

## On the Precision in the Determination of the Movement Features by Doppler Radio–Telemetry

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**Abstract** – For determining the features of a movement at low speed in the atmosphere, the authors have proposed a Doppler radio-telemetry system with active fixed referential. This paper deals with the precision in the determination of the movement features at low speed in the atmosphere, using electromagnetic waves. The establishing of the systematic errors and the appreciating of the random errors, represents the authors' contribution in this paper.

**Keywords:** Doppler signal, systematic errors, random errors.

### I. INTRODUCTION

The authors have proposed the structure of a Doppler radio-telemetry system with active fixed referential for the determination of the movement features at low speed in the atmosphere, using electromagnetic waves. The authors' contributions in the establishing of this structure, presented in Fig.1, are detailed in [1]. The main contribution consists in the selection of the movement to be evaluated from other movements taking place simultaneously in the same space by the activation of the fixed referential in that it produces the change of frequency  $f_1$  into  $f_2$  and its modulation in DSB-SC with the low frequency  $f_j$ .

The signal  $s_{e1}(2\pi f_1 t + \varphi_0)$ , generated by the frequency synthesizer with PLL1, is emitted by the antenna A1

on the mobile referential (MR) to the active fixed referential (AFR). The signal received by the antenna A2, on the AFR, is phase modulated by the longitudinal Doppler effect caused by the movement of the MR in relation to the AFR; the Doppler shift in frequency,  $\pm\Delta f_1$ , is retrieved into the signal  $s'_{e2}[2\pi(f_2 \pm \Delta f_2)t + \varphi_0]$  through  $\Delta f_2 = (f_2 - \Delta f_1)/f_1$ . The signal  $s_{e2}[2\pi(f_2 \pm \Delta f_2)t \pm 2\pi f_j t \pm \varphi_j + \varphi_0]$  is emitted by the antenna A2 to the antenna A1 and the signal received by the antenna A1 is again phase modulated by the longitudinal Doppler effect. At the output of the Doppler signal extractor, DSE, the signals:  $s_D$  - Doppler signal and  $s_{axi}$  - Doppler shift in frequency axing signal are obtained. The two signals are represented by the relations:

$$s_D = A_D \sin(8\pi\Delta f_2 t) \quad (1)$$

$$s_{axi} = 2A_m \cos[2\pi(f_j \pm 2\Delta f_2)t + \varphi_j] \quad (2)$$

The movement features – instant speed, average speed and distance covered in the time interval  $\Delta t$  are determined by indirect measurement based on the relations:

$$v = \frac{v_f}{f_2} \cdot \frac{1}{8\pi} \cdot \frac{\partial}{\partial t} \left[ \arcsin\left(\frac{s_D}{A_D}\right) \right] \quad (3)$$

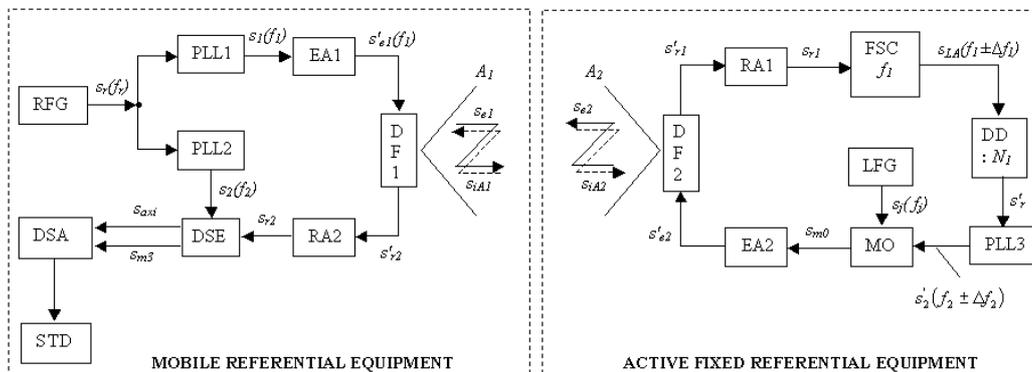


Fig.1. The structure for the Doppler radio-telemetry system with active fixed referential: RFG – reference frequency signal generator; LFG – low frequency signal generator; PLL1, 2, 3 – frequency synthesizers; EA1, EA2 – emission amplifiers; DF1, DF2 – duplex filters; RA1, RA2 – reception amplifiers; FSC $f_j$  –  $f_j$  frequency selection circuit; DD – digital frequency divider; MO – DSB-SC modulator; DSE – Doppler signal extractor; DSA – Doppler signal analyzer; STD – block for showing and transmitting data.

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$$v_{\text{med}} = \frac{1}{T} \int_0^{\Delta t} v \cdot dt \quad \text{and} \quad D = v_{\text{med}} \cdot \Delta t \quad (4)$$

The movement sense related to the AFR, approach or getting away, is indicated by the increase of the  $f_j$  frequency with  $2\Delta f_2$  for approach and the decrease of the  $f_j$  frequency with  $2\Delta f_2$  for removing from the fixed referential. The determination of these frequencies is made based on the axing signal  $s_{axi}$  of the Doppler shift in frequency in the Doppler signal analyzer, DSA.

The structure of the Doppler radio-telemetry system with active fixed referential for the determination of the movement features, presented in Fig.1, can be reduced to the block diagram given in Fig.2.

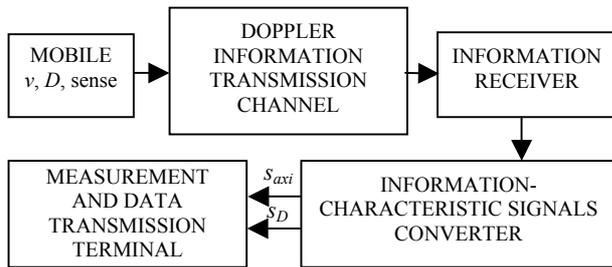


Fig. 2. The block diagram of the telemetering process of the movement features with a Doppler radio-telemetry system with an active fixed referential

The transmission channel of the Doppler information contains the whole chain, presented in Fig.1, from the  $f_1$  frequency signal generator to the antenna A2 which plays the role of receiving the electromagnetic signal with the  $f_2$  carrier frequency. In other words, the transmission channel of the Doppler information contains:

- the blocks on the mobile referential with the role of emitting electromagnetic signal with  $f_1$  frequency (noted in Fig.1 with RFG, PLL1, EA1, DF1, A1);
- the propagation space of the electromagnetic signal with the  $f_1$  carrier frequency from the antenna A1 to the antenna A2;
- the blocks on the active fixed referential with the role of receiving, frequency changing and emitting electromagnetic signal with  $f_2$  carrier frequency (noted in Fig.1 with A2, DF2, RA1, FSC $f_i$ , DD, PLL3, LFG, MO, EA2, DF2, A2);
- the propagation space of the electromagnetic signal with the  $f_2$  carrier frequency from the antenna A2 to the antenna A1.

The information receiver contains the blocks on the mobile referential which plays the role of receiving the electromagnetic signal with the  $f_2$  carrier frequency. The information-characteristic signals converter is, in fact, the extractor of Doppler signal, DSE, and the measurement and data transmission terminal is the Doppler signal analysis block, DSA.

The Doppler information transmission channel, the information receiver, the information-characteristic signals converter and the measurement and data transmission terminal contain professional and fast

electronic blocks, meaning that the response speed of the electronic circuits is very high, resulting a very short response time, less than 10 ns. In these conditions it can say that the speed at certain moment can be considered constant during  $\tau_p$  of a measurement series with the duration of one measurement  $\tau_m \ll \tau_p$ .

The measurement error  $\Delta v_i$  is defined as

$$\Delta v_i = v_i - v_0 \quad (i = 1, 2, \dots, n) \quad (5)$$

where  $v_i$  is the value obtained by the measurement  $i$ , while  $v_0$  is real value of the physical parameter measured, generally unknown.

An estimate of the real value using a  $n$  measuring number can be the arithmetic mean value:

$$\bar{v} = \frac{1}{n} \sum_{i=1}^n v_i \cong v_0 \quad (6)$$

If the errors are random and uncorrelated it is demonstrated that by mediation the dispersion of the measurement error is reduced by  $n$  times.

Between the errors that occur from the measurements and the parameters that characterize the measurement physic process appear bindings with functional and random character. Therefore, the measuring error  $\Delta v_i$  must be perceived as a sum of systematic, random and sometimes even rough errors.

In conclusion, the determination precision of the movement features is given by establishing the systematic and random errors that occur.

## II. SYSTEMATIC ERRORS

The systematic errors can be: *instrumental*, *methodical*, *exterior* and *rough*. They have a cumulative character and therefore it is necessary to establish carefully and accurately the components and to take measures for eliminating or at least reducing their substantially.

The **instrumental errors** are due to, on the one hand, that it is made an indirect measurement of the velocity by using an electromagnetic wave phase-modulated by longitudinal Doppler effect, and, on the other hand, the relations (3) and (4) are numerically solved in the block DSA.

The instrumental error due to the indirect measurement depends on the form of the electromagnetic wave and the signal-to-noise ratio at the output of the reception amplifiers RA1 and RA2. The electromagnetic signal is generated by the frequency synthesizer with PLL1, respective, PLL3 and has a correct sinusoidal waveform. Through the quality diplexer filters, through the band-pass amplifiers RA1 and RA2 and through the band-selector filters of the modulators and demodulators, the signal waveform remains sinusoidal, the distorting components being insignificant.

The signal-to-noise ratio at the output of the reception amplifiers being of minim 40 dB, so, the influence of the noise is, also, insignificant.

The instrumental errors due to the DSA block refer to: a) the quantization error of the analog/digital conversion of the signal  $s_D$ ; b) the error in knowing the  $f_2$  frequency and of its stability in time and with temperature; c) the error in knowing the propagation speed of the electromagnetic wave in the atmosphere,  $v_f$ ; d) the error in knowing the time interval  $\Delta t$ ; e) the error of the digital/analog conversion of parameters  $v$ ,  $v_{med}$ ,  $D$ .

The propagation speed  $v_f$  of the electromagnetic wave in the atmosphere is known with an accuracy  $\gamma_{vf} = 0.033\%$ . The carrier frequency  $f_2$ , due to the frequency synthesizer with PLL3, is known with the accuracy of the assigned frequency  $f_r$ ; this is given by a quartz oscillator and it is known with an accuracy of more than 0.05%, by using a quartz resonator of 10 MHz with three decimals.

Taking into account the quantization errors  $\gamma_{qd}$ , the accuracy of the analog/digital conversion  $\gamma_{ad}$  and the solving by digital method of the equation of the movement speed (relation (3)), results  $\gamma_v$  - the precision in determination of the instant speed of the mobile (mobile referential):

$$\gamma_v = \gamma_{vf} + \gamma_{f_2} + \gamma_{qd} + \gamma_{ad} < 0.5\% \quad (7)$$

When determining the covered distance  $D$  we also must consider the error in knowing the time interval  $\Delta t$ . As  $\Delta t$  is determined digital, considering the tact frequency given by the reference frequency  $f_r$ , it can appreciate that the distance  $D$  can be determined with an accuracy better than 0.5%.

For estimating the error due to the temperature variation, greater than the error in knowing the time interval, we must take into account that:

$$v = F(v_f, f_2, s_D, A_D) \quad (8)$$

The influence of the temperature  $\theta$  on the measurement accuracy can be determined taking into account the variation of these parameters with temperature [2, 3, 4]. This leads to:

$$v_\theta = F(v_{f_0}, f_{20}, s_{D0}, A_{D0}) + \gamma \cdot \theta \quad (9)$$

where

$$\gamma = \alpha \cdot v_{f_0} \frac{\partial F}{\partial v_f} + \beta \cdot f_{20} \frac{\partial F}{\partial f_2} + \delta \cdot s_{D0} \frac{\partial F}{\partial s_D} + \mu \cdot A_{D0} \frac{\partial F}{\partial A_D} \quad (10)$$

representing the coefficient of temperature for the block Doppler signal analyzer, DSA. The parameters -  $v_{f_0}$ ,  $f_{20}$ ,  $s_{D0}$  and  $A_{D0}$  are the values for normal temperatures, and  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\mu$  are the temperature coefficients of these parameters.

The variation with temperature of the propagation speed of the electromagnetic waves in the atmosphere is less than 0.1% and therefore it is insignificant [3, 4]. The variation with temperature of the frequency  $f_2$

is given by the variation with temperature of the quartz resonator, which is kept at the same temperature. So, we can say that the variation with temperature of the frequency  $f_2$  is insignificant. The other two terms,  $\delta$  and  $\mu$ , are also very low due to using the professional integrated circuits with a good rejection of the temperature influence.

However, as a preventive measure, the electronic equipment on the RM is shelters into a temperature-controlled oven, and the electronic equipment on the RFA is build in compact construction and maintained within certain limits of temperature.

In these conditions, it can consider that the influence of the temperature on the precision in the determination of the movement parameters is insignificant.

The **methodical errors** are due to the fallibility of the measurement method used and they appear especially when the measurement is indirect. It is consider theoretical that the method of measurement is described by the function

$$y = F(v, l, m, \dots, q) \quad (11)$$

where  $v$  is the measuring parameter,  $y$  is the result of the measurement and  $l, m, \dots, q$  are physical parameters which can vary during the measurement process, and this leads to the methodical errors.

When calibrating the Doppler radio-telemetry system with active fixed referential, it is considering the reference values  $l_0, m_0, \dots, q_0$  such as

$$y_0 = F(x, l_0, m_0, \dots, q_0) \quad (12)$$

$$y = y_0 + \Delta y = F(x, l_0 + \Delta l, m_0 + \Delta m, \dots, q_0 + \Delta q) \quad (13)$$

and by the development in Taylor series and neglecting the infinitely minute of superior order of 1, the methodical errors  $\Delta y$  is obtained:

$$\Delta y = \frac{\partial F}{\partial l} \cdot \Delta l + \frac{\partial F}{\partial m} \cdot \Delta m + \dots + \frac{\partial F}{\partial q} \cdot \Delta q \quad (14)$$

The relation (14) shows that the value of the methodical error is determined both the offsets  $\Delta l$ ,  $\Delta m, \dots, \Delta q$  of the physical parameters taken into account and the character of the variation of the function  $F$  in respect with these parameters.

In the event of the Doppler radio-telemetry system with active fixed referential these parameters are: the linearizing action of the Doppler effect; the Doppler shift of the low frequency  $f_j$  of the signal  $s_{e2}$ ; the directivity of the antennas on the two referentials; the quality of the diplexer filters; the parasitic signals from the official frequency field near of the working frequencies  $f_1$  and  $f_2$ ; the offset of the reference frequency  $f_r$  due to the Doppler shift in frequency  $\Delta f_i/N_1$  in PLL3 on AFR. The first two parameters are more important and they will be studied below.

The methodical error due to the linearizing action of the Doppler effect appears by using relation (16) instead of relation (15):

$$\Delta f_1 = f_1 \left( \sqrt{\frac{1 - v/v_f}{1 + v/v_f}} - 1 \right) \quad (15)$$

$$\Delta f_1 = -f_1 \cdot \frac{v}{v_f} \quad (16)$$

At low speeds,  $v \leq 62$  m/s, the speed ratio  $v/v_f \leq 2.07 \cdot 10^{-7} \ll 0.02$ . In [1] it is shown that for  $-0.02 < v/v_f < +0.02$ , the dependence of the Doppler shift in frequency on the movement speed is linear; this means that, for low speeds taken into consideration, the error due to the linearizing action of the Doppler effect is insignificant.

The methodical error due to the Doppler shift for the frequency  $f_j$  given the situation for  $f_2$ , can be appreciated taking into account the usual values for  $f_j$  and  $f_2$  presented in [1]. From relation (14), for  $f_j = 400$  Hz and  $f_2 = 160$  MHz, it obtains  $\Delta f_j / \Delta f_2 = f_j / f_2 = 2.5 \cdot 10^{-6}$ , so that, it can consider the methodical error by the neglect of  $\Delta f_j$  as being insignificant.

In the choice of the structure presented in Figure 1 and the electronic equipment on the two referentials it kept in view also the minimization of the methodical errors due to the other parameters mentioned above. Therefore it can say that methodical errors in the proposed telemetering system are very small compared to instrumental errors.

The **external errors** are due to the external factors of the Doppler radio-telemetry system, such as atmospheric changes. These do not change the propagation speed of the electromagnetic waves, the longitudinal Doppler effect and the work of the electronic equipment. When the wind speed is considerable (winds are powerful), the movement speed of the mobile is higher or lower, depending on the direction in which the wind blows: whether it's in the direction of the motion or in the opposite direction. In these conditions the movement of the mobile is strong affected by the external factors.

The **rough errors** are caused by some break-downs in the electronic equipment, in the transmission and reception aerials or the encroachment of the general measurement principles. The identification criterion of these errors is based on the fact that the measurement results are very different from the results of the other measurements.

These values must be eliminated from the performed-measurement series. If they persist, it must fix the problem with the electronic equipment.

### III. RANDOM ERRORS

The random errors are due to different causes whose individual influence is not easy to see, the experiments putting into evidence only their random appearance. These errors may be positive or negative,

grouped around the zero value, and they are according to the known statistical laws [3, 4]. The random errors are very important in the telemetering precision of the movement features and this is why they have been carefully researched.

The parameter that must be measured is the movement speed  $v$ , which can be considered practically constant during  $\tau$ , of a measurement series with the duration of a measurement  $\tau_m \ll \tau$ . Consequently, it can admit that during  $\tau$ , there are made  $n$  individual measurements  $v_i$  with  $v_i \in R$  ( $i = 1, 2, \dots, n$ ). Due to the random errors, the most probable value of the speed  $v$  it is considered as being  $v_p$  and the random errors in the  $n$  individual measurements can be estimated by  $\alpha_i \in R$  ( $i = 1, 2, \dots, n$ ), where

$$\alpha_i = v_i - v_p \quad (17)$$

It is considered  $\xi$  as being the stochastic-continuous variable, with the repartition function  $F$  and the repartition density  $f$ . It is very important to define the variation limits of the random errors  $\alpha_i$  and the probability that  $\alpha_i$  to be between these limits; in these conditions it can take into account the random errors, the causes of their appearance and it can give solutions for diminishing them. In order to do that, it must determine the repartition function  $F$  and the density of the random errors repartition  $f$ .

It is established an infinitesimal variation  $h$  and it is considered the probability  $P$  as to the stochastic variable to be find in the interval  $[v, v + h)$ . It can say that:

$$P(v \leq \xi < v + h) = \int_v^{v+h} f(v) dv \quad (18)$$

which means that, the stochastic variable being in the variation interval of the speed, the density function of the stochastic-variable repartition becomes the density function of the speed repartition,  $f(v)$ , and the repartition function  $F(\xi)$  becomes  $F(v)$ .

For  $h$  low enough and for the continuity of the function  $f(v)$ , in the conditions in which  $\alpha_i \in (v, v + h)$  with  $i = 1, 2, \dots, n$ , it can appreciate that the probability  $P$  as to the stochastic variable  $\xi$  to be find in interval  $[v, v + h)$  is:

$$P(v \leq \xi < v + h) = h \cdot f(v) \quad (19)$$

The probability  $p$  as to the  $n$  random errors  $\alpha_i$  to be determined with an error  $\beta \in (\alpha_i, \alpha_i + h_i)$  with  $i = 1, 2, \dots, n$ , it can put in the relation:

$$p = \prod_{i=1}^n h_i \cdot f(\alpha_i) \quad (20)$$

where  $f(\alpha_i)$  is the repartition density of the random errors  $\alpha_i$ .

Because  $h_i > 0$ , but very small (infinitesimal),  $p$  may represent, concomitantly, the probability as the speed  $v$  to be near of the most probable value  $v_p$ . Therefore:

$$p(v) = \prod_{i=1}^n h_i \cdot f(v_i - v_p) \quad (21)$$

In hypothesis that the function  $f(v_i - v_p)$  is derivable, the probability  $p$  is maximum if the derivative of the function  $p(v)$  in report with  $h_i$  is nullified for  $v_p = v_0$ , where  $v_0$  is expressed with relation (6).

The repartition density function of the random errors  $\alpha_i = u_i = v_i - v_0$  [2], is:

$$f(u_i) = f(\alpha_i) = \frac{1}{r\sqrt{2\pi}} \cdot \exp\left(-\frac{u_i^2}{2r^2}\right) \quad (22)$$

It is noted that de relation (22) represents, in fact, the density of normal repartition  $N(0, r)$ . The function of normal repartition  $N(m, r^2)$  is the same with  $F(v)$ , namely

$$F(v) = \frac{1}{r\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{(t-m)^2}{2r^2}\right) dt \quad (23)$$

$$\text{with } m = \int_{-\infty}^{+\infty} v \cdot f(v) dv$$

and it satisfies the Laplace and Gauss Theorem. The function

$$\overline{\Phi}(v) = \frac{2}{\sqrt{\pi}} \int_0^v \exp(-u^2) du \quad (24)$$

is called the error function, and the constant  $h$ , from the relation (19), takes the value

$$h = \frac{1}{r\sqrt{2}} \quad (25)$$

and is called the measurement precision (precision module) from the point of view of the random errors.

For the evaluation of the dispersion of variable  $v$ , noted with  $r^2$ , are used the  $n$  individual values  $v_i$ , obtained thorough the measurement, keeping account of the definition of the dispersion  $r^2$  as the dispersion in report with the mean value  $m$ . It results [2, 3]:

$$\begin{aligned} r^2 &= \sum_{i=1}^n P(v = v_i) \cdot (v_i - v_0)^2 = \\ &= \frac{1}{n} \sum_{i=1}^n (v_i - v_0)^2 \end{aligned} \quad (26)$$

With  $r$  thus obtained is determined the precision  $h$  and the probability as the absolute value of the apparent error to be contained in the interval  $[a_0, b_0]$  with  $a_0 \geq 0$  is calculated with the relation

$$P(a_0 \leq |\alpha_i| < b_0) = \overline{\Phi}(hb_0) - \overline{\Phi}(a_0) \quad (27)$$

For a number  $n$  of sufficient great measurements ( $n \rightarrow \infty$ ) it can be appreciated as good the equalization of the probable value of the measuring parameter with the mean value of measurements.

Practically speaking, however, the number of measurements is finite, and as result, the arithmetic mean  $v_0$  is departed off the probable value  $v_p$  with an random error. On the basis of the previous relations, it may be deduced relations for the parameters that characterize this real state, the error  $S_r$  on the result, the probable error  $R$  of the result, the average error  $T$  and the value of the value of the limit random error  $\alpha_{lim}$ . These relations are:

$$\begin{aligned} S_r &= \pm \sqrt{\sum_{i=1}^n \frac{u_i^2}{n(n-1)}}, & R &= \frac{2}{3} S_r, \\ T &= \pm \sqrt{\frac{2}{\pi}} \cdot S_r, & \alpha_{lim} &= 3S_r = 4,5R \end{aligned} \quad (28)$$

Choosing the interval for the variation of the random error  $[-3r, +3r]$ , the probability that this error to be inside this interval will be obtained by the integration of the relation (27) between these limits. It results:

$$P(-3r \leq \alpha_i < +3r) = 0.9972$$

This means that the random errors will be inside the interval  $[-3r, +3r]$  with a probability of 99.72 %.

The dispersion  $r$  of the measured value, of the speed  $v$  around the average value  $v_0$ , is, concomitantly, also the random error calculated with the little squares method. According to this method of determination of the measured error, the best estimate of the parameter  $v$  is the value for which the sum of the squares of the random errors is minimum. This means that the derivative of the sum of the squares of the random errors in report with the probable random value  $v_p$  is zero.

In these conditions,  $v_p = v_0$  and  $\alpha_i = u_i = v_i - v_0$ . It is seen that the value  $v$  for which the sum of the squares of the random errors is minimum represents the arithmetical mean  $v_0$  of the  $n$  experimental values  $v_i$ . The mean square error of the measurements, taking into account the random variable  $\xi$ , is

$$\sigma_v = \sqrt{\frac{1}{n} \sum_{i=1}^n (v_i - v_0)^2} = r \quad (29)$$

and it can be appreciated that, the limits between which are situated the true value of the instant movement speed is given by the relation:

$$v = v_0 \pm r \quad (30)$$

In reality, the speed  $v$  is not directly determined, but through the measurement of the Doppler signal  $s_D$  and of its amplitude  $A_D$  (relation (3)). Because it has been supposed that for an instant speed  $v$ ,  $n$  individual measurements are made, it is possible to say that the  $n$  values of the speed have been obtained through measuring directly  $n$  signals  $s_{Di}$  and  $n$  amplitudes  $A_{Di}$ , with  $i = 1, 2, \dots, n$ . In this case the calculation of the random errors is modified.

First of all, it has to determined the arithmetic mean value for each one of the two measurable variable directly:

$$s_{D0} = \frac{1}{n} \sum_{i=1}^n s_{Di} \quad \text{and} \quad A_{D0} = \frac{1}{n} \sum_{i=1}^n A_{Di} \quad (31)$$

and then the mean square errors of the random errors are determined with the relations:

$$\sigma_{SD} = \sqrt{\frac{1}{n} \sum_{i=1}^n (s_{Di} - s_{D0})^2} \quad \text{and} \quad (32)$$

$$\sigma_{AD} = \sqrt{\frac{1}{n} \sum_{i=1}^n (A_{Di} - A_{D0})^2}$$

According to relation (3), the dependency of  $v$  to the two measurable variable,  $s_D$  and  $A_D$ , is not a linear function but unlinear one (arcsin); in these conditions:

$$v = V(s_D, A_D) \quad (33)$$

where

$$V(s_D, A_D) = \frac{v_f}{f_2} \cdot \frac{1}{8\pi} \cdot \frac{\partial}{\partial t} \left[ \arcsin \left( \frac{s_D}{A_D} \right) \right] = (34)$$

$$= C \cdot \frac{\partial}{\partial t} \left[ \arcsin \left( \frac{s_D}{A_D} \right) \right]$$

because the parameters  $v_f, f_2$  are not related to any random errors, they are determined from the beginning with a very good precision.

The errors at the two measurable variable directly are noted with

$$\alpha_i = s_{Di} - s_{D0} \quad \text{and} \quad \beta_i = A_{Di} - A_{D0} \quad (35)$$

with the mention as that  $\sum \alpha_i = \sum \beta_i = 0$ .

It results:

$$v_i = V(s_{Di}, A_{Di}) \quad \text{and} \quad (36)$$

$$V(s_{D0} + \alpha_i, A_{D0} + \beta_i)$$

and developing in Taylor series the relation (36), in hypothesis that the  $\alpha_i$  and  $\beta_i$  errors have values smaller than 1, it is obtained :

$$\sigma_{pv} = \sqrt{s_{D0}^2 \cdot \sigma_{pSD}^2 + A_{D0}^2 \cdot \sigma_{pAD}^2} \quad (37)$$

So, in the conditions of indirect measuring of momentary speed using the two measurable variable directly,  $s_D$  and  $A_D$ , the value of the speed is:

$$v = V(s_{D0}, A_{D0}) \pm \sigma_{pv} \quad (38)$$

#### IV. CONCLUSIONS

The theoretical consideration of the systematic and random errors, through experimentation proved to be correct and it demonstrated the good knowledge of the performing electronic equipment used, as well as the

correct choice of the structure of the Doppler radio-telemetry system with active fixed referential for the telemetering of the small speed movements characteristics.

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