

# Recognition of OFDM modulations : approach based on high-order time-frequency methods

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**Abstract** – Nowadays a deep interest for processing of different types of digital modulations (FSK, PSK, QAM and OFDM) exists. This paper studies a technique for discrimination of the OFDM modulation. The approach is based on the high-order time-frequency representation and filtering process with orthogonal filters.

**Keywords:** OFDM, time-frequency, orthogonal filters.

## I. INTRODUCTION

The identification of the digital modulation type of a signal has found applications in many areas, including electronic warfare, surveillance and threat analysis. Recognition of communication signals became an independent discipline in the electronic warfare. The goal is to intercept, analyze, classify and, eventually, understand the message carried out by the communication signals.

In this paper a potential method is proposed to classify different types of numeric modulations. In section II the characteristics of the PSK, QAM, FSK, and OFDM modulations are described. In section III the time-frequency representations based on polynomial phase signal processing are presented. The new method for recognition of the OFDM modulations is illustrated in section IV. Furthermore, in section V some simulation results are depicted. Section VI will close this communication.

## II. DIGITAL MODULATIONS: PSK, QAM, FSK, OFDM

The class of PSK (Phase Shift Keying) modulations is widely used in numeric satellite TV broadcasting, being very robust to perturbations. The general equation of PSK is given by:

$$m_{PSK}(t) = \sum_k \Omega(t - kT) \cos(2\pi f_c t + \psi_k) \quad (1)$$

where the symbol  $\psi_k$  is  $\psi_0 + (2m-1)\frac{\pi}{M}$  with  $0 \leq m \leq (M-1)$  (the common value of  $\psi_0$  is 0).

The QAM (Quadrature Amplitude Modulation) modulation is defined as:

$$m_{QAM}(t) = \sum_k \Gamma_k \cos(2\pi f_c t + \psi_k) \quad (2)$$

where:

$$\Gamma_k = \sqrt{a_k^2 + b_k^2} \text{ et } \psi_k = \arctan\left(\frac{b_k}{a_k}\right) \quad (3)$$

It is clear that QAM signals contain a phase as well as an amplitude modulation. Compared to PSK signals, one distinction in QAM signals is that it does not have constant amplitude. This type of modulation is used in high-speed modems.

The FSK (Frequency Shift Keying) modulation can be considered as a non-linear frequency modulation. It can be represented as:

$$m_{FSK}(t) = \cos(2\pi f_c t + \psi(t)) \quad (4)$$

where  $\psi(t)$  depends on the integral of modulator signal. Thus, the frequency varies with the message and the information is carried out by the instantaneous frequency.

In comparison with the modulations described previously, which are single-carrier modulation techniques, the OFDM (Orthogonal Frequency Division Multiplex) modulation is a multiple-carrier technique. OFDM is a method that allows to transmit high data rates over extremely hostile channels at a comparable low complexity.

The OFDM spread spectrum technique distributes the data over a large number of carriers that are spread regularly over a frequency band. This spacing provides the "orthogonality" in this technique resulting in a high spectral efficiency. Thus, the complex signal is represented by:

$$m_{OFDM}(t) = \sum_{l=0}^{M-1} \sum_{k=0}^{N-1} X_k \cdot e^{j2\pi f_k t} \cdot e^{j2\pi f_c t} \cdot p(t - lT) \quad (5)$$

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where  $X_k$  represents the  $k$ -th information symbol,  $f_k = f_0 + k\Delta f$  stands for the  $k$ -th subcarrier and  $p(t)$  represents the pulse shaping function.  $N$  and  $M$  are the total number of subcarriers and total number of transmitted blocks respectively. OFDM has become very popular as it is used in such major applications as digital subscriber lines (DSL) and digital audio broadcasting (DAB).

### III. HIGH-ORDER TIME-FREQUENCY METHODS

It is well known that there is no transformation from the Cohen's class that can produce the complete concentration along the instantaneous frequency (IF) when this one is a nonlinear function of time. Therefore, different high order distributions have been developed in order to better match the non-linear time-frequency behavior of the analyzed signal. On the other hand, the polynomial phase signal (PPS) constitutes a good model in a variety of applications, e.g. radar imagery, mobile communication systems, etc.

As it was illustrated in [1], [2], the classical HAF algorithms present some limitations, related to the noise robustness, the cross-terms presence and the effect of the error propagation. In order to solve the first two aspects, the multi-lag HAF (ml-HAF) concept has been initially proposed in [2]. In fact, the ml-HAF is based on the generalization of the high order instantaneous moment HIM [2]:

$$M_K[x(t); \tau_{K+1}] = M_{K-1}[x(t + \tau_{K-1}); \tau_{K+2}] M_{K-1}[x(t - \tau_{K-1}); \tau_{K+2}] \quad (6)$$

where  $\tau_i = (\tau_1, \tau_2, \dots, \tau_i)$ . Applying the FFT to (1), we obtain the ml-HAF of the signal  $x(t)$ :

$$mlHAF_K[s; \alpha, \tau] = \int_{-\infty}^{\infty} HIM_K[s(t); \tau] e^{-j\alpha t} dt \quad (7)$$

Assuming a PPS model for the analyzed signal, i.e.:

$$s(t) = A \exp j\phi(t) = A \exp \left[ j \sum_{k=0}^K a_k t^k \right] \quad (8)$$

the main property of HIM is that, the  $K^{\text{th}}$  order HIM is reduced to a harmonic with amplitude  $A^{2^{K+2}}$ , frequency  $\tilde{\omega}$  and phase  $\tilde{\phi}$ :

$$M_k[y(t); \tau] = A^{2^{K-1}} \exp j(\tilde{\omega}_k \cdot t + \tilde{\phi}_k) \quad (9)$$

where  $\tilde{\omega}_k = k! \tau^{K-1} a_k$  (10).

Based on these results, Porat [1] has proposed an algorithm, which estimates sequentially the coefficients  $\{a_k\}$ . At each step, using a spectral analysis method, we estimate the spectral peak and, using the HAF, we compute an estimation value ( $\hat{a}_k$ ) of  $a_k$ . With this value, the effect of the phase term of the higher order is removed:

$$s^{(k-1)}(t) = s^{(k)}(t) \exp \left\{ -j \hat{a}_k t^k \right\} \quad (11)$$

Using the ml-HIM concept (relation (6)), Barbarossa and al [3] introduced the *Product* HAF: the ml-HAFs computed, via relation (7), for different lag sets:

$$\mathbf{T} = \left\{ \tau_{K-1}^{(l)} \right\}_{l=1, \dots, K-1}, \tau_{K-1}^{(l)} = \left\{ \tau_i \right\}_{i=1, K-1} \quad (12)$$

are multiplied, obtaining also a more robust method and a cross-terms free representation:

$$PHAF(\alpha, \mathbf{T}) = \prod_{l=1}^K mlHAF_K \left[ s, \frac{\prod_{i=1}^{K-1} \tau_i^{(l)}}{\prod_{i=1}^{K-1} \tau_i^{(l)}} \alpha, \tau_{K-1}^{(l)} \right] \quad (13)$$

Still, the effect of error propagation remains a serious limitation of the PHAF when we try to estimate a deeply non-linear IF laws (underwater transitory signals, digital modulations, etc). Therefore, in [5] is proposed a new procedure for polynomial order compensation, the WarpCom method, based on unitary transform phase reducing [4].

Let consider a signal modeled by a  $K^{\text{th}}$  order PPS (relation (8)). Using a modern version of the HAF (PHAF operator or the approach proposed in [3]), we obtain an accurate estimate of the  $K^{\text{th}}$  order polynomial coefficient, denoted by  $\hat{a}_K$ . With this value, we construct the following time axis warping function:

$$\mathbf{w}_K : t \xrightarrow{\mathbf{w}_K} t_w^{(K)} = \mathbf{w}_K(t) = \left( \frac{t}{|\hat{a}_K|} \right)^{1/K} \quad (14)$$

The effect of the associated unitary operator  $\mathbf{U}$  (14) on the PPS is depicted by:

$$\begin{aligned} (\mathbf{U}_K y)(t_w^{(K)}) &= \tilde{A} \exp \left\{ j \hat{a}_K \left[ \left( \frac{t}{|\hat{a}_K|} \right)^{1/K} \right]^K \right\} \cdot \exp \left\{ j \sum_{n=0}^{K-1} a_n \left[ t_w^{(K)} \right]^n \right\} \\ &= \tilde{A} \exp \left\{ j \sum_{n=0}^{K-1} a_n \left[ t_w^{(K)} \right]^n \right\} \cdot \exp \left\{ j \frac{a_K}{|\hat{a}_K|} t \right\} \quad (15) \\ &\quad \text{SPF of order } (K-1)^{\text{th}} \quad \text{residual} \end{aligned}$$

where  $\tilde{A} = A \sqrt{K! \hat{a}_K} \left( \frac{t}{|\hat{a}_K|} \right)^{1/(K-1)}$  (16). Since all the terms

in (16) are known and non-random, the induced amplitude modulation can be compensated, for example, through an amplitude weighting using the inverse of relation (16).

Therefore, the result of the warping transform of a  $K^{\text{th}}$  order PPS consists in a  $(K-1)^{\text{th}}$  order PPS for the new temporal variable  $t_w^{(K)}$ . The  $(K-1)^{\text{th}}$  order PHAF of this signal, with respect to  $t_w^{(K)}$ , peaks to a frequency location related, via relation (10), to the  $a_{K-1}$  coefficient. Once  $a_{K-1}$  is estimated, we construct the  $(K-1)^{\text{th}}$  order unitary operator  $\mathbf{U}_{K-1}$ :

$$\mathbf{w}_{K-1} : t_w^{(K)} \xrightarrow{\mathbf{w}_{K-1}} t_w^{(K-1)} = \mathbf{w}_{K-1}(t_w^{(K)}) = \left( \frac{t_w^{(K)}}{|\hat{a}_{K-1}|} \right)^{1/(K-1)} \quad (17)$$

which removes the  $(K-1)^{th}$  order component. The process is iterated (see figure 1) until all polynomial coefficients are estimated.

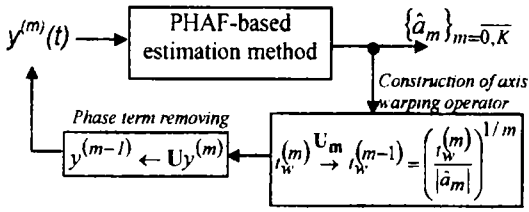


Fig. 1. Polynomial coefficient estimation based on unitary transform phase reducing

IV. METHOD FOR RECOGNITION OF OFDM MODULATION

For separation of modulation OFDM the method proposed is based on the filtering process with an orthogonal filter-bank and a high-order time-frequency representation. The time-frequency space of an OFDM signal is very complex due to numerous carrier frequencies included in the signal. The main idea is to recover and to locate these carrier frequencies in the frequency band of interest.

Therefore, this technique is based as follows:

1. Determination of the frequency band of interest in the time-frequency plane of the signal using a detection method based on cumulates of order 4 [6].
2. Construction of an orthogonal filter-bank in order to perform a sub-bands filtering operation in the frequency band of interest. These filter batteries are composed of  $2^k$ ,  $k=0, N$  filters considering that each bank of filters introduces a level of decomposition.
3. Stationarization and Extraction of the signal using the WarpCom time-frequency representation [5] for each filtered signal obtained at step 2.
4. Calculation of the frequency marginal and the standard deviation  $\{\sigma_{i,k}\}$ , where  $i$  is the decomposition level and  $k$  is the position of the filter in the filter-bank, for the normal distribution (Pdf - Probability density function) corresponding to each stationary signal obtained in step 3.
5. Construction of a decomposition tree and location of the minima  $\{\sigma_{i,k}\}$  for each branch of the decomposition tree.

Steps 1 and 2 are considered to be a filtering process and steps 3 and 4 a time-frequency characterization process. The types of the filters implied are : Chebyshev Type II, Morlet, Sinc. Other types of filters could be taken into account.

At first to test the efficiency of the proposed method, the signal analyzed is of type OFDM obtained by the sum of four QPSK (Quaternary Phase Shift Keying) quasi-analytic signals. The spacing of the carrier frequencies is constant and the normalized values are spread in the interval  $(0 \div 0.5)$ : 0.2, 0.224, 0.248, 0.273. In this case, the frequency band of interest is in range of 0.2 to 0.3. The orthogonal filter-banks considered are Chebyshev Type II filters. These filter batteries are composed of  $2^k$ ,  $k=0, 1, 2$  filters. Thus, the decomposition tree will have two decompositions levels: first, composed by the output of the battery with 2 filters and the second composed by the output of the battery with 4 filters.

In this case the number of carrier frequencies and their spectral localization is known. Thus the design of the filter batteries is progressive : with 2 filters and finally with 4 filters. The second bank of filters is constructed so that each filter is centered on each carrier. Normally, taking into account the procedure described above in step 5, the carriers (the minima for each branch) in the second level of decomposition of the tree are retrieved. In other words, the detection of the carrier frequency is accurate. This situation is illustrated in figure 2. The minima are represented with encircled stars.

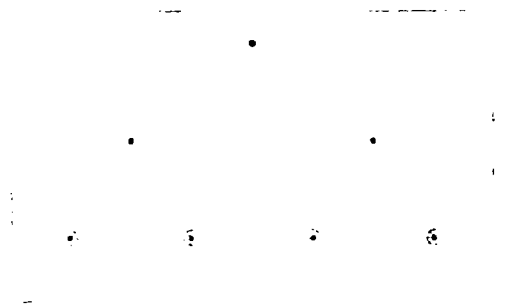


Fig. 2. Decomposition tree for an OFDM signal with 4 carriers: 0.2, 0.224, 0.248, 0.273, in normalized frequency

Because of the possible errors that could appear in the method proposed, it's possible to detect not all the 4 carriers in the last level of the tree: only 3, 2, 1 or none, in the worse case. They could appear in the upper levels.

Consequently, we apply this algorithm taking into account the simulation method of Monte Carlo for  $N=50$  signals described above. The results obtained are presented in table 1.

Table 1

4 carriers detected	3 carriers detected	2 carriers detected
35 times	14 times	1 times

The results show good performances: from 50 times, the proposed method have detected all the carriers 35 times in the second level of decomposition (the situation depicted in figure 2), 14 times the algorithm

have picked up 3 carriers and 1 time only 2 carriers. The efficiency of the technique is very dependent on the construction of the batteries of filters and the band of interest covered by these batteries. The performances could be improved with a battery of filters more centered on the carrier frequencies as well as worse results are obtained with a battery of filters that are not precisely centered on the carriers.

The next signal analyzed is of type QPSK with a normalized frequency of 0.248. The batteries used are the same as before. In this case, the localization of the minima in the decomposition tree has to be different. Normally, on the second level of decomposition it has to detect only one carrier.

The results using the simulation method of Monte Carlo for  $N=50$  such signals are presented in table 2.

Table 2

4 carriers detected	3 carriers detected	2 carriers detected	1 carrier detected
0 times	0 times	17 times	27 times

From the results obtained is evident the different structure of the decomposition tree, being possible to discriminate the OFDM modulation from others types of numeric modulation.

Next, the signals analyzed are real communication numeric signals : an OFDM signal issued from a multi-path channel and an FSK signal. The first signal has the following characteristics: 250 kHz sampling frequency, 32 carrier frequencies (base carrier located at 12.5 kHz), signal-to-noise (SNR) 5 db, 3201 samples and 9 symbols. The algorithm generates with the following parameters : band of interest (10÷41 kHz), 5 levels of decomposition (with the batteries composed of  $2^k$ ,  $k=0, 1, 2, 3, 4, 5$  filters of type Chebyshev Type II). The decomposition tree resulted is presented in figure 3.

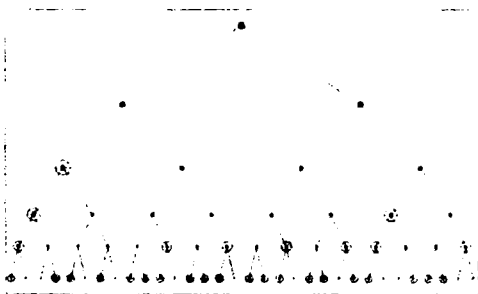


Fig. 3. Decomposition tree for a real OFDM signal

The second real signal is a FSK signal with the following characteristics : 250 kHz sampling frequency, single-carrier frequency located at 25 kHz, signal-to-noise (SNR) 0 db, 2000 samples and 200 symbols. The algorithm parameters are : band of interest (2.9÷47.1 kHz), 5 levels of decomposition (with the batteries composed of  $2^k$ ,  $k=0, 1, 2, 3, 4, 5$  filters of type Chebyshev Type II). The result is showed in figure 4.



Fig. 4. Decomposition tree for a real FSK signal

We can remark that for the FSK signal, is not needed many levels of decompositions : the carrier can be detected in the upper levels as shown in figure 4.

## VI. CONCLUSIONS

In this paper, a new method for the discrimination of the OFDM modulation was proposed. The principle is to filter the carrier frequencies in order to obtain, via WarpCom time frequency-method, the stationarized version of the signal. Furthermore, the corresponding standard deviation of the frequency is involved for the construction of the decomposition tree. A further research direction will be needed to improve and study the factors that affect the effectiveness of the method.

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