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Space and frequency analyze of an LSI optical system with different input signals

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Abstract-Our goal in this paper is to develop a calculus algorithm beginning from Optical Fourier Transform and then to generalize to optical linear shift invariant systems. We begin from the premise that an optical system can be seen like a black box with an input and an output. Our system property analyze can be made in space with the help of convolution and in frequency by multiplying specters, these operation are complementarily. Optical system also can be seen from the perspective of components which is made. So in our example we choose components which can made Optical Fourier Transform: a square aperture and a convergent lens.

INTRODUCTION

We consider an optical system which is characterized by input signal $x(u_1,u_2)$ and output signal $y(u_1,u_2)$. We will find a relation between input signal and output signal by the means of system impulse response and transfer function. In optics we utilize linear shift invariant systems. By linear optical systems we understand that the system has a linear relation between input signal and output signals by the relation:

$$F[a x_1(u_1,u_2)+b x_2(u_1,u_2)]= aF[a x_1(u_1,u_2)]+bF[x_2(u_1,u_2)] \quad (1)$$

If at the input we apply a 2D optic impulse $\delta(u_1-\mu_1,u_2-\mu_2)$ then output image will be called system response to optical impulse with the notation $h(u_1,u_2;\mu_1\mu_2)$ so we will have the relation:

$$F[\delta(u_1-\mu_1,u_2-\mu_2)]=h(u_1,u_2;\mu_1\mu_2) \quad (2)$$

The system is linear shift invariant if system response to optical impulse is independent of impulse input position or more clearly an input impulse variation produce the same variation at the output.

$$F[\delta(u_1-\mu_1,u_2-\mu_2)]=h(u_1-\mu_1,u_2-\mu_2) \quad (3)$$

Knowing the impulse system response we can calculate in space with convolution relation between input and output signals:

$$y(u_1,u_2)=\iint x(v_1,v_2)h(u_1-v_1;u_2-v_2)dv_1dv_2 \quad (4)$$

If in (4) we apply Fourier Transform then we will have $Y(\omega_1,\omega_2)=X(\omega_1,\omega_2)H(\omega_1,\omega_2)$ were:

$$X(\omega_1,\omega_2)=\iint x(u_1,u_2)\exp[-j(\omega_1u_1+\omega_2u_2)]du_1du_2$$

input spectrum (5)

$$H(\omega_1,\omega_2)=\iint x(u_1,u_2)\exp[-j(\omega_1u_1+\omega_2u_2)]du_1du_2$$

frequency response (6)

In conclusion convolution in space is:

$$y(u_1,u_2)=f(u_1,u_2)*g(u_1,u_2) \quad (7)$$

And equivalently in spatial frequency by the means of Fourier transform we have:

$$Y(\omega_1,\omega_2)=X(\omega_1,\omega_2)H(\omega_1,\omega_2) \quad (8)$$

In optical linear systems (7) and (8) can be generalized:

$$y(u_1,u_2)=f(u_1,u_2)*g(u_1,u_2)*\ldots*j(u_1,u_2) \quad (9)$$

$$Y(\omega_1,\omega_2)=X(\omega_1,\omega_2)H(\omega_1,\omega_2)\ldots J(\omega_1,\omega_2) \quad (10)$$

In conformity with result in *eq.9* and *eq.10* we will make a simulation of an optical system made by a square aperture and a convergent lens.

FIRST EXAMPLE

In table 1 we will se mathematical relation in parallel for the two situations. We have as input signal a harmonically plane wave. Results are presented in Fig. 1

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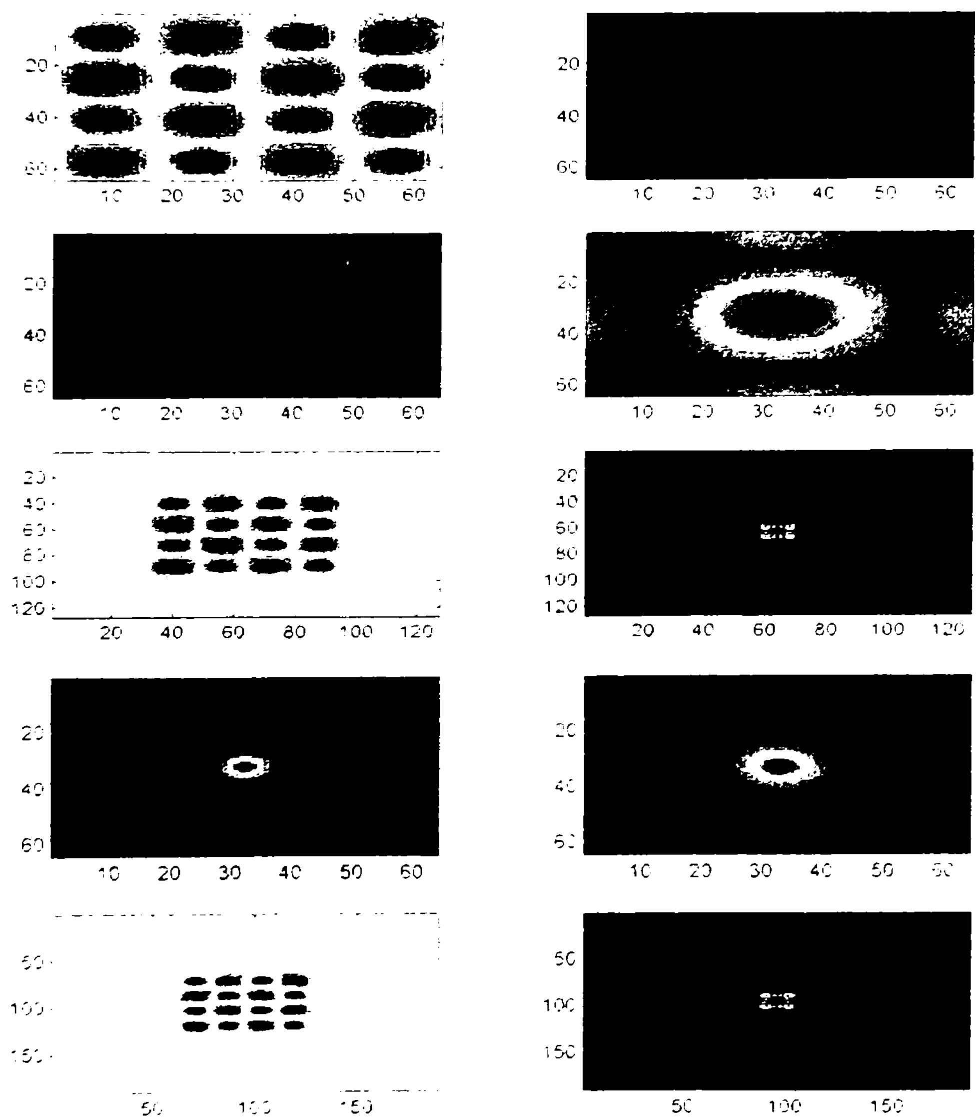
	Spatial domain analyses	Frequency domain analyses
Input signal	$A(x,y)=\sin(x)\sin(y)$	$A(\omega_1,\omega_2)=(\pi)^2[\delta(\omega_1-\Omega_1)-\delta(\omega_1+\Omega_1)][\delta(\omega_2-\Omega_2)-\delta(\omega_2+\Omega_2)]$
Aperture transfer function	$B(x,y)=rect(x)rect(y)$	$B(\omega_1,\omega_2)=sinc(\omega_1)sinc(\omega_2)$
convolution	$C(x,y)=A(x,y)*B(x,y)$	$C(\omega_1,\omega_2)=A(\omega_1,\omega_2)\,B(\omega_1,\omega_2)$
Lens transfer function	$D(x,y)=p2\exp(-\frac{x^2}{2}-\frac{y^2}{2})$	$D(\omega_1,\omega_2)=\exp(-\frac{\omega_1^2}{2}-\frac{\omega_2^2}{2})$
convolution	$E(x,y)=C(x,y)*D(x,y)$	$E(\omega_1,\omega_2)=C(\omega_1,\omega_2)\,D(\omega_1,\omega_2)$

Table 1



The first column present analyses in space domain for the case considered in Table 1, on the x and y axes have spatial dimentions. The second column present frequency domain analyses on x and y axes we have spatial frequency dimension.

SECOND EXAMPLE

We will make a more complex analysis with the help of an optical binary signal, made by the next optical Fourier string:

A(x,y)=[sin(x)+sin(3x)/3+sin(5x)/5+sin(7x)/7] [sin(y)+sin(y)/3+sin(5y)/5+sin(7y)/7]

This means that we have at the input 16 harmonics plan waves and equivalent spectrum consisting in 64 points. The rest of the calculus is in conformity with situation in Table 1

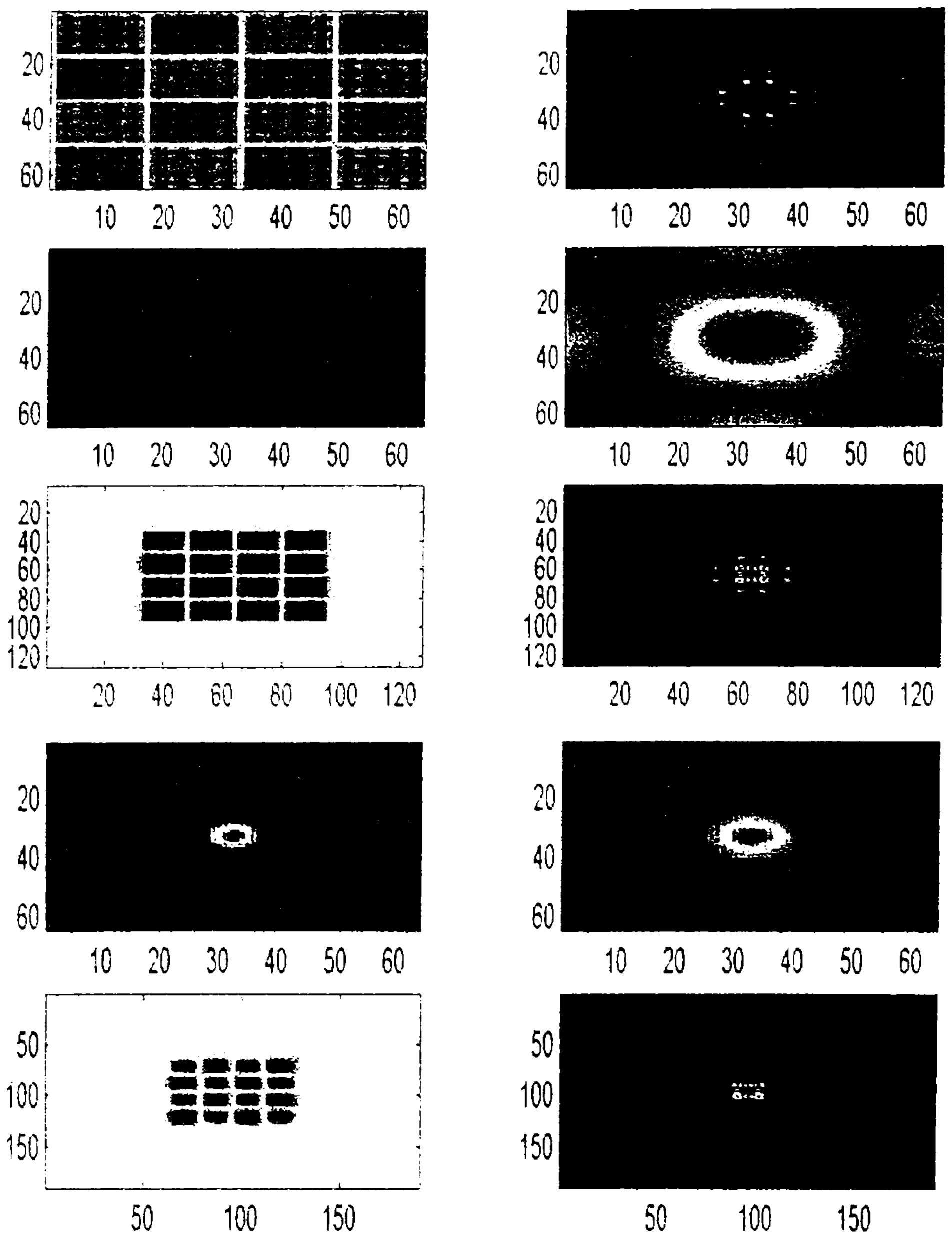


Fig. 2 The first column present analyses in space domain, on the x and y axes we have spatial dimensions. The second column present frequency domain analyses on x and y axes we have spatial frequency dimension.

THE THERD EXAMPLE

We have as input signal an image. It is know that an image can be decomposed in a very large number of harmonically plane waves of diverse spatial frequency and the spectrum will be a sum of points with random distributions in spatial frequency domain as in Fig. 3

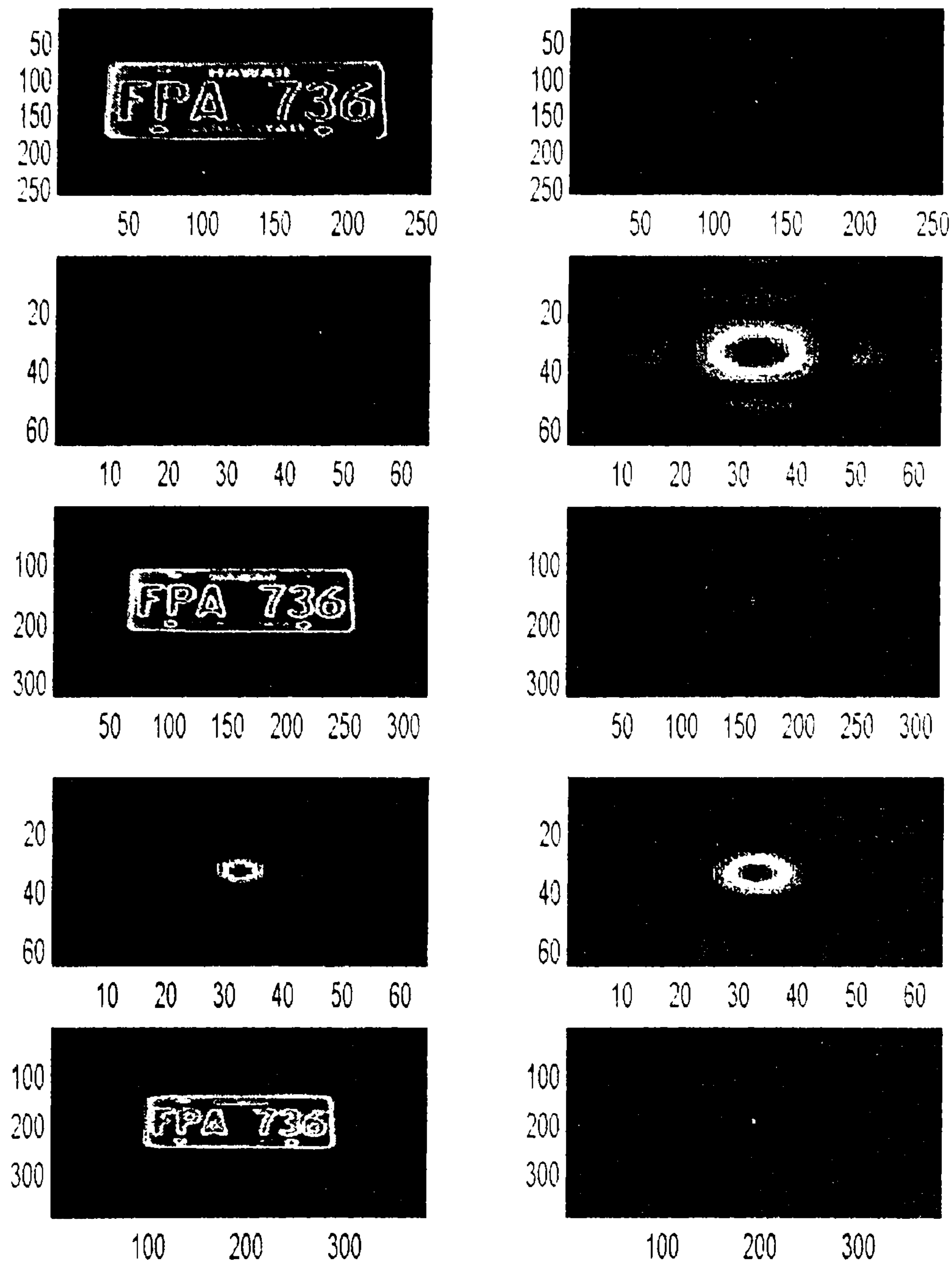


Fig. 3 The first column present analyses in space domain, on the x and y axes we have spatial dimensions. The second column present frequency domain analyses on x and y axes we have spatial frequency dimension.

CONCLUSION

In this paper we try to develop a calculus algorithm for an optical linear shift invariant system, made by optical component which can make optical Fourier transform. We try to have a global view of the process both in space and frequency. In spatial domain analyze we observe that after we convolve an image with an aperture the image change space position, without distortion. Convergent lens reduces signals but produce noise. In frequency analyze we observe that input signal spectrum is made by 4 points then 64 points and finally from a very large number. This spectrum is multiplied with transfer function and as result we have the same number of sample but other amplitude and function of their spatial position lens let to pass only that component that intersect lens (low frequency components).

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