Determining the appropriate architecture for information transfer between the nodes of a wireless network and the computation of amount of information that can be transported, i.e. the information capacity of the wireless network are fundamental problems for which the solutions is given by the network information theory.

In connection with this approach, in the field of communication systems, Multiple Input Multiple Output (MIMO) channels have recently become a popular means to increase the spectral efficiency and quality of wireless communications by the use of spatial diversity at both sides of the link [2, 3]. In fact, the MIMO concept is much more general and embraces many other scenarios such as wireline networks (LAN's) and single antenna frequency-selective channels [4, 5].

This paper summarizes the state of the art in MIMO channels. We begin by reviewing some well-known definitions and results on MIMO channel models, then we present a summary of the information capacity computing in MIMO systems. The core of this paper is dedicated to determine the information capacity for some particular MIMO channels, which can be the theoretical basis for a MIMO system implementation.

II. THE MIMO CHANNEL MODEL

The model of a MIMO channel is used in applications like the transmission between two nodes, where are many propagation paths, i.e. wireless and wireline (LAN) networks. We will consider a communication system where *N* sample signals x_i are transmitted from *N* input nodes simultaneously. Each transmitted signal x_i goes through the channel to arrive at each of the *M* output nodes.

In a MIMO communication channel with *N* input nodes and *M* output nodes, each output of the channel is a linear superposition of the faded versions of the inputs. Each pair of transmit and receive nodes provides a signal path from the transmitter *i* to the receiver *j*, described by the path gain h_{ij}. Based on this model, the signal y_j, which is received at node *j*, is given by:

$$y_j = \sum_i h_{ij} x_i + n_j , \qquad (1)$$

where n_j is the AWGN noise sample, with variance σ_n^2, of the receive antenna *j*.

We represent the signals that are transmitted from *N* transmit nodes as $\mathbf{x} = (x_1, x_2, ..., x_N)$, the signals received from *M* receive nodes as $\mathbf{y} = (y_1, y_2, ..., y_M)$, the path gains in a channel matrix $\mathbf{H} = \left[h_{ij}\right]_{N\times M}$, resulting the following matrix form of (1):

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} , \qquad (2)$$

where $\mathbf{n} = (n_1, n_2, ..., n_M)$ is the AWGN noise matrix.

[1] Faculty of Electronics,Telecommunication and Information Technology, Applied Electronics and Information Enginnering Departament, University Politehnica of Bucharest, ROMANIA, rodicastoian2004@yahoo.com, lperisoara@yahoo.com, www.orfeu.pub.ro

For wireless systems, different path gains may be independent from each other, that is h_{ij} is independent from $h_{i'j'}$ for $i \neq i'$ or $j \neq j'$. The path gain h_{ij} represents the attenuation of a signal from antenna *i* to antenna *j* and can be written as:

$$h_{ij} = \frac{ce^{-\gamma d_{ij}}}{d_{ij}^{\delta}}, \quad (3)$$

where $d_H(i, j) = d_{ij}$ is the Euclidian distance between two specific nodes *i*, *j*, δ is the path loss exponent and γ is the medium absorption constant [7]. Each node is also assumed to have a power constraint *P*.

Until now, we used the term of distance in two ways: the distances between nodes and the attenuation as a function of distance. The third way, distance will explicitly enter our model is through the choice of the performance measure "the information capacity".

For a network with *N* input nodes and *M* output nodes, having *NM* possible source-destination pairs, we use the rate vector denoted by the matrix $\mathbf{R} = [R_{ij}]$, with $i = \overline{1, N}$, $j = \overline{1, M}$, [6, Cap. 14]. Let $\Re$ be the set of feasible rate vectors. We define the information capacity as the supremely distance weighted sum of rates, described by the relation:

$$C = \max_{\mathbf{R} \in \Re} \sum_i \sum_j R_{ij} d_{ij}, \text{ [bps/Hz].} \quad (4)$$

III. THE INFORMATION CAPACITY OF MIMO CHANNELS

For the information capacity of a particular MIMO channel, we assume that the receiver knows the realization of the channel, i.e. it knows both **y** and **H** or it has a perfect channel state information. For the transmitter, we study two cases:

A) the transmitter does not know the realization of the channel; however, it knows the distribution of **H**.

B) the transmitter knows the realization of the channel.

The resulting capacity of the channel is a random variable because the capacity is a function of the channel matrix **H** and has the distribution of the channel matrix **H**. For the input signal **x**, we assume that it is a zero-mean circularly symmetric complex Gaussian vector with covariance matrix $\mathbf{C}_X$, [2, 3].

A. *Capacity of a deterministic MIMO Channel*

We use a quasi-static block fading model. Under such a model, the channel path gains are fixed during a large enough block such that information theoretical results are valid. The values of path gains change from block to block based on Rayleigh fading channel model. First, we assume that the realization of the channel **H** is fixed, i.e. the channel matrix is deterministic and its value is known at the receiver.

The capacity is defined as the maximum of the mutual information between the input and output given a power constraint *P* on the total transmission power of the input, that is $\mathrm{Tr}(\mathbf{C}_X) \leq P$ [2].

The mutual information *I*, between input and output for the given realization is:

$$I = \log_2 \det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\mathbf{H}\mathbf{C}_X\mathbf{H}^H\right), \text{ [bps/Hz],} \quad (5)$$

Then, the information capacity is the maximum of the mutual information over all inputs satisfying $\mathrm{Tr}(\mathbf{C}_X) \leq P$. In other words,

$$C = \max_{\mathrm{Tr}(\mathbf{C}_X) \leq P} \log_2 \det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\mathbf{H}\mathbf{C}_X\mathbf{H}^H\right), \text{ [bps/Hz],} \quad (6)$$

The unit bps/Hz represents the fact that for a bandwidth of *W*, the maximum possible rate for a reliable communication is *CW* bps.

The capacity from (6) can be computed in terms of the positive eigenvalues λ_i of $\mathbf{HH}^H$, using a water-filling algorithm [6] for the power allocation of the nodes, as:

$$C = \max_{\sum_i \gamma_i = \gamma} \sum_i \log_2(1 + \gamma_i \lambda_i), \text{ [bps/Hz],} \quad (7)$$

where $i = \overline{1, rank(\mathbf{H})}$.

B. *Capacity of a random MIMO Channel*

Since the channel matrix **H** is random in nature, the capacity in (6) is also a random variable. Let us assume an equal distribution of the input power that transforms matrix $\mathbf{C}_X$ to a multiple of identity matrix $\mathbf{I}_N$ and with the constraint $\mathrm{Tr}(\mathbf{C}_X) \leq P$, we have $\mathbf{C}_X = \mathbf{I}_N$ and:

$$C = \log_2 \det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\mathbf{HH}^H\right), \text{ [bps/Hz].} \quad (7)$$

IV. INFORMATION CAPACITY OF MIMO PARTICULAR CHANNELS

In this section, we study the capacity of a particular MIMO channels for different values of *M* and *N*.

- The graphical model of the particular MIMO channel with $M = 2$, $N = 2$ is illustrated in Fig. 1.

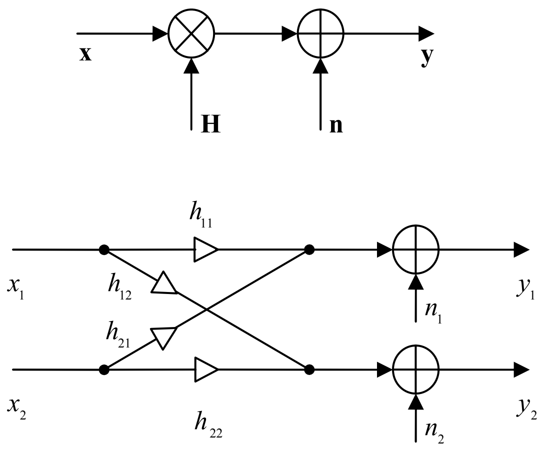

Fig. 1. The model of a MIMO Channel for $M = 2, N = 2$

The signal model can be written as:

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}, \quad (8)$$

where $\mathbf{x} = [x_1 \quad x_2]^T \in \mathbb{C}^{2\times1}$ are the input data samples with the covariance matrix $\mathbf{C}_X = E[\mathbf{xx}^H]$, $\mathbf{y} = [y_1 \quad y_2]^T \in \mathbb{C}^{2\times1}$ are the output data samples, $\mathbf{n} = [n_1 \quad n_2]^T \in \mathbb{C}^{2\times1}$ are the Gaussian noise samples and $\mathbf{H} = [h_{ij}]_{M\times N} = [|h_{ij}| e^{j\alpha_{ij}}]_{M\times N}$ is the fading channel matrix.

The mutual information is defined as ($\mathbf{I}_2$ is the unitary matrix):

$$I = \log_2 \det\left(\mathbf{I}_2 + \frac{1}{\sigma_n^2}\mathbf{HC}_X\mathbf{H}^H\right), \text{[bps/Hz]}. \quad (9)$$

The information capacity of the MIMO channel, from Fig. 1, is:

$$C = \max_{\mathbf{C}_X} \log_2 \det\left(\mathbf{I}_2 + \frac{1}{\sigma_n^2}\mathbf{HC}_X\mathbf{H}^H\right), \text{[bps/Hz]}. \quad (10)$$

For a flat fading channel, characterized by the coefficients $h_{ij} = 1$, the information capacity for a signal to noise ratio $SNR = \sigma_X^2 / \sigma_n^2$ can be determined for the two cases of an unknown or known channel at the transmitter side (see Table I, column 2). For a fading channel, if the attenuations are omitted, $|h_{ij}| = 1$, the matrix channel is $\mathbf{H} = [e^{j\alpha_{ij}}]_{M\times N}$. For an unknown channel, the coefficients α_{ij} are calculated so that $\mathbf{H} = 2\mathbf{I}_2$. The expression of information capacity for these cases is also shown in Table I.

- For the MISO channel with $M = 2$ and $N = 1$, the signal model can be written as:

$$y = \mathbf{Hx} + n, \quad (11)$$

where $\mathbf{x} = [x_1 \quad x_2]^T \in \mathbb{C}^{2\times1}$ is the input data samples with the covariance matrix $\mathbf{C}_X$, $y \in \mathbb{C}$ is the output data sample, $n \in \mathbb{C}$ is the Gaussian noise samples and the fading channel matrix is $\mathbf{H} = [h_1 \quad h_2]$.

The information capacity of a MISO channel is calculated in the same manner as for MIMO channel and can be written as in Table I, column 3, for a flat fading channel with matrix $\mathbf{H} = [1 \quad 1]$ and for a fading channel, in the case of a known or unknown channel at the transmitter.

- For the SIMO channel with $M = 1$ and $N = 2$, the signal model is:

$$\mathbf{y} = \mathbf{H}x + \mathbf{n}, \quad (12)$$

where $x \in \mathbb{C}$ is the input data sample with the covariance σ_X^2, $\mathbf{y} = [y_1 \quad y_2]^T \in \mathbb{C}^{2\times1}$ are the output data samples, $\mathbf{n} = [n_1 \quad n_2]^T \in \mathbb{C}^{2\times1}$ are the Gaussian noise samples and $\mathbf{H} = [h_1 \quad h_2]^T$ is the fading channel matrix. The information capacity of this channel is shown in Table I, column 4.

Table I. The particular expressions of information capacity for flat fading / fading MIMO channels determined for a known / unknown channel at the transmitter.

Information Capacity, [bps/Hz]	**MIMO**	**MISO**	**SIMO**	**SISO**
flat fading, unknown	$\log_2(1+4\cdot SNR)$	$\log_2(1+2\cdot SNR)$	$\log_2(1+2\cdot SNR)$	$\log_2(1+SNR)$
flat fading, known	$\log_2(1+8\cdot SNR)$	$\log_2(1+4\cdot SNR)$	$\log_2(1+2\cdot SNR)$	$\log_2(1+SNR)$
fading, unknown	$2\log_2(1+2\cdot SNR)$	$\log_2(1+2\cdot SNR)$	$\log_2(1+2\cdot SNR)$	$\log_2(1+SNR)$
fading, known	$2\log_2(1+2\cdot SNR)$	$2\log_2(1+SNR)$	$\log_2(1+2\cdot SNR)$	$\log_2(1+SNR)$

- For the SISO channel with $M = 1$ and $N = 1$, the signal model can be written as:

$$y = hx + n, \quad (13)$$

where $x \in \mathbb{C}$ is the input data sample with the covariance σ_x^2, $y \in \mathbb{C}$ is the output data sample, $n \in \mathbb{C}$ is the Gaussian noise sample and h is the fading channel coefficient.

For a flat fading channel, the capacity is shown in Table I, column 5 and it coincides with the standard Shannon capacity of a Gaussian channel for a given value of SNR.

V. CONCLUSIONS

In the previous section we determined the particular expressions of information capacity of MIMO channels for different channel model parameters: number of input / output nodes, flat fading / fading and for a known / unknown channel at the transmitter.

Based on the results from Table I, we make the following dependences analysis:

- **The information capacity increases as the number of input or output channel nodes increases** at the same SNR, in the following order SISO, SIMO, MISO, MIMO. If the number of input and output nodes are the same, the capacity increases at least linearly as a function of number of antennas. This dependence is not influenced if the channel is with or without fading or if it is known at the input nodes. Normally, the SISO channel has the lowest capacity and the MIMO channel with two input and output nodes has the highest capacity.

Fig. 2 illustrates the dependence of information capacity of SNR for different numbers of input and output nodes and for flat fading channel, known at the transmitter.

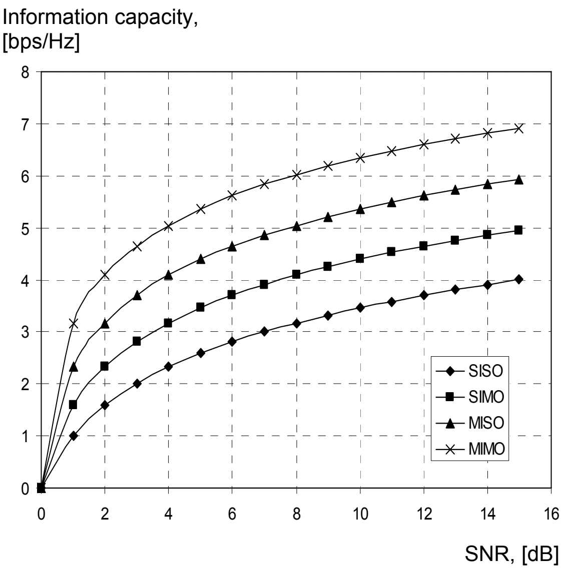

Fig. 2. The dependence of information capacity of SNR for different channel models.

At high SNR, the curves tends to be flat, because the information capacity for a MIMO channel without errors is limited and depends on various parameters, i.e. the system structure, channel parameters.

In this paper, the information capacity of a MIMO channel with different number of input and output nodes is analyzed. These results confirm the work from [8] and are the base for choosing the strategy by which nodes cooperates and for choosing the best architecture for information transport. These will be generalized, in further works, for MIMO channels with $M, N \geq 3$, in applications like wireless networks and LANs communications.

REFERENCES

[1] C. E. Shannon, "A Mathematical Theory of Communication", Bell Syst. Tech. Journal, vol. 27, pp. 379-423, 1948.

[2] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," Eur. Trans. Telecomm.., vol. 10, no. 6, pp. 585–595, Nov-Dec. 1999.

[3] G. Foschini, M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Pers. Commun., vol. 6, pp. 311–335, 1998.

[4] M. L. Honig, K. Steiglitz and B. Gopinath, "Multichannel signal processing for data communications in the presence of crosstalk," IEEE Trans. Commun., vol. 38, no. 4, pp. 551–558, Apr. 1990.

[5] A. Scaglione, G. B. Giannakis and S. Barbarossa, "Redundant filterbank precoders and equalizers. Part I: Unification and optimal designs," IEEE Trans. Signal Process., vol. 47, no. 7, pp. 1988–2006, Jul. 1999.

[6] T. M. Cover, J. A. Thomas, "Elements of Information Theory", John-Wiley, 1991.

[7] E. A. Jorswieck, H. Boche, "Performance Analysis of Capacity of MIMO Systems under Multiuser Interference Based on Worst-Case Noise Behavior", *EURASIP Journal on Wireless Comm. and Net*., no. 2, pp. 273-285, Dec. 2004.

[8] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-time block codes for high data rate wireless communication: performance results", *IEEE Journal on Selected Areas in Communications*, vol. 17(3), pp. 451–460, Mar. 1999.