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Image filtering and enhancement using directional and anisotropic diffusion techniques

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Abstract – A novel diffusion filter for low-level image processing is proposed. Analyzing the drawbacks of Gaussian convolution based regularization of partial derivatives equations, we propose an alternate method that employs anisotropic diffusion techniques to pre-smooth an image. The new technique is developed within the framework of previously proposed directional diffusion processes. Through a statistical interpretation we prove that the new filter produces consistently better results than the original version, especially when dealing with oriented textures having different spatial frequencies. Application samples are also provided in the final part of the paper.

Keywords: diffusion, anisotropic, orientation

I. DIFFUSION FILTERS FOR IMAGE SMOOTHING AND ENHANCEMENT

In recent years a lot of research was done for proposing various diffusion based image filtering and enhancement techniques. The simplest diffusion equation is the isotropic filter that relies on the classical heat equation to smooth an image. Let *U(x,y,t)* denote the gray level of a pixel of coordinates *(x,y)* at some instant *t*. The partial derivatives equation (PDE) that drives the diffusion process is:

$$\frac{\partial U}{\partial t} = div(\nabla U) = \Delta U \quad (1)$$

(1) is usually solved by iterative means and, as time advances, smoothed versions of the original image $U(x,y,0) = U_0(x,y)$ are produced. As pointed out by Koenderink [4], the solution of the isotropic diffusion equation at time $t = \frac{\sigma^2}{2}$ is equivalent with a convolution between the original image and a Gaussian kernel of standard deviation σ:

$$U_\sigma = U(x,y,\frac{\sigma^2}{2}) = G_\sigma * U(x,y,0) \quad (2)$$

Perona and Malik were the firsts to consider anisotropic behavior for diffusion processes. They proposed in [5] an anisotropic diffusion equation that is driven by a non constant diffusivity $c(|\nabla U(x,y,t)|)$. $c(.)$ plays the role of an edge detector that penalizes the intensity of the smoothing process in regions where gradient norms $|\nabla U|$ are large (e.g. edges):

$$c(|\nabla U(x,y,t)|) = g(|\nabla U|) = \frac{1}{1+(|\nabla U|/K)^2} \quad (3)$$

The behavior of their anisotropic diffusion equation:

$$\frac{\partial U}{\partial t} = div[c(|\nabla U(x,y,t)|)\nabla U], \quad (4)$$

can be more easily understood if its directional interpretation [6] is considered:

$$\frac{\partial U}{\partial t} = g(|\nabla U|)U_{\xi\xi} + [g(|\nabla U|) + |\nabla U| g'(|\nabla U|)]U_{\eta\eta} \, . \quad (5)$$

For the type of diffusivity functions proposed by Perona and Malik, in the edge directions - $\vec{\xi} = (-\frac{U_y}{|\nabla U|}, \frac{U_x}{|\nabla U|})$ - the diffusion process will always have a smoothing action ($g(.)>0$) , whereas in the direction of gradient vectors - $\vec{\eta} = (\frac{U_x}{|\nabla U|}, \frac{U_y}{|\nabla U|})$ - smoothing can take place ($g'(.)>0$) or, for gradient norms greater than the diffusion threshold *K,* the equation can behave like an inverse diffusion filter that enhances edges ($g'(.)<0$).

The results obtained by the authors are impressive, edges are kept better and noise is eliminated.

Even if edge enhancement is desired in the original model, Catte et al. pointed out [1] that negative diffusion coefficients can make the diffusion equation instable. They also argued the fact that if noise is important, edge enhancement can amplify also the

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noise level to theoretically unbounded levels. The solution proposed by Catte et. al consists in pre-smoothing the image prior to the estimation of the diffusivities:

$$c(|\nabla U(x,y,t)|) = g(|\nabla(G_\sigma * U)|) \qquad (6)$$

The benefit of the Gaussian convolution is twofold: from a practical point of view influence of noise is diminished and, from a theoretical point of view, the diffusion equation becomes well posed and admits a unique solution. The same idea is encountered in more elaborate diffusion filters that were proposed since: the edge [8] or coherence enhancing diffusion [9] filters proposed by Weickert, the flow coherence diffusion filter we proposed in [7] etc.

II. PROPOSED METHOD

Addressing specific problems appearing when filtering, restoring or enhancing images composed of oriented patterns, we proposed in [6] an efficient method for low level processing of this type of images. The PDE model for our filter was:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial \xi}[g^{\xi}(U_{\xi})U_{\xi}], \qquad (7)$$

where $\vec{\xi}$ denotes the eigenvector corresponding to the smallest eigenvalue of the gradient autocorrelation matrix:

$$M = \frac{1}{N}\begin{pmatrix} \sum_{i=1}^{N}(U_i)_x^2 & \sum_{i=1}^{N}(U_i)_x(U_i)_y \\ \sum_{i=1}^{N}(U_i)_x(U_i)_y & \sum_{i=1}^{N}(U_i)_y^2 \end{pmatrix}. \qquad (8)$$

$\vec{\xi}$ points to a direction orthogonal to the mean direction of the gradient vectors and its orientation *(θ)* represents the mean orientation of a structure passing through the pixel under study. As shown theoretically in [3], orientation estimation using (8) is highly robust against additive Gaussian like noise.

When additive noise is considered, provided that the orientation of the underlying oriented textures can be correctly estimated, the values of the directional derivatives U_ξ are depending only on the noise level and not on the local signal to noise ratio. Local maxima of $|U_\xi|$ are characterizing abrupt orientation changes (e.g. corners and junctions) whereas on region like areas $|U_\xi|$ depends only on the noise level. The directional interpretation of.(7):

$$\frac{\partial U}{\partial t} = [g^{\xi}(U_{\xi}) + g^{\xi}{}'(U_{\xi})U_{\xi}]U_{\xi\xi} = c_{\xi}U_{\xi\xi}, \qquad (9)$$

shows that, in each pixel, the equation acts as a one-dimensional diffusion process that smoothes *($c_\xi>0$)* oriented patterns with energy independent speed and can enhance corners and junctions for negative diffusion coefficients *($c_\xi<0$)*. The above discussion and described behavior of our filter are of course valid only if noise has low values. Only under this assumption maxima of $|U_\xi|$ can be directly associated to corners and junctions.

For dealing with images composed both of regions and oriented textures we proposed also a 2D version for our filter:

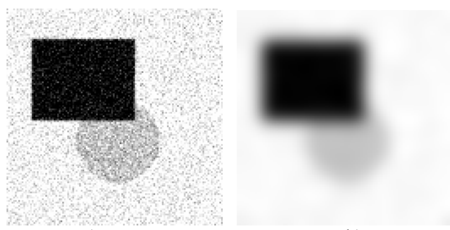
$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial \xi}[g^{\xi}(U_{\xi})U_{\xi}] + \frac{\partial}{\partial \eta}[g^{\eta}(U_{\eta})U_{\eta}] \qquad (10)$$

In (10) $\vec{\eta}$ denotes the eigenvector associated to the biggest eigenvalue of (8) i.e. the mean direction of the gradient vectors. In contrast to (7) the new equation allows smoothing of regions like areas and is capable of enhancing edges. (10) is essentially a superposition of 1D diffusion processes that, unlike divergence equations as (4), allows a complete control of its behavior. Different thresholds can be chosen on the two directions, different functions can be employed for $\vec{\eta}$ and $\vec{\xi}$ etc.

Influence of heavy tailored noise on the results obtained by diffusion processes can be diminished using Gaussian convolution (or equivalently an isotropic diffusion) in a pre-processing step. This was the solution we employed in [6] when proposing a regularized version for the equation:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial \xi}[g^{\xi}(\frac{\partial}{\partial \xi}U_{\sigma\xi})U_{\xi}] + \frac{\partial}{\partial \eta}[g^{\eta}(\frac{\partial}{\partial \xi}U_{\sigma\xi})U_{\xi}] \qquad (11)$$

Convolution with a Gaussian kernel is essentially a low pass filter and when embedding it in anisotropic diffusion processes some precautions have to be taken. Larger kernel sizes are efficiently filtering out spurious noise (Fig.1), but on the same time they are eliminating objects with spatial dimensions inferior to the standard deviation of the associated Gaussian function. Another drawback of Gaussian convolution is the fact that it produces inherent edge displacement (Fig.2) and, due to the edge enhancing term, a diffusion model based on (11) could produce artifacts such as false edges that can be further enhanced as time advances. If diffusion thresholds are chosen to have large values, by diminishing the gradient norms, Gaussian convolution forbids any edge enhancement process.



a) b)

Fig.1 Isotropic diffusion a) Original image b) Smoothed image

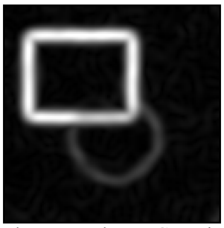


Fig.3 Edge displacement due to Gaussian convolution

For avoiding the above-mentioned effects we propose a method that employs a different pre-smoothing technique, based on a Perona and Malik filter. By denoting the solution of (4) at some instant t with $U_{PM,\,t}$ we consider the following evolution equation:

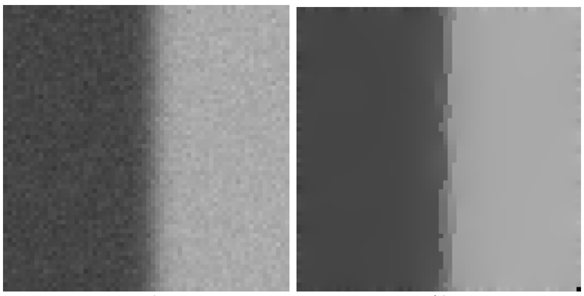
$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial \xi}[g^{\xi}(U_{PM,t_{\xi}})U_{\xi}] + \frac{\partial}{\partial \eta}[g^{\eta}(U_{PM,t_{\eta}})U_{\eta}] \quad (12)$$

Since an anisotropic diffusion process outperforms an isotropic one, we expect better results when filtering an image with the modified equation (12).

However, the influence of the two extra parameters for the Perona and Malik process – the scale t and the threshold K – is still to be discussed.

We showed in [6], [7] that our original filter produced better results than classical filters. We found experimentally out that the optimal results were obtained for limited sizes of the Gaussian kernel: $\sigma = 0.75 \div 1$. The pre-smoothing scale t for (12) can be thus chosen according to Koenderink's observation: $t = \sigma^2/2$. When implementing diffusion equations with explicit discrete schemes, a time step $dt = 0.2$ satisfies the stability constraints for the 2D case; this leads to a number of about 5 iterations that will introduce the same amount of smoothing but in an anisotropic way.

The choice of the threshold K can also influence strongly the results. An undesired effect that might appear in the so-called staircase effect, well documented for the anisotropic diffusion equation [11]. For particular choices of K, due to edge enhancement term, some contours might get irregular when processing the image with (4) (Fig.3).



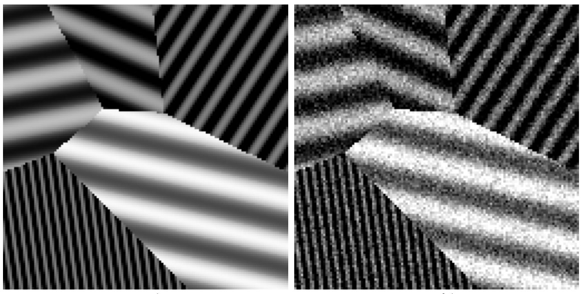
a) b)

Fig.3 Anisotropic diffusion. a) Original image
b) Irregular contours (staircase effect)

For the diffusion function (4) the effect appears only for gradient norms inferior to $\sqrt{3}K$ [11] and, thus, it can be avoided for sufficiently large K's. Following the technique indicated by Perona and Malik we choose to set K equal to some percentage (60 % in all our experiments) of the integral value of the gradient norms histogram [5].

III. EXPERIMENTAL RESULTS

PDE based models have a large number of parameters and comparisons between them are not always straightforward. A particular choice of parameters may suit well an image and could be less optimal for others. For solving this problem we took an experimental approach to solve this problem: we considered 15 randomly generated images, composed of oriented patterns, affected by Gaussian white noises and, for a given method and for each image, we searched for a best filtered result by allowing all parameters to vary. To quantify objectively the results we used the classical *PSNR* measure. Results from [10] are indicating that a 0.5dB improvement in terms of PSNR should be visible on the processed images. As processing methods we consider the new approach (12), its previous version (11) and the classical Perona and Malik filter (PM). The nature of the image we are interested in is shown on Fig.4.



a) b)

Fig.4 Image composed of oriented patterns a) Noise free image
b) Degraded image (*PSNR=15dB)*

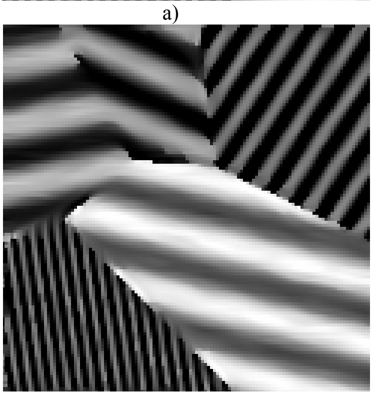
The noise levels and the *PSNR's* corresponding to the best filtered results for each considered method are shown on Table 1.

Table 1 PSNR values obtained for the 15 images under study

Image	Noise levels [dB]	Best filtered results - [dB]		
		PM	Original [6]	Proposed
1	16.66	25,3617	30,4611	30,8752
2	14.07	21,6275	24,7432	25,234
3	14.67	24,2511	25,8878	26,1256
4	15.60	23,6651	26,7267	26,9073
5	15.16	24,4292	26,3235	26,53745
6	13.68	24,4584	25,3846	26,0875
7	15.00	25,2881	27,8959	28,852
8	14.95	24,2299	26,3201	27,2197
9	14.64	26,1899	27,689	28,256
10	14.39	23,8604	26,0956	26,6131
11	14.85	24,6417	27,0079	27,5345
12	16.10	24,5689	26,4134	26,93394
13	13.27	22,0977	25,0166	25,485
14	16.65	26,2491	27,4969	28,0894
15	14.14	25,5648	25,7799	26,7116

Both our filters are outperforming the classical anisotropic diffusion equation. In terms of relative performances between our approaches, we obtain systematically better results with the new filter. Quantitatively, the improvements in terms of PSNR are ranging from 0.18dB (for the fourth image) to 0.95 dB (for the seventh image). The following PSNR's are obtained for the set of images: 24.43dB for the Perona and Malik filter, 26.61dB for the original approach and 27.16dB for the new equation. In terms of visual results, we are showing bellow the best filtered images corresponding to both our methods.

a)



b)

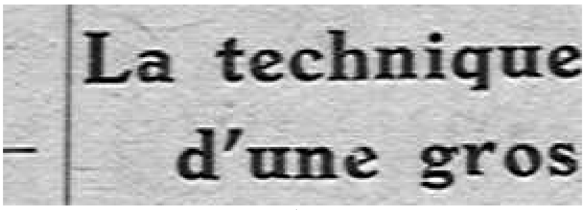
Fig.5 Best filtered results for the image from Fig.4.
a) Result obtained using (11): PSNR=27.89dB;
b) Result obtained using (12): PSNR=28.85dB

The improvement of almost 1dB is clearly visible and can be explained theoretically. The high frequency region placed on the bottom left hand side of the image strongly limits the Gaussian kernel's size. Consequently, on lower frequency regions Gaussian of the image pre-smoothing is unable to diminish the noise influence and, besides corners and junctions, local maxima of directional derivatives are appearing also on the oriented part of the image. Anisotropic diffusion based pre-smoothing does not suffer from the above mentioned effects and better results are obtained.
A question that may arise is related to the variability of the results from Table 1. A non-parametric two-way rank analysis of variance (ANOVA) [2] (Table 2) allows us to isolate the two sources of variability: the nature of the images and the different behavior of each method.
As the results from Table 2 are showing, more than 93 % of the variability between the obtained results is due to both the choice of the processing method and to the nature of the images. The two-way ANOVA allows us to isolate and investigate only the method effect. The extremely low probability *($p=4.9*10^{-13}$)* associated to a Fisher-Snédécour test *(F)* allows us to conclude that the processing method has a very significant influence over the quality of the processed result.

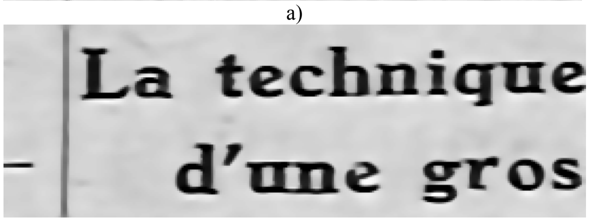
Table 2 Two way non-parametric ANOVA [2]

Source of variance	Sum of squares	Degrees of freedom	Mean squares	F	P
Total	7890.00	44	172.5		
Between images	3076.00	14	219.7		
Between methods	3917.73	2	1959	91,99	$4.9*10^{-13}$
Residual	596.27	28	21.30		

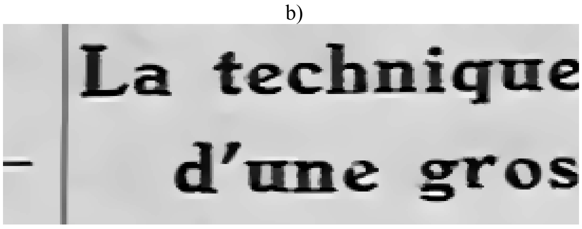
We are also interested in building a hierarchy for the analyzed methods. The mean ranks, computed for each method over the 45 measurements, are: 10.06 for the Perona and Malik filter, 27.2 for our original method and 31.73 for the improved one. The three ranks can be compared using a classical Student-Newman-Keuls post-hoc test (SNK)[2]. Its critical values, computed for a 5% risk, are: 3.45 for comparing two consecutive ranks and 4.16 for comparing two values spanning three ranks. Using the SNK test we can that conclude that the new method is better that the original one and that both our methods are significantly better than the original anisotropic diffusion equation.
Some results obtained for a real gray scale image are shown in Fig.6. Starting from the original image (Fig. 6.a), containing both oriented patterns and region like areas, we first processed it with the improved filter (Fig. 6.c). We then considered the filtered result as an original noise free image and searched for the choice of parameters for the original version that produces the closest result in terms of PSNR (Fig. 6.b).



a)



b)



c)

Fig.6 Results for a real image. a) Original image
b) Result obtained using (11) c) Result obtained using (12)

A different effect is observable. The Gaussian regularization employed by our original formulation smoothes edges of the processed image and leads to a slightly blurred result. The new formulation does not suffer from this effect since edges are kept better when using anisotropic diffusion pre-smoothing. The filtered result is not blurred and background is also more efficiently filtered.

A third experiment deals with a color image shown in Fig.7. The degradations are much more severe than for the image in Fig.6. and they are consisting in moiré effects and blocking artifacts, dues to the insufficient resolution of the scanning device and to the presence of high frequency details.

Fig.7 Color image

For processing the image we implemented a straightforward extension of the algorithm presented in section II. The image is first decomposed on the constituent red, green and blue channels; each channel is then processed with the same set of parameters and the filtered results are then recomposed.

In Fig. 8, Fig. 9.a and Fig. 9.c we are presenting respectively the original red channel image and the filtered results corresponding to both our approaches. We used the same approach for establishing the parameters: first we computed a result with the proposed method that we judged the best and then we searched for those parameters of the Gaussian formulation of our filter that are producing the closest results in terms of PSNR values.

Fig.8 Green channel of the image in Fig. 7

Once again the new approach proves to be more efficient than its Gaussian formulation. Even if edges are more regular when preprocessing with the Gaussian filter, the anisotropic diffusion formulation allows true edge enhancing and efficient background restoration.

a)

b)

Fig.9 Results for the green channel a) Original image Result obtained using (11) c) Result obtained using (12)

a)

b)

Fig.10 Results for a color image a) Result obtained using (11) b) Result obtained using (12)

The combined effect of processing all the color channels is illustrated in Fig.10. The same behavior is observable; the anisotropic diffusion based preprocessing allows a better restoration of both regions like areas and of important edges in the image.

IV. CONCLUSIONS

We proposed a technique that employs anisotropic pre-smoothing of an image prior to the estimation of the diffusivity function of outer diffusion processes. Considering this technique in the framework of directional diffusion we showed that better results can be obtained when compared to classical Gaussian regularization. The described technique can be employed to any diffusion equation that requires a pre-smoothing step.

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