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Controller Design for Single Phase APF Circuits

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Abstract – In the present paper is developed a small signal averaged model for a single phase APF circuit. The model can be applied to both unipolar and bipolar topologies. Based on the APF model, a controller design strategy is given, controller that can be implemented in a simple way with good dynamic performances. There are also presented some simulation results for a single phase APF circuit with integration unipolar control.

Keywords: power factor, active power filter, small signal averaged model

I. INTRODUCTION

Active Power Filters (APF) are part of the unity power factor converters. The APF circuits, placed in parallel with the load, compensates the loads current reactive component and harmonics, excepting the fundamental, in order to provide a mains current averaged valued in phase and proportional with the mains voltage. Due to the parallel connection, only a fraction of the load power will be processed by the APF circuit, leading to smaller components stress and switching losses compared with the series PFC circuits.

There are various control methods for the APF circuits, many of them based on a current loop witch implies the existence of a reference current signal. But, the integration control method, presented in this paper, does not need a reference current, being more simple to be implemented [1], [2]. The basic structure of a single phase APF circuit is presented in Fig.1.

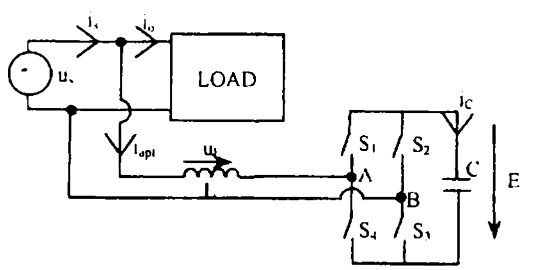

Fig.1. Single phase APF circuit

The central element of the circuit is represented by the controlled bridge, characterized by an alternative, i_{apf}, absorbed current which compensates the, i_o, load current phase shift and harmonics. This will finally lead to a, i_s, input current in phase and proportional with the main voltage. The capacitors value, C, could be considered large enough in order to admit the voltage across it, E, as a constant value. The voltage waveform shape between the A and B bridge terminals depends on the type of control.

In the case of bipolar control the switches are diagonally synchronous controlled at, f_s, switching frequency (S_1 and S_3, respectively S_2 and S_4), the u_{AB} voltage value varies between E and $-E$ in both half-periods of the mains voltage.

In the unipolar control method, in the first half-period of the mains voltage S_3 is permanently on and S_2 is off, the other two switches (S_1 and S_4) are controlled at the, f_s, switching frequency, u_{AB} varies between 0 and E. On the second half-period, S_2 is on and S_3 is off and S_1 respectively S_4 are controlled at f_s, u_{AB} varies between 0 and $-E$.

II. SINGLE PHASE APF CIRCUITS WITH INTEGRATIVE CONTROL

For the purpose of integration control the switching frequency must be chosen much higher than the mains frequency in order to admit that the output voltage can be considered to be constant during a switching period. The voltage across the inductor, L, averaged value is:

$$\overline{u_L} = L\frac{d\overline{i_{apf}}}{dt} \qquad (1)$$

If the input current, i_s, does not have steep transitions during a mains voltage period, the i_{apf} current will share the same properties and the averaged value on a switching period of this current can be considered to be constant as it follows:

$$\frac{d\overline{i_{apf}}}{dt} = 0 \Rightarrow \overline{u_L} = 0 \qquad (2)$$

For both integration control methods, unipolar and bipolar, there can be developed control circuits witch are presented in Fig.2a for bipolar control and Fig.2b for unipolar control, [1], [2].

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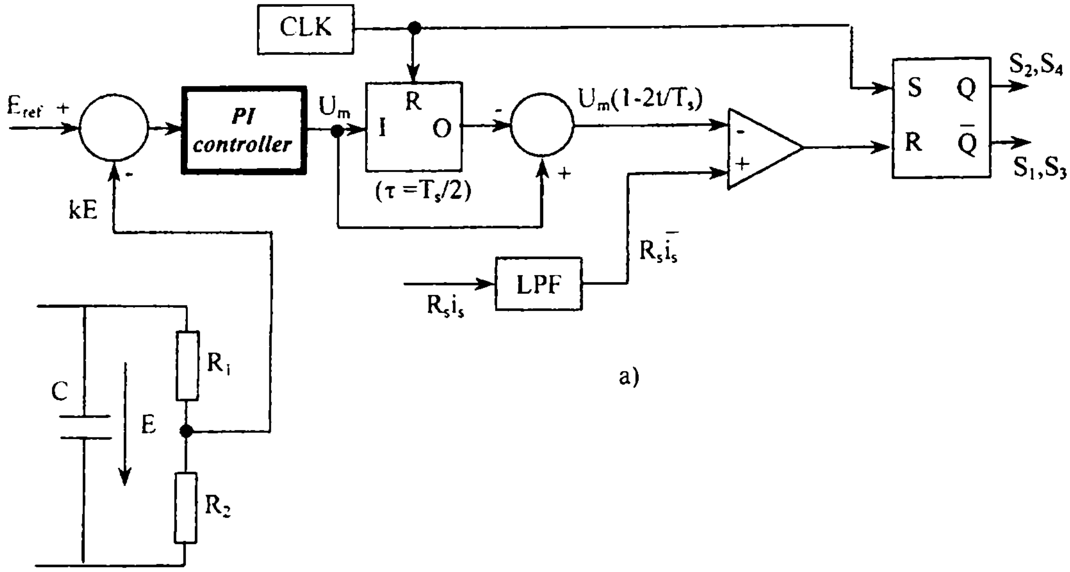


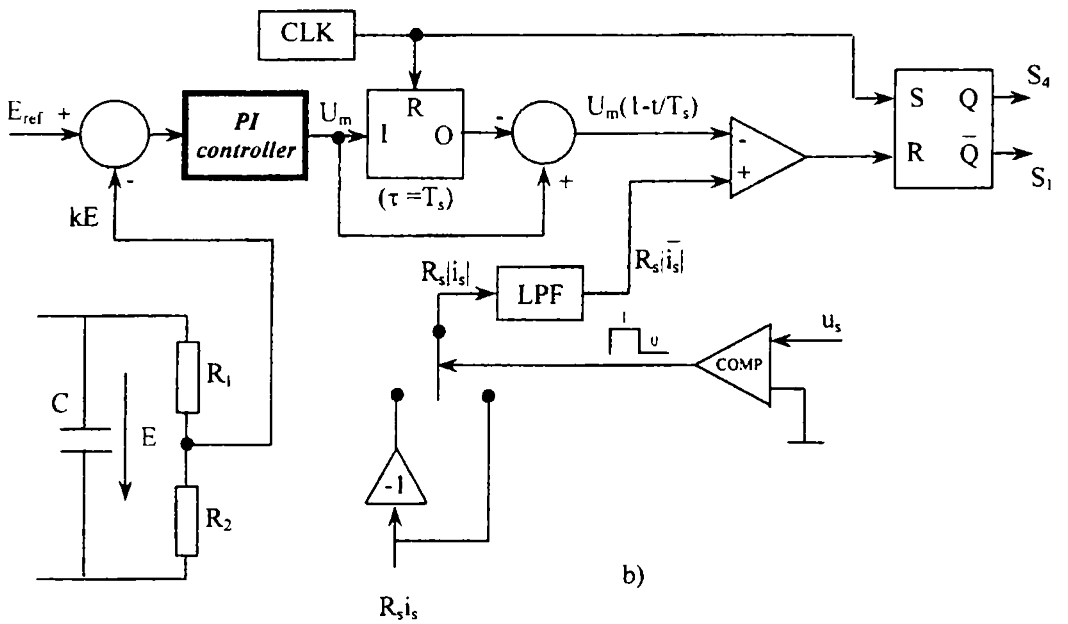


Fig.2a-Block diagram for integration bipolar control
b- Block diagram for integration unipolar control

In the block diagrams presented above, E_{ref} is the APF bridge reference voltage, R_s is the current sensing resistor, U_m is the control voltage (depending on E, R_s and the emulated resistance R_e) and the mains supply voltage is $u_s = U_M sin\omega t$, $\omega = 2\pi/T_{ac}$.

III. CONTROLLER DESIGN FOR INTEGRATION CONTROL

For a classical design of the controller, the transfer function between the voltage across the capacitor and the control voltage must be known. Because of the APF circuits non-linearity in the first step it must be developed a large-signal non-linear time invariant model which will be then linearized considering some simplified assumptions. It is generally admitted that the efficiency equals the unity. Writing the instantaneous powers equality (neglecting the power across the inductor, because the inductors value is small, being designed for the high switching frequency) it results:

$$p_i = p_o + p_L + p_C \Leftrightarrow \frac{u_s^2}{R_e} = p_o + ei_C \quad (3)$$

and knowing that $R_e = eR_s/u_m$, [1], it leads to:

$$\frac{u_s^2 u_m}{R_s} = ep_o + e^2 i_C \quad (4)$$

The relation above can be averaged with respect to T_s and knowing that u_s, u_m and e have slow variations it will result:

$$\frac{u_M^2(1-\cos 2\omega t)u_m}{2R_s} = e\overline{p_o} + e^2\overline{i_C} \quad (5)$$

If the last relationship is averaged with respect to the mains half-period, the term containing $\cos 2\omega t$ will disappear and the result will be:

$$\left\langle \frac{u_M^2 u_m}{2R_s} \right\rangle_{\frac{T_{ac}}{2}} = \left\langle e\overline{p_o} \right\rangle_{\frac{T_{ac}}{2}} + \left\langle e^2 \overline{i_C} \right\rangle_{\frac{T_{ac}}{2}} \quad (6)$$

Overimposing on the stationary state values small perturbations, characterized by the "^" symbol, the following equations can be written:

$$\begin{cases} \langle u_M \rangle_{\frac{T_{ac}}{2}} = U_M + \hat{u}_M \\ \langle u_m \rangle_{\frac{T_{ac}}{2}} = U_m + \hat{u}_m \\ \langle e \rangle_{\frac{T_{ac}}{2}} = E + \hat{e} \\ \langle p_o \rangle_{\frac{T_{ac}}{2}} = P_o + \hat{p}_o \\ \langle i_C \rangle_{\frac{T_{ac}}{2}} = I_C + \hat{i}_C \end{cases} \quad (7)$$

Replacing (7) in (6) and neglecting the terms that contain second order perturbations it will result:

$$\frac{U_M U_m}{R_s}\hat{u}_M + \frac{U_M^2}{2R_s}\hat{u}_m = E\hat{p}_o + P_o\hat{e} + 2EI_C\hat{e} + E^2\hat{i}_C \quad (8)$$

The major interest in the controller design is represented by the input-output transfer function. Usually in many applications the output power is constant and knowing that in steady state the capacitors average current value equals zero it can be written:

$$\begin{cases} \hat{u}_M = 0 \\ \hat{p}_o = 0 \\ P_o = \dfrac{U_M^2}{2R_c} = \dfrac{U_M^2 U_m}{2R_s E} = ct. \\ I_C = 0 \\ \hat{i}_C = C\dfrac{d\hat{e}}{dt} \end{cases} \quad (9)$$

Replacing (9) in (8) gives:

$$\frac{U_M^2}{2R_s}\hat{u}_m = \frac{U_M^2 U_m}{2R_s E}\hat{e} + E^2 C\frac{d\hat{e}}{dt} \quad (10)$$

By converting the last equation in the Laplace domain it results:

$$\frac{U_M^2}{2R_s}\hat{u}_m(s) = \frac{U_M^2 U_m}{2R_s E}\hat{e}(s) + sE^2 C\hat{e}(s) \quad (11)$$

From (11) the control-output transfer function can be revealed as it follows:

$$\frac{\hat{e}(s)}{\hat{u}_m(s)} = \frac{\dfrac{U_M^2}{2R_s}}{\dfrac{U_M^2 U_m}{2R_s E} + E^2 Cs} = \frac{\dfrac{E}{U_m}}{1 + \dfrac{E^2}{P_o}C} \quad (12)$$

So, the control-output transfer function of the APF circuit with integration control is:

$$G_c(s) = \frac{\hat{e}(s)}{\hat{u}_m(s)} = \frac{K}{1+\tau s} \quad (13)$$

in which:

$$K = \frac{E}{U_m},\ \tau = \frac{2E^3}{U_M^2 U_m}R_s C = \frac{E^2}{P_o}C \quad (14)$$

Because the above transfer function contains one pole, it is normal to adopt for the error amplifier a second type amplifier, having a zero, ω_z, and a pole, ω_p, in the left half-plane and a pole in the origin, resulting:

$$G_{AE}(s) = k_{AE}\frac{1+\dfrac{s}{\omega_z}}{s\left(1+\dfrac{s}{\omega_p}\right)} = \frac{1}{\dfrac{s}{\omega_{UGF}}}\,\frac{1+\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_p}} \quad (15)$$

where $\omega_{UGF}=k_{AE}$, represents the unity gain frequency at which the amplitude characteristic from the low frequency zone may intersect by extrapolation the frequency axis. Imposing the cut-off frequency, f_c, the following equality must be fulfilled:

$$\left|G_{AE}(j2\pi f_c)\right| = \frac{1}{\left|G_c(j2\pi f_c)\right|} \quad (16)$$

Replacing (13) and (15) in (16) will lead to:

$$\left|\frac{1}{j\dfrac{f_c}{f_{UGF}}}\cdot\frac{1+j\dfrac{f_c}{f_z}}{1+j\dfrac{f_c}{f_p}}\right| = \left|\frac{1}{\dfrac{K}{1+j2\pi f_c\tau}}\right| \quad (17)$$

By expressing the absolute values from the last equation it is obtained that:

$$\frac{f_{UGF}}{f_c}\sqrt{\frac{1+\left(\frac{f_c}{f_z}\right)^2}{1+\left(\frac{f_c}{f_p}\right)^2}}=\frac{\sqrt{1+4\pi^2 f_c^2\tau^2}}{K} \qquad (18)$$

From (28) results the expression for f_{UGF}:

$$f_{UGF}=\frac{f_c\sqrt{1+4\pi^2 f_c^2\tau^2}}{K}\sqrt{\frac{1+\left(\frac{f_c}{f_z}\right)^2}{1+\left(\frac{f_c}{f_p}\right)^2}} \qquad (19)$$

The cut-off frequency, f_c, imposed value must be small enough compared with half of the minimum averaging frequency. In order to have a higher value for f_c (an acceptable response speed), the pole and the zero from the error amplifier will be chosen as in the following relations:

$$\begin{cases} f_c=\frac{1}{4}f_{ac} \\ f_p=\frac{3}{4}f_{ac} \\ f_z=\frac{1}{12}f_{ac} \end{cases} \qquad (20)$$

Replacing (20) in (19) will lead to:

$$f_{UGF}=\frac{\frac{1}{4}f_{ac}\sqrt{1+4\pi^2\frac{1}{16}f_{ac}^2\tau^2}}{K}\sqrt{\frac{1+\frac{1}{9}}{1+9}}= \qquad (21)$$
$$=\frac{1}{12}\frac{f_{ac}}{K}\sqrt{1+\frac{\pi^2}{4}f_{ac}^2\tau^2}$$

By substituting in (21) the expressions for K and τ, given by (14), the final expression for f_{UGF} it will be obtained as it follows:

$$f_{UGF}=\frac{1}{12}f_{ac}\sqrt{\left(\frac{\pi}{2}f_{ac}\frac{U_m E}{P_o}C\right)^2+\left(\frac{U_m}{E}\right)^2} \qquad (22)$$
$$=\frac{1}{12}f_{ac}\sqrt{\left(\frac{\pi}{2}f_{ac}\frac{U_m E}{P_o}C\right)^2+\left(\frac{U_m}{E}\right)^2}$$

From the equations (20) it can be expressed the systems phase margin:

$$\varphi_M=arctg\frac{f_c}{f_z}-arctg\frac{f_c}{f_p}= \qquad (23)$$
$$=arctg3-arctg\frac{1}{3}=53{,}13^o$$

According to [2], if $f_p=f_{ac}/4$ a phase margin of $26{,}56^o$ is obtained, value that is inferior than the value of the phase margin obtained with (23) in which $f_p=f_{ac}3/4$.

IV. HARDWARE IMPLEMENTATION OF THE CONTROLLER

According to the considerations from the last paragraph the controller can be hardware implemented with an operational amplifier as it can be seen in Fig.3.

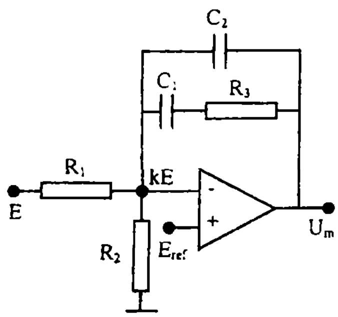


Fig.3-Hardware implementation of the controller

For the circuit from Fig.3. it can be expressed the control-output transfer function as it follows:

$$\frac{U_m(s)}{E(s)}=\frac{\dfrac{\frac{1}{sC_2}\left(R_3+\frac{1}{sC_1}\right)}{\frac{1}{sC_2}+R_3+\frac{1}{sC_1}}}{R_1} \qquad (24)$$

The above equation can be in a simple way rearranged in the following form:

$$\frac{U_m(s)}{E(s)}=\frac{1+sC_1R_3}{sR_1(C_1+C_2)\left(1+sR_3\frac{C_1C_2}{C_1+C_2}\right)} \qquad (25)$$

Comparing (25) with (15), there can be easily identified the values for the poles and zero frequencies, depending on the circuit parameters:

$$\omega_{UGF}=\frac{1}{R_1(C_1+C_2)},$$
$$\omega_z=\frac{1}{R_3C_1},\ \omega_p=\frac{1}{R_3\frac{C_1C_2}{C_1+C_2}} \qquad (26)$$

Usually R_2 is chosen around $1k\Omega$ and R_1 is imposed to match the condition $ER_2/(R_1+R_2)=E_{ref}$, where E_{ref} is known. The cut-off, zero and pole frequencies must obey the (20) equations and according to them, f_{UGF} can be determined from (22), assuming that C, E, P_o and U_m are given. Finally from (26) there can be determined the values for C_1, C_2 and R_3.

V. SIMULATION RESULTS

In order to verify the correct behavior of the designed controller, a single phase APF circuit with integration unipolar control was simulated in the CASPOC medium (Simulation Research). The rms mains voltage is *230V* at *50Hz* frequency, the value of the *L* inductor is *1.5mH* and the load is represented by an uncontrolled bridge rectifier supplying a constant current of *2.6A*. The circuit used for simulation is presented in Fig.4.

The input current waveform and the current absorbed by the APF circuit, together with the main supply voltage are presented in Fig.5. The spectral analysis of the input current performed in MATLAB, presented in Fig.6. reveal very good values for the merit

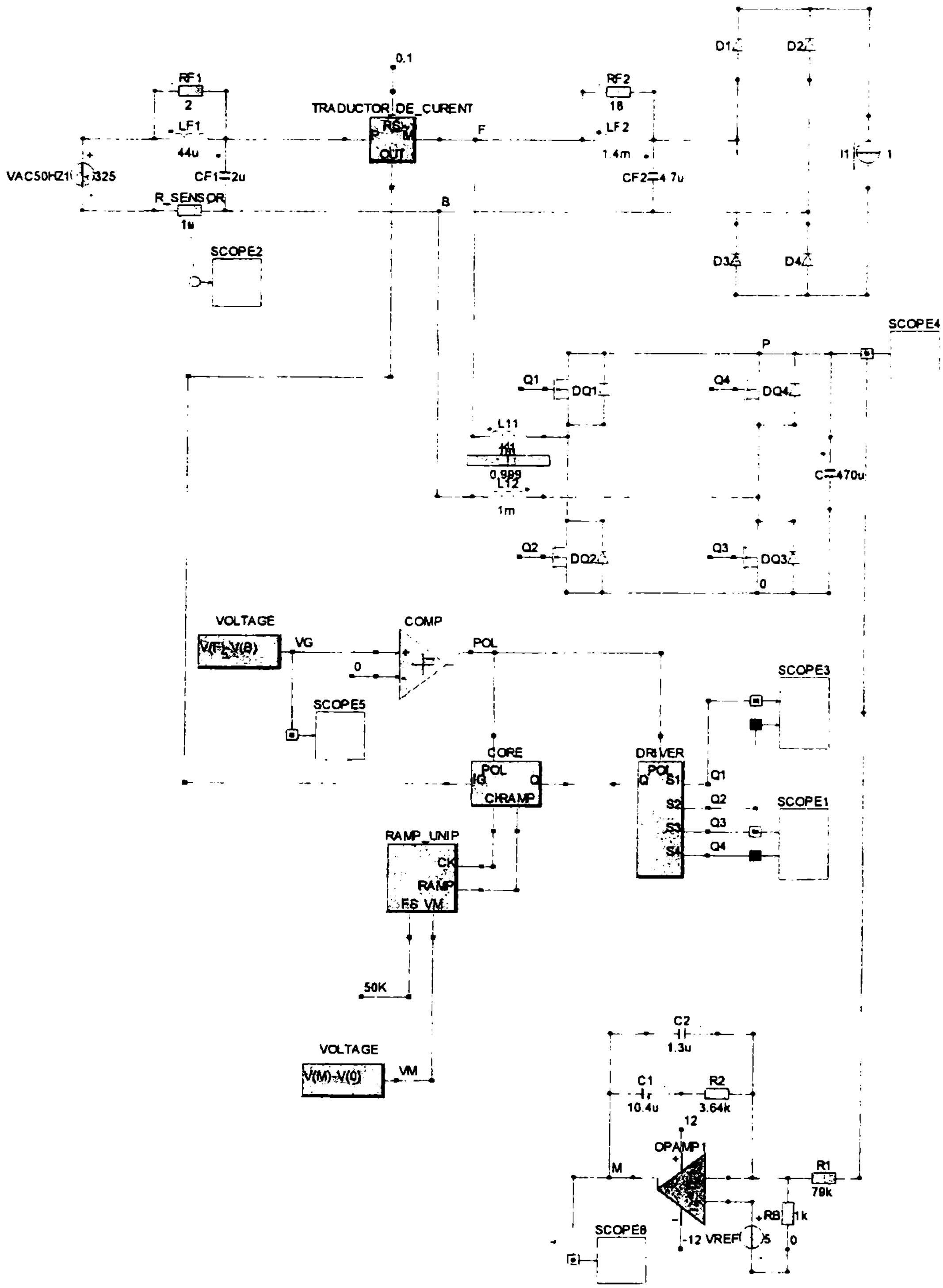
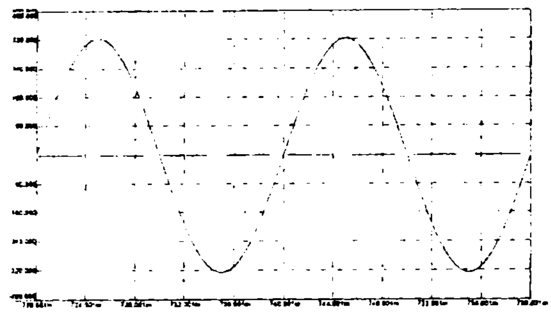
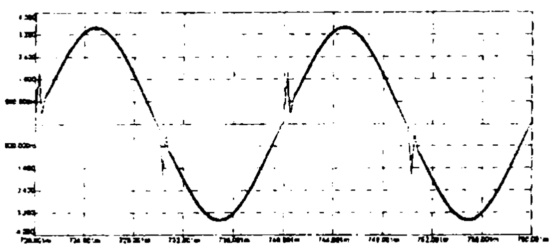


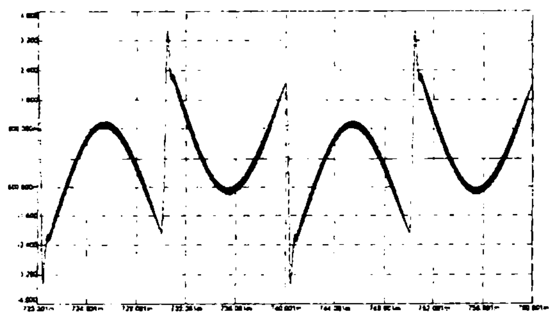
Fig.4.-Simulation circuit in CASPOC for a single phase APF circuit with integration unipolar control

parameters as it can be seen from Table1 and Tabel2.



(a)





(c)

Fig.5. Simulation waveforms:
(a) – mains supply voltage, u_s
(b) – mains input current, i_s
(c) – absorbed current by the APF, $i_{apf} = i_o - i_s$

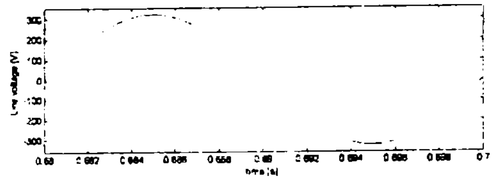
Table 1

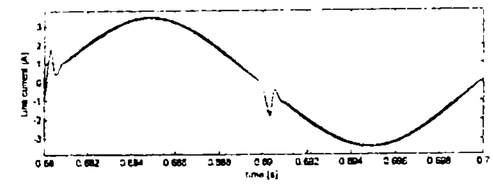
THD_tr [%]	*Kd_tr*	*Φ1 [deg]*	*Kφ*	*PF_tr*
5.1813	0.9987	-3.1886	0.9985	0.9971

Table 2

THD_tot [%]	*Kd_tot*	*Φ1 [deg]*	*Kφ*	*PF_tot*
8.3129	0.9966	-3.1886	0.9985	0.9950

The main merit parameters: total harmonic distortion factor *THD*, distortion factor K_d, the angle between the input current fundamental and the supply voltage φ_1, displacement factor K_φ and the power factor *PF* were determined in two ways. In the first case, there were taken in consideration only harmonics until the 20th order (Table1) and then were taken in consideration all the harmonics (Table2).





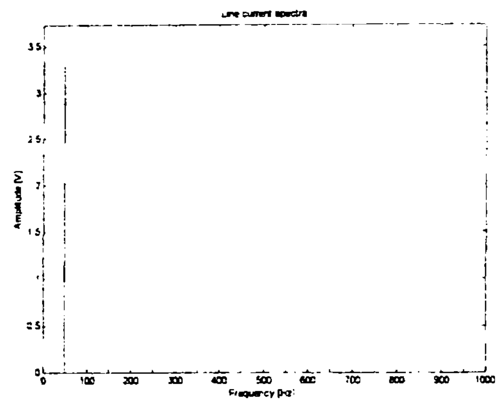


Fig.6. Spectral analysis for the APF circuit
(a) – mains supply voltage, u_s
(b) – mains input current, i_s
(c) – input current spectra

For bipolar control simulation results also confirmed the good behavior of the controller.

VI. CONCLUSIONS

A general controller suitable to be used both with unipolar and bipolar controlled APFs is developed. Reasonable dynamics with a good phase margin is achieved. The controller is attractive by its simplicity, the hardware implementation requiring only an operational amplifier and some passive components.

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