

# Reconstruction Methods for Missing Portions in Signals

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**Abstract** – On the storage medium the information can be lost from a portion of a signal. The missing or disturbed portion of the audio signal is replaced by a weighting average of the forward and backward extrapolated signals. A new weighting function is proposed for the signal reconstruction method. A method for the detection of the impulsive noise, using two thresholds, is presented.

**Keywords:** extrapolation, disturbance, reconstruction

## I. INTRODUCTION

There are many situations when the long portions in the recorded audio signals are removed or affected by the impulsive noise. A method for the restoration based on the separation of autoregressive processes is proposed. The corrupted samples are replaced by a weighted average of the signals extrapolated from areas preceding and corrupted area. For the detection of the impulsive noise a method with two thresholds is used.

## II. RESTORATION METHOD FOR LONG PORTIONS IN AUDIO SIGNAL

The impulse response function is obtained from  $M = 2N$  samples of the known signal by using the equation:

$$\mathbf{X}\mathbf{h}' = \mathbf{x} \quad (1)$$

where

$$\mathbf{h}' = [h_1', h_2', \dots, h_N']^T, \quad (2)$$

and

$$\mathbf{x} = [x_{N+1}, x_{N+2}, \dots, x_{2N}]^T. \quad (3)$$

The matrix  $\mathbf{X}$  contains the shifted samples of the signal:

$$\mathbf{X} = \begin{pmatrix} x_N & x_{N-1} & x_{N-2} & x_1 \\ x_{N+1} & x_N & x_{N-1} & x_2 \\ \vdots & \vdots & \vdots & \vdots \\ x_{2N-1} & x_{2N-2} & x_{2N-3} & x_N \end{pmatrix} \quad (4)$$

The one-step forward extrapolation equation is:

$$x_n' = \sum_{k=1}^N h_k' x_{n-k}, \quad (5)$$

The backward extrapolation equation is

$$x_n'' = \sum_{k=1}^N h_k'' x_{n-k} \quad (6)$$

The impulse response vectors  $\mathbf{h}'$  and  $\mathbf{h}''$  are obtained by Burg method [1]. The forward and backward extrapolated signal is given by

$$x_n = w_n x_n' + (1 - w_n) x_n'' \quad (7)$$

where  $w_n$  is a weighting function

$$w_n = \begin{cases} 1 - \frac{1}{2}(2u_n)^a, & u_n \leq \frac{1}{2} \\ \frac{1}{2}(2 - 2u_n)^a, & u_n > \frac{1}{2} \end{cases} \quad \text{and} \quad u_n = \frac{n - n_s}{n_e - n_s}$$

Another proposed weighting function is:

$$w_{n1} = 2u_n^3 - 3u_n^2 + 1 \quad (8)$$

This function is obtained using the polynomial:

$$g(x) = ax^4 + bx^3 + cx^2 + dx + e$$

with the conditions:  $g(0) = 1$ ,  $g(1) = 0$ ,  $g'(0) = 1$ ,  $g'(1) = 0$ . The solution is:  $a = 0$ ,  $b = 2$ ,  $c = -3$ ,  $d = 0$  and  $e = 1$ .

In Fig.1 the original and the bilateral extrapolated signal are given for 1000 extrapolated samples using  $N=1000$  coefficients of extrapolation.

The general weighting functions  $w_n$ ,  $1-w_n$ , for  $a=3$ , and  $w_{n1}$ ,  $1-w_{n1}$  are illustrated in Fig. 2.

The relation for signal to noise ratio is:

$$RSZ = 10 \lg \frac{\sum_{n=0}^{W-1} x_n'^2}{\sum_{n=0}^{W-1} (x_n - x_n')^2} \quad (9)$$

where  $x_n$  is the original signal and the term  $x_n - x_n'$  represents the error of extrapolation. The mean square error (MSE) between the estimate and the desired signal is given by:

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$$MSE = \frac{1}{2W} \sum_{n=0}^{W-1} (x_n - x'_n)^2 \quad (10)$$

In Fig. 3 the values of RSZ and MSE are plotted as functions of number of impulse response coefficients  $N$ , by using comparatively the weighting functions  $w_n$  and  $w_{n1}$ . The original and the bilateral extrapolated signal are given for 1000 extrapolated samples. The length of the impulse response is varied from 100 to 2000.

The spectral distortion is used to evaluate the quality of the restoration and is defined as:

$$M_{sd}[dB] = \left[ \frac{1}{N_{sd}} \sum_{k=1}^{N_{sd}} (10 \log |X(k)|^2 - 10 \log |\hat{X}(k)|^2) \right]^{1/2} \quad (11)$$

where  $N_{sd}$  is the length of the discrete Fourier transforms of the reference and the processed signals, respectively.

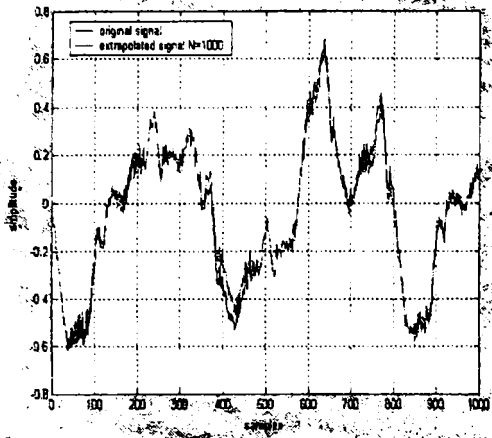


Fig. 1. Original signal and bilateral extrapolated signal for  $N=1000$

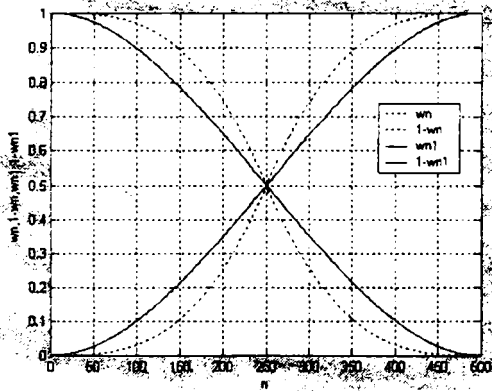


Fig. 2. Weighting functions  $w_n$ ,  $1-w_n$  for  $a=3$  and  $w_{n1}$ ,  $1-w_{n1}$ .

Fig. 4 illustrates the variation of  $M_{sd}$  as function of number of impulse response coefficients  $N$ , for the signal 1 [a)] and signal 2 [b)]. For the signal 2 we have better restoration results.

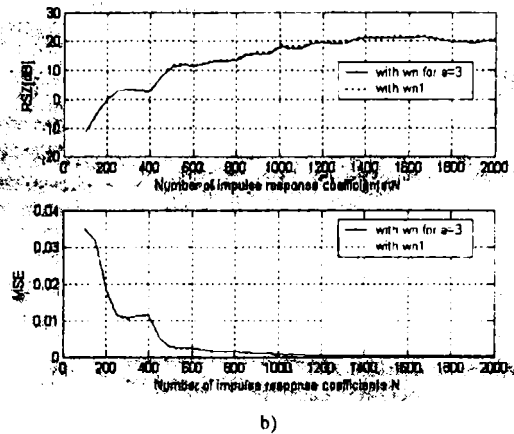
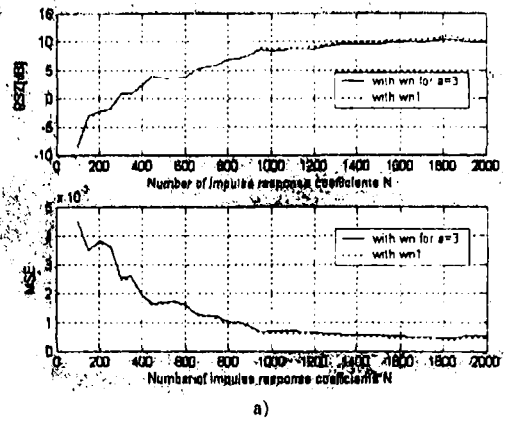


Fig. 3. RSZ and MSE as functions of number of impulse response coefficients  $N$ , by using comparatively the weighting functions  $w_n$  and  $w_{n1}$  [a) signal 1; b) signal 2]

The illustrations from Fig. 5 justifies the choice of the parameter  $a$ .

Fig. 6 presents the parameters RSZ, MSE as functions of bilateral extrapolation length. The quality is very high at the beginning and at the end of the extrapolated section.

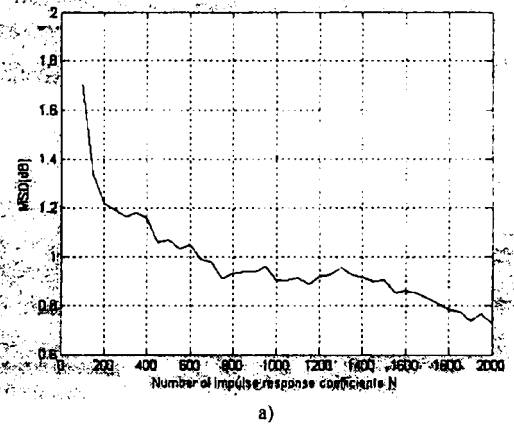


Fig. 4. (to be continued)

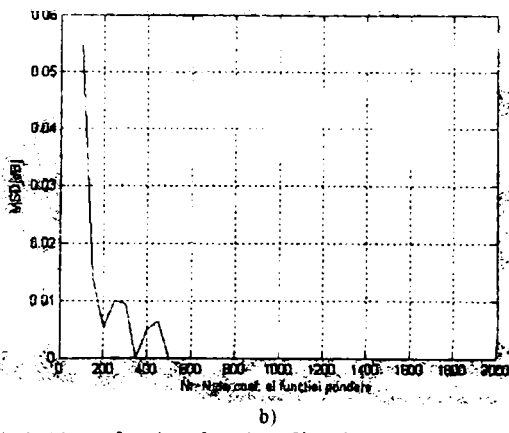


Fig. 4.  $M_{e,d}$  as function of number of impulse response coefficients  $N$ , for the signal 1 [a]) and the signal 2 [b]).

### III. CANCELLATION OF THE IMPULSIVE NOISE

For the classical method, the audio signal affected by perturbations,  $y(k)$ , is segmented in frames of  $N$  samples. Every frame is modeled as an autoregressive process (AR) of order  $p$ :

$$y(k) = x(k) + d(k),$$

where  $x(k)$  is affected by the additive impulsive noise,  $d(k)$ , and

$$x(k) = \sum_{j=1}^p a(j)x(k-j) + e(k), \quad k = \overline{p, N-1}. \quad (12)$$

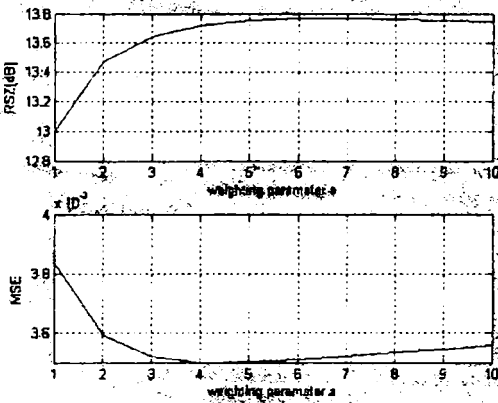


Fig. 5. The choice of the parameter  $a$ .

In the relation (12)  $a(j)$  are the model parameters and  $e(k)$  is the input signal associated with  $x(k)$ . The linear prediction can't model correctly signals with very rapid variations and so the detection of the impulsive noise will be made (Fig. 7). It's necessary to estimate the parameters of the autoregressive model of the signal and then to pass the perturbed signal  $y(n)$  through the error filter of the forward prediction with the transfer function:

$$I(z) = 1 - \sum_{j=1}^p a(j)z^{-j} \quad (13)$$

The samples of the signal  $y(n)$  which corresponds to the samples of the signal

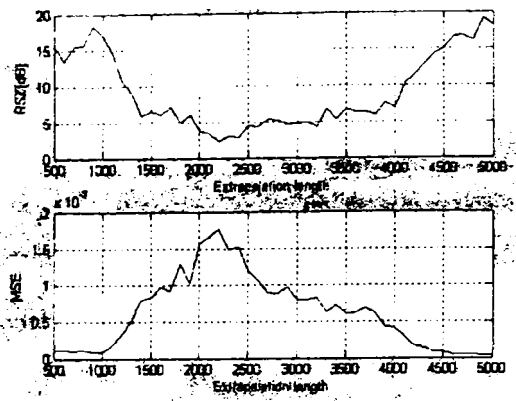


Fig. 6. RSZ and MSE as functions of extrapolation length measured for last 500 extrapolated samples, for a bidirectional extrapolation

$e(k)$  with the amplitude greater than the threshold  $\lambda$  which is given by the relation:

$$\lambda = K\hat{\sigma}_e \quad (14)$$

where  $\hat{\sigma}_e$  is the estimated value of the variance for the signal  $e(k)$ . The affected samples will be replaced by an interpolation algorithm.

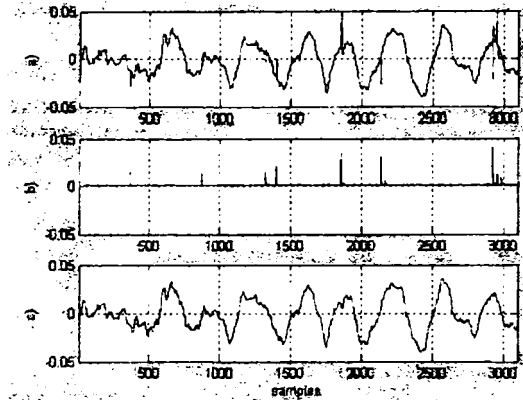


Fig. 7. a) Corrupted signal; b) Input signal; c) Restored signal

This method of detection can be improved by using two thresholds, by unifying adjacent perturbations and by changing the detection threshold in the reprocessing of the signal in a frame.

The two thresholds are: the detection threshold (the initial threshold)  $\lambda_D$  and the location threshold  $\lambda_L$  which are in the relation:

$$\lambda_L = b\lambda_D, \quad 0 < b < 1 \quad (15)$$

The threshold  $\lambda_L$  is used to localize the perturbations, as is shown in Fig. 8.

To reject the wrong time interval for the small amplitude noise perturbations, the location threshold is expressed as a function of the current iteration number:

$$\lambda_{L_i} = b_i \lambda_D, \quad i = 1, 2, \dots, i_{\max} \quad (16)$$

where

$$b_i = r^{i-1} b_1 \quad (17)$$

In the relations (14) and (15)  $r$  is the reduction factor,  $f$  is a parameter that controls the reduction speed of  $b$  and  $i_{max}$  is the maximum number of iterations to pass to the next step (Fig. 9). The choice of the parameters  $i_{max}$ ,  $f$  and  $r$  is based on the experimental observations.

With this method more iterations could be necessary to detect all the noise pulses from a frame (especially for the small amplitude noise pulses). This algorithm contains also stop criterions for the processing of a frame.

The Table 1 give comparative measurements between the results of the modified (MOD) and conventional (CONV) methods. The parameters were the same for all the six signals.

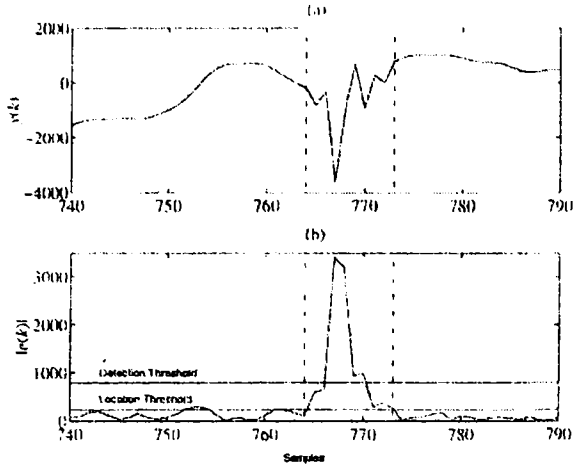


Fig. 8. a) Original signal affected by impulsive noise; b) Detection and location thresholds in the input signal.

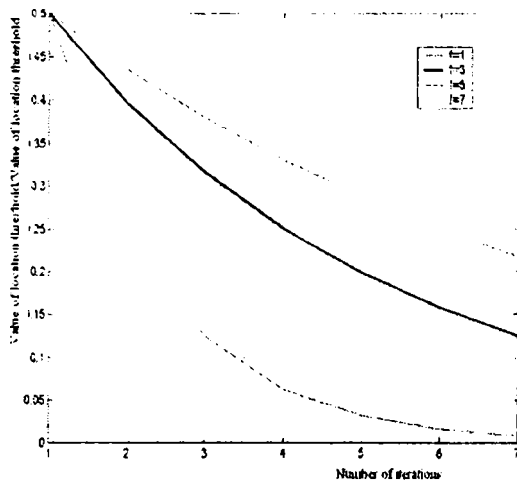


Fig. 9. Reduction of the location threshold value as a function of iteration number, with the parameter  $f$ .

This method has a good sensitivity for the parameter variations of the signal and the noise.

For the modified method the next parameters have been used:  $N=1024$ ,  $p=40$ ,  $K=5$ ,  $b_i=0.5$ ,  $r=0.5$ ,  $n=3$ ,  $f=3$ ,  $i_{max}=7$ .

#### IV. CONCLUSIONS

This paper presented methods for the correction of the disturbances in audio signals and for the estimation and removing for long pulses from old recordings. These methods can be implemented in

real-time applications. The information in the preceding and in the following data sections can be used to recover the lost information. This recovering is not perfect. The parameters RSZ, MSE,  $M_{sd}$  and its proposed graphical representations, for different situations, give the appreciation of this recovering. The final judgement is determined by the human ear, because audio signals do not exactly satisfy the requirements of being fully predictable, and therefore they cannot be perfectly extrapolated.

Tab. 1: Comparative measurements between the results of the modified (MOD) and conventional (CONV) methods

	Undetected pulses [%]		Wrong detections [%]	
	MOD	CONV	MOD	CONV
Signal 1	1.56	16.41	3.26	1.17
Signal 2	2.1	20.39	1.56	0.68
Signal 3	1.56	10.71	1.83	0.87
Signal 4	4.21	21.52	2.40	0.93
Signal 5	1.63	14.86	6.65	3.54
Signal 6	1.85	12.11	8	4.45

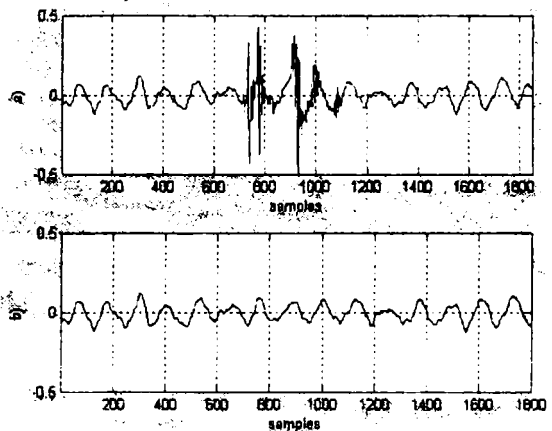


Fig. 10. a) Audio signal affected by a perturbation; b) Restored audio signal.

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