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Considerations about the design requirements for analog anti-aliasing filters

Adrian Stoica, Constantin I. Vizitiu, Ioan Nicolaescu, Lucian Anton[1]

Abstract - This is an overview on theoretical aspects of anti-aliasing filters, which are used together with high speed sampling Analog to Digital Converters (ADCs) in many applications. The main purpose is to define and to identify the particularities for analog filters designed for baseband and undersampling applications, in order to relieve the contributions of filtering on ADC dynamic range improvements.

Keywords: sampling process, Analog to Digital Conversion and Filtering, Data Acquisition Systems.

I. INTRODUCTION

For the ADCs used in data acquisition systems, in measurement instruments, and signal processing applications, are specified the time-domain performances (full scale, resolution, slew rate, accuracy, precision, integral or differential non-linearity, time conversion per bit etc.). In addition, the applications of ADCs in digital communications and high definition television require comprehensive frequency-domain specifications (gain and bandwidth, signal to noise ratio–SNR, sampling rate, effective number of bits-ENOB, spurious free dynamic range–SFDR etc.).

An important aspect of using ADCs in digital communications systems is the understanding of the sampling process and the possible causes of distortions, which limit the system performances.

The sampling process can be discussed from either the time or frequency domain or both. In the digital communications applications has more results the frequency-domain analysis, and for this reason we assume that is the right way to study the basic aspects of high speed sampling process.

II. THE ALIASES SIGNALS

The unwanted signals for ADCs, which result in the signal sampling process, are named *images frequencies* or *alias signals*.

The anti-aliasing filter is a good choice to assure an unambiguously high speed sampling process into the ADCs, in order to remove the unwanted signals, which have frequencies outside of the desired Nyquist zone of sampler.

There are two types of sampling:

- Baseband sampling, for signals without modulation;
- Undersampling, for signals with modulation and carrier frequency.

For baseband sampler we consider a single harmonic signal, with frequency noted f_a, sampled at a frequency noted f_s by ideal Dirac pulses. It must to respect the Shannon condition $f_s \geq 2 \cdot f_a$. But in the frequency domain it can be identified the images frequencies of the original signal, around every multiple of frequency f_s like in fig. 1, and these frequencies are given by the relation:

$$f_{ikm} = \left| \pm k \cdot f_s - f_a \right| \; ; \qquad f_{ikp} = \left| \pm k \cdot f_s + f_a \right| \; ; \quad k = 1,\ 2,\ 3,\ \ldots \tag{1}$$

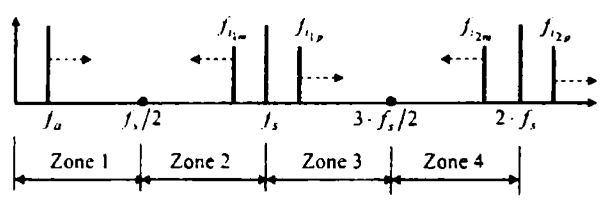


Fig. 1 The Nyquist zones for ideal sampler and the images frequencies

The first Nyquist zone (Zone 1), defined like the basic bandwidth (baseband), has the left limit on zero frequency and the right limit on first half of sampler frequency.

In the frequency domain there is an infinite number of Nyquist zones, each of them having bandwidth $f_s/2$.

If the analog signal frequency increases up to $f_s/2$ value, the alias signal with frequency f_{i1m} decrease direct to $f_s/2$ value, and the frequency f_{i1p} increase up to $3 \cdot f_s/2$ value. It can obtain $f_a = f_s/2 = f_{i1m}$ as a critical case for correct sampling process.

[1] Communications and Electronic Systems Department, Military Technical Academy, Bucharest, Address 81-83, Bd George Cosbuc, Sector 5 Bucuresti, Romania, Tel 0213354660 / 0197, e-mail astoica@mta.ro, vic@mta.ro

If f_a still increase, and $f_a \geq f_s/2$, theoretically the $f_{l_m} = f_s - f_a$ component falls inside the first Nyquist zone, and certainly this gives an unwanted signal at the output of ADC. This case is similar to the analog mixing process and the alias signal $f_{l_m} = f_s - f_a$ is like an intermediary frequency for radioreceiver.

For undersampling we consider the signal bandwidth $B_a = f_2 - f_1 = 2 \cdot \Delta f$, which is symmetrical around carrier frequency, noted f_{cr}. In fig. 2 is shown a signal in the third Nyquist zone, centered around a carrier frequency.

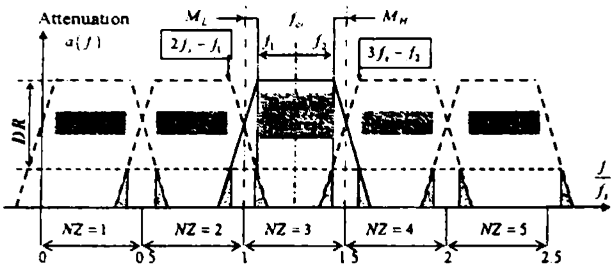


Fig 2 The aliases signals for undersampling process

In this case the signal bandwidth must be $B_a < f_s$, and $\Delta f = f_2 - f_{cr} = f_{cr} - f_1$ given from the Nyquist criteria with the centered carrier frequency.

III. THE ANTI-ALIASING FILTERS

A generally block diagram for a data conversion system is shown in the fig. 3, with the analog filtering at input and digital filtering of outputs in order to reject all unwanted signals.

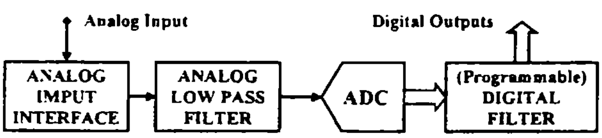


Fig. 3 The block diagram for an analog-to-digital conversion system

In the concrete applications it can be used only the analog filtering, or the digital filtering (Fast Fourier Transform FFT, Finite or Infinite Impulse Response FIR or IIR), or both in very sophisticated systems. There are some differences between the filtering in the analog domain and this filtering process in the digital domain.

The analog filtering is more suitable for high frequency, and can remove the extraneous noise spikes which can saturate the sample-and-hold amplifier (SHA) at the input of ADC, and the alias signals which can produce distortion in the conversion process.

The digital filtering has a proper action after analog-to-digital conversion and for this reason can't reduce the influence of analog noise in the conversion process, but it can remove the noise injected in ADC parts during the conversion. In addition, the digital filter may be programmable by software to optimize the circuit respond and the filtering characteristics.

For practical signal processing the ideal sampler can be obtained with an ADC followed by a FFT processor, which only provide an output in the frequency interval $[0, f_s/2]$, even for useful signal or alias one. For this reason it is necessary to use the filtering ahead the sampler circuit in order to remove the frequency components which are outside the first Nyquist zone whose aliased components fall inside it.

The anti-aliasing analog filter is a good choice to assure an unambiguously high speed sampling process into the ADCs both for baseband sampling and for undersampling.

Generally, for analog filter circuits, the transition bandwidth is smaller if more poles are used in the filter design. In these conditions it is necessary to use filters with high complexity, and for most of ADC producers the *Elliptic filters* with more than 10 poles are a popular choice. The *Bessel filters* or the *Chebyshev lters* (with the ri le error under 1 dB), both with minimum 8 poles, represent another options to implement the analog anti-aliasing filter. The *Butterworth filters*, which achieve 6dB attenuation per octave for each pole of filter transfer characteristic (or 20 dB per decade), is the most usual type of filter used in low cost practical applications.

For the baseband sampled signals are used the low-pass analog anti-aliasing filter (LP-AAF), but for the undersampled signals must use the band-pass analog anti-aliasing filter (BP-AAF).

IV. THE DESIGN RECQUIREMENTS

The main characteristics of LP-AAF are the followings: *the pass band, the attenuation value in the pass band* (a_{Pass}, ideally 0 dB), *the attenuation value in the stop band* (a_{Stop}, usually more than 60 dB), *the dynamic range DR* ($DR = |a_{Pass} - a_{Stop}|$), *the cut-off frequency (the corner frequency)*, simply noted f_a, which is equal to the highest frequency in analog input signal spectrum, and *the transition band*, which is the interval $[f_a, (f_s - f_a)]$.

Only the frequencies from transition band have the aliases signals in the band pass $[0, f_a]$, but the aliases components have the levels under the limit of dynamic range, regarding fig. 4.

The band pass of LP-AAF is lower than the width of first Nyquist zone, and in these conditions the aliased components between f_a and $f_s/2$ are not interest and do not limit the desired dynamic range for ADC.

The dynamic range of LP-AAF is chosen based on the requirements for signal fidelity and ADC resolution. Usually the stop band attenuation is between –60 dB and –80 dB, and the transition band is the interval $[0.45 f_s, 0.6 f_s]$.

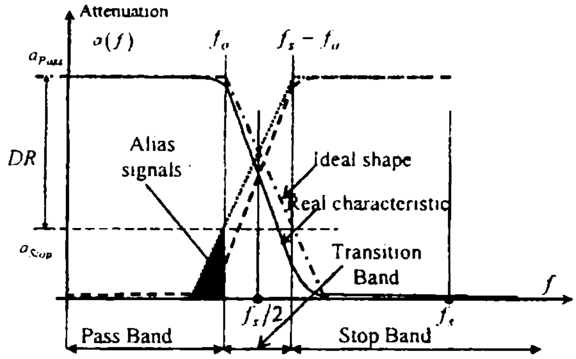


Fig. 4 The main parameters for LP-AAF

Choosing a higher rate of sampling it can reduce the requirements for filter sharpness and therefore the number of poles for filter characteristic can be reduce (and the filter complexity too). This aspect is illustrated in fig. 5, which shows how the increasing of the sampling frequency decreases the sloping rate of filter characteristic, while maintaining the analog corner frequency and the dynamic range requirements.

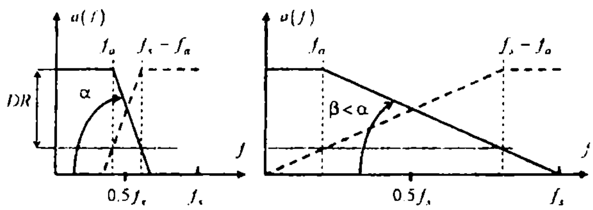


Fig. 5 The increasing sampling frequency reduces the order of LP-AAF

The transfer function for Butterworth filter is given by relation:

$$Y(s)=\frac{G}{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1}s^2 + a_n s + 1}, \qquad (2)$$

where G is the desired gain for filter circuit, usually $G=1$, and the coefficients a_0, a_1, a_{n-1}, a_n are given in reference [1], [2].

The amplitude response in dB is given by relation:

$$a=-20\cdot\log|H(j\omega)|=-20\cdot\log\frac{G}{\left(\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}\right)^n}, \qquad (3)$$

where the cut off frequency is $f_c = \omega_c/2\pi \geq f_a$, and n is the order of filter, the number of poles for the transfer function.

In these conditions we have a single equation and two unknown variables: n and ω_c.

Usually the design process is started with the attenuation $a \geq DR$, and with $n \geq 4$. If the sampling frequency is imposed in the digital system, it must to calculate the corner frequency of low pass filter from the relation (3) as well:

$$f_c = f_s \cdot \frac{1}{2\cdot\sqrt{10^{\frac{2|DR|\,dB|}{20n}}-1}}. \qquad (4)$$

F r he Butterworth filter with $DR=-80$ dB and $n=5$ ... obtain $f_c = 0.16\cdot f_s/2$.

If the analog signal is imposed, it mu... ...l.c. $f_c \geq f_a$, and f r the sig..al-t.-n..se rat.o (*SNR*) which i. al.. imp..sed $a \geq SN.. = DR$, choosing $n \geq 4$, it can be calculated the minimum sampling frequency for ADC:

$$f_s \geq 2\cdot f_a \cdot \sqrt{10^{\frac{2|DR|\,dB|}{20n}}-1}, \qquad (5)$$

For N bits ADC with Gauss noise, the minimum *SNR* value is given by the next relation:

$$SNR=\frac{RMS_{ADC\ signal}}{RMS_{Gauss\ noise}}=2^{N-1}\cdot\sqrt{6}, \qquad (6)$$

This equation is valid if the Gauss noise is measured over the entire first Nyquist zone, from DC to $f_s/2$. It can be expr.ssi.l. in d ci ls by relation:

$$SNR=[6.02\cdot N+1.76]\ \text{dB}. \qquad (7)$$

If the bandwidth of analog signal, $B_a = \Delta f$, is less than $f_s/2$, than the N ncrease ecause t e quantization noise within the signal bandwidth is smaller. In these situations the *SNR* must be calculate with the relation:

$$SNR=\left[6.02\cdot N+1.76+10\log\left(\frac{f_s}{2\cdot\Delta f}\right)\right]\ \text{dB}. \qquad (8)$$

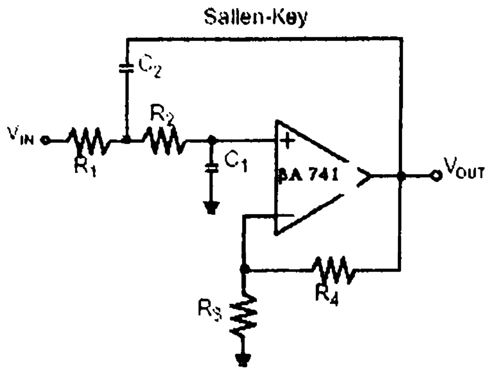
For ADC with 8, 12, or 16 bits it can obtain the desired *SNR* at 50 dB, 74 dB, or 98dB, which mean the desired dynamic range *DR* at 60 dB, 80 dB, or 100dB. If B_a is six times less than $f_s/2$, the correction term in (8) increase *SNR* with 7.8 dB.

In reference [1] are presented some interesting results of LP-AAF simulation with *Microchip FilterLab* software, and the possible dynamic range with usually filters order is presented in table I.

Table I The dynamic range with various LP-AAF

Filter order	**Dynamic range / Maximum attenuation a_{max} [dB]**		
n	**Butterworth**	**Bessel**	**Chebyshev**
4	80	66	90
5	100	79	117
6	120	92	142
7	140	104	169

The implementation of analog anti-aliasing filter may use the active cells with op amp, like successive Salen-Key low pass filters or Multiple Feedback low pass filters (MFB cells). These solutions are most popular and offer a good satisfaction for ADCs which process signals in the first Nyquist zone.



$$\frac{V_{OUT}}{V_{IN}} = \frac{K/(R_1 R_2 C_1 C_2)}{s^2 + s(1/R_1 C_2 + 1/R_2 C_2 + (1-K)/R_2 C_1) + 1/R_1 R_2 C_1 C_2}$$

$K = 1 + R_4/R_3$

Fig. 6 The Sallen-Key double pole low pass cell

For the ADCs suitable for digital communications applications it must to allow dynamic performances into a higher order Nyquist zones. In these cases it is necessary to use ADCs with the dynamic range given by the stopband attenuation of the BP-AAF, recommended for undersampling process.

In order to allow the placement of the carrier frequency in the middle of the NZ order Nyquist zone of undersampling process it must to select the sampling frequency based on the relation:

$$f_s = \frac{4 \cdot f_{cr}}{2 \cdot NZ - 1}, \quad NZ = 1,2,3,\ldots \qquad (9)$$

The NZ order must be an integer, and it is normally chosen as large as possible while is satisfied the Nyquist condition $f_s > 2 \cdot \Delta f$.

For example, we consider an analog FM signal with bandwidth $B_a = 8\,\text{MHz}$, centered around carrier frequency. If we assume only $M_L = M_H = 1\,\text{MHz}$ margins for band of interest in the Nyquist zone, then it must use a minimum sampling frequency $f_s = 10\,\text{Msps}$ (Mega samples per second).

The desired NZ order can be calculated as the lowest integer value, which results from the next relation:

$$NZ = \left\lfloor \frac{4 \cdot f_{cr} - f_s}{2 \cdot f_s} \right\rfloor. \qquad (10)$$

By calculus on obtain $NZ = \lfloor 23.5 \rfloor = 23$, and from relation (9) result $f_s = 10{,}666\,\text{Msps}$.

For the filter the attenuation in the stop band is $a_{Stop} \geq SNR$ given by relation (8), and the width of transition bands is smaller than $2 \cdot M_L$ or $2 \cdot M_H$.

In reference [2] are recommended some variants for active and passive circuits which can be used to realize the band pass filters. Still remain a problem the availability of high speed op. amp. with the maximum frequency over 300 MHz.

Now the fabricants of IC offer five types of ADCs architecture for high resolution (more than 8 bits) and high speed conversion (more than 100 Ksps):

- Flash (Parallel) ADC;
- Successive approximation ADC;
- Sigma-Delta ADC;
- Pipeline ADC;
- Bit per stage (Serial) ADC.

Based on our study and applications result a recommendation for pipeline ADCs in various applications, because the producers offer numerous variants of these ADCs, and their price isn't so high..

V. CONCLUDING REMARKS

The analog filtering can be a critical part of a data conversion system, because the anti-aliasing filters remove the ambiguity in the data conversion process of ADCs.

A good design for this filtering part is a warranty for the good functionality of digital systems.

There are a lot of dynamic characteristics (key specifications) for all high speed ADCs as well: power supply, maximum power dissipation, input range, input impedance, signal to noise and distortion ratio (SINAD), effective number of bits (ENOB), analog bandwidth, which can be specified FPBW (full power bandwidth) or SSBW (small signal bandwidth). Some of them are implied by filtering features and these characteristics must be analyzed in the context of the desired application with ADCs.

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