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Optimal Chaotic Asynchronous DS-CDMA Communications over Frequency-Nonselective Rician Fading Channels

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Abstract – The use of some chaotic sequences is a new approach for spread-spectrum systems, ofering better BER (Bit Error Rate) performances and system capacity than conventional sequences. This paper presents some optimal sets of chaotic sequences for the A-DS-CDMA (Asynchronous Direct-Sequence Code-Division Multiple Access) system under the SGA (Standard Gaussian Approximation) condition. The A-DS-CDMA communication over frequency-nonselective Rician fading channel with AWGN (Additive White Gaussian Noise) is considered.

Keywords: A-DS-CDMA systems, BER, SGA, Chebyshev chaotic sequences, Rician fading channel.

I. INTRODUCTION

In A-DS-CDMA system using the single user matched-filter receiver the average BER performances depend mainly on the correlation properties of the spreading sequences assigned to the users. Hence, the problem of guaranteeing minimum performances to each user is equivalent to choosing spreading sequences with respect to an adequate correlation criterion.

Most DS-CDMA systems presented to date have used binary PN sequences, including Gold sequences and Kasami sequences, which prove some quasi-orthogonality correlation properties. These sequences present crosscorrelation values that depend on the generator polynomial degree [1].

A new family of spreading sequences is represented by chaotic sequences generated from the orbits of some dynamical discrete systems. These sequences present noise-like features that make them good for DS-CDMA systems. A single system, described by its discrete chaotic map, can generate a very large number of distinct chaotic sequences, each sequence being uniquely specified by its initial value [2]. This dependency on the initial state and the nonlinear character of the discrete map make the DS-CDMA system using these sequences more secure.

It was shown that sequences generated by chaotic Chebyshev polynomial maps are exact thus mixing and ergodic, and show good correlation properties [6], [7], and [9].

This paper is organized as follows. The second paragraph is presenting the BER estimation for the A-DS-CDMA system for optimal spreading sequences and perfectly random (white) spreading sequences, assuming a frequency non-selective fading channel with AWGN noise. The third part of the paper describes the design method for optimal sets of chaotic sequences for the A-DS-CDMA system based on the ergodicity of the dynamical systems with Chebyshev polynomial maps. The fourth part presents some simulation results compared to the theoretical average values for both optimal and white sequences. Finally, some conclusions are taken.

II. OPTIMAL BER COMPUTATION FOR A-DS-CDMA SYSTEM AND RICIAN NONSELECTIVE FADING CHANNEL

According to [3], [4], [4] and [9] the overall (non-faded) interference variance for the desired *i*th user from all other users in an A-DS-CDMA system can be computed as:

$$\sigma_A^2(i) = \frac{PT^2}{12N^3}\left(\sum_{\substack{k=1\\k\neq i}}^{K} r_{k,i}\right), \qquad (1)$$

where P represents the common signal power, T is the user's data signal symbol duration, N is the spreading factor, which is also identical to the period of the spreading sequences, $N = T/T_C$, T_C is the chip interval duration, $r_{k,i}$ is the interference term

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corresponding to the interfering user k, and K is the total number of users.

The interference term $r_{k,i}$ is written in terms of the autocorrelation function C_k as [3], [4]:

$$r_{k,i} = 2C_k(0)C_i(0) + 4\sum_{l=1}^{N-1} C_k(l)C_i(l) + \sum_{l=0}^{N-1}\left[C_k(l)C_i(l+1) + C_i(l)C_k(l+1)\right] \quad (2)$$

It is known that when perfectly random sequences (white noise-like sequences) are employed the overall interference variance from (1) can be written as [3], [6]:

$$\sigma^2_{A,\,white}(i) = \frac{PT^2(K-1)}{6N}, \quad (3)$$

where the normalization $C_i(0) = 1$ was considered.

According to [6] and [7] assigning the same interference variance to every user can attain the lower bound of average BER for all the users. This can be done with $C_k(l) = C_i(l) = C(l)$, for all k,i, and l. With the normalization $C_i(0) = 1$, the solution that minimizes the interference power in (1), considering the expression (2) for each term in the sum, leading to minimum BER under the SGA assumption is given by:

$$C_k(l) = (-1)^l \frac{r^{l-N} - r^{N-l}}{r^{-N} - r^{N}}, \ l = 0,1,2,\ldots,N-1, \ \forall k \quad (4)$$

where $r = 2 - \sqrt{3}$.

Note that when $l << N$, $C_k(l) \approx (-r)^l$, which decays exponentially with alternate sign. By introducing relation (4) into (2), the minimum interference power is obtained for user i:

$$\sigma^2_{A,\,optimum}(i) = \frac{PT^2(K-1)\sqrt{3}}{12N}\,\frac{r^{-2N} - r^{2N}}{r^{-2N} + r^{2N} - 2}, \quad (5)$$

For large values of N, the second term in (5) rapidly decreases to 1. It is important to note that the second term in (5) is very close to 1 even for N=5 (it differs from 1, starting with the sixth decimal value). With this approximation the minimum interference variance is:

$$\sigma^2_{A,\,optimum}(i) = \frac{PT^2(K-1)\sqrt{3}}{12N}, \quad (6)$$

Comparing the optimum case with the case when white sequences are employed, the first one offers an increase in the system BER performances which increases the number of users accommodated for the same mean BER, by

$$\frac{K_{optimum}}{K_{white}} \to 1.1547, \quad (7)$$

for large numbers of users. It is obvious from (7) that the optimum case increases the number of users by more than 15% than the white sequences case, for the same mean BER.

The output signal of a Rician nonselective fading channel is the sum of a non-faded version of the input signal (specular component) and a non-delayed Rayleigh faded version of the input signal (scatter component). All communications links are assumed to fade independently. We also assume that all users have the same faded power ratio γ^2.

Under the SGA assumption, the average BER for any user i over the Rician fading channel is given by [6], [7], [8]:

$$BER_A(i) = Q\left(\frac{\sqrt{\frac{P}{2}}T}{\sqrt{\gamma^2\frac{PT^2}{4} + (1+\gamma^2)\sigma^2_A(i) + \sigma^2_n}}\right), \quad (8)$$

where $\sigma^2_A(i)$ is the overall (non-faded) interference power for the desired ith user from all other users in an A-DS-CDMA, $\sigma^2_n = \frac{N_0 T}{4}$ is the variance for the additive Gaussian noise with two-sided PSD (Power Spectral Density) $N_0/2$ [3], [5], and the numerator is the is the useful component (the desired contribution from any user i). The Q-function is given by $Q(z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$.

The theoretical BER for both optimum and white sequences is presented in Fig. 1 for the following system parameters values: $K \in \{3, 5, 7, 9\}$, N=31 and $\gamma^2 = 0.1$.

III. OPTIMAL SETS OF CHAOTIC SEQUENCES FOR THE A-DS-CDMA SYSTEM

A second-order time-averaged statistic of the spreading sequences is needed for the sequence design and performance analysis. According to the ergodic theory the autocorrelation function of a sequence generated by a measure-preserving and ergodic transformation can be estimated statistically [6].

An example of ergodic transformation is the nth degree Chebyshev polynomial defined by:

$$T_n(x) = \cos(n \cdot \arccos(x)), \quad (9)$$

where x takes values from the interval [-1, 1]. It was shown that the Chebyshev polynomials of degree $n \geq 2$ are mixing and thus ergodic and they have an invariant measure.

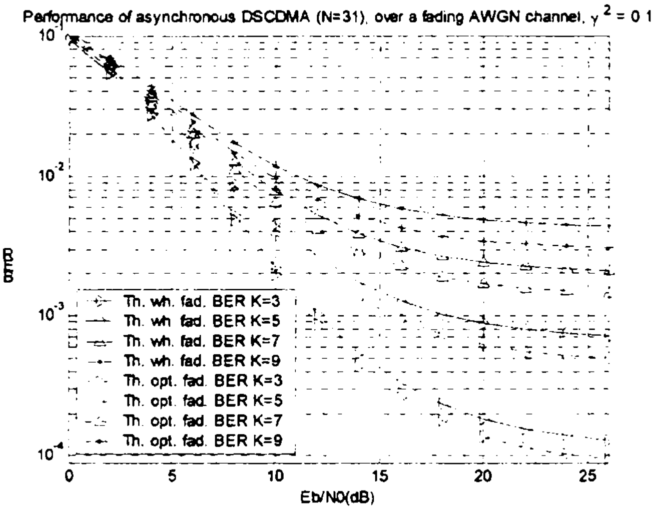

Fig.1. The theoretical BER performances for optimum sequences and white sequences A-DS-CDMA system over frequency-nonselective Rician fading channel with AWGN. The common faded power ratio is $\gamma^2 = 0.1$.

The dynamical systems given by Chebyshev polynomial maps are from a special class of dynamical systems with Lebesque spectrum [6]. The Lebesgue spectrum systems denoted as $\varphi(x)$ are associated with a special set of orthonormal basis functions $\{f_{\lambda,j}(x) | \lambda \in \Lambda, j \in F\}$ for Hilbert space L_2, where λ labels the classes and j labels the functions within each class. These particular basis functions $f_{\lambda,j}(x)$ have an important property such that, $f_{\lambda,j} \circ \varphi = f_{\lambda,j-1}, \forall \lambda \in \Lambda, j \in F$. This property states that all other basis functions in the same class can be generated from one basis function by using compositions with powers of the dynamical system $\varphi(x)$.

For the particular case of Chebyshev polynomial maps, we consider the pth degree polynomial map, i.e., $\varphi(x) = T_p(x)$, where $p \geq 2$ is prime. The associated basis functions for $L_2\{[-1,1]\}$ are also Chebyshev polynomials $\{T_i(x)\}_{i=0}^{\infty}$.

Let us consider the polynomial function $P(x)$ in the Hilbert space $L_2\{[-1,1]\}$ with the following expression:

$$P(x) = \sum_{j=1}^{N} (-r)^j T_{p^j}(x), \qquad \forall x \in [-1,1] \tag{10}$$

By using the ergodic theory the average of $P^2(x)$ for a sequence generated by a Chebyshev transformation $T_p(x)$ is given by

$$C(0) = \frac{1}{N}\sum_{i=1}^{N} P^2(x_i) \overset{N \to \infty}{\to} \int_{-1}^{1} P^2(x)\rho(x)dx = \frac{1}{2}\frac{r^2\left(1-r^{2N}\right)^{not}}{\left(1-r^2\right)} = A \tag{11}$$

and the normalized autocorrelation function of such a sequence is given by

$$\frac{C(l)}{A} = \frac{1}{A}\frac{1}{N-l}\sum_{i=1}^{N} P(x_i)P(x_{i+l}) \overset{N \to \infty}{\to} \frac{1}{A}\int_{-1}^{1} P(x)P\left(T_{p^l}(x)\right)\rho(x)dx = (-1)^l \frac{\left(r^{l-N} - r^{N-l}\right)}{\left(r^{-N} - r^{N}\right)}, \text{for finite } l \tag{12}$$

It is easy to note that the average value of the left-side term, $E_{x_0}\left[\frac{C(l)}{A}\right]$, is equal to the right-side term in (12), even for finite N if the initial condition x_0 is selected randomly according to the corresponding invariant measure.

Considering the same value for the parameter $r = 2 - \sqrt{3}$ as in (4) the output sequences $\{y_1, y_2, \ldots, y_N\}$ generated by

$$y_i = \frac{1}{\sqrt{A}} P(x_i), \quad x_{i+1} = T_p(x_i) \tag{13}$$

are the optimal spreading sequences for the A-DS-CDMA system. Hence, each user is assigned a different spreading sequence generated by the same Chebyshev map but having a different initial condition or generated by a different-degree Chebyshev map.

IV. SIMULATION RESULTS

The A-DS-CDMA system using optimal Chebyshev polynomial maps of degree $p = 3$ and Gold sequences generated by primitive polynomials of degree $n = 5$, having the period $N = 2^n - 1 = 31$, was considered.

The estimated average BER was evaluated for $K = 10$ users and the energy-per-bit to noise DSP ratio E_b / N_0 taking values from 0 to 30 dB. The common faded power ratio is taking the value $\gamma^2 = 0.1$.

The resulting BER as a function of the ratio E_b / N_0 is depicted in Fig. 2.

The A-DS-CDMA system capacity is also an important parameter to measure. The average BER was estimated considering several values for the number of users $K \in \{12, \ldots, 30\}$ having the same value of the energy-per-bit to noise DSP ratio E_b/N_0=18dB. The resulting BER values as a function of the number of users K is depicted in Fig. 3.

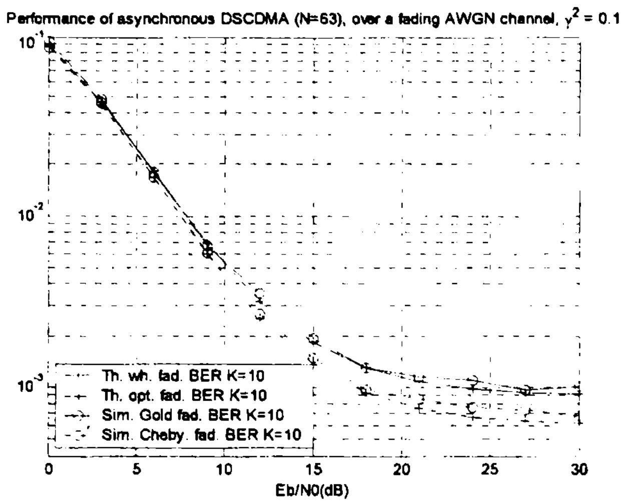

Fig.2. Theoretical and simulated BER for A-DS-CDMA system over frequency-nonselective Rician fading channel with AWGN, using Chebyshev and Gold sequences (N=63, K=10). The common faded power ratio is $\gamma^2 = 0.1$.

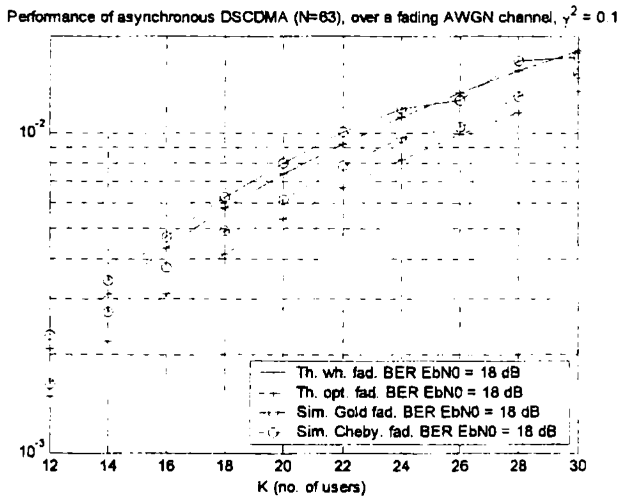

Fig.3. Theoretical and simulated BER for A-DS-CDMA system over frequency-nonselective Rician fading channel with AWGN, using Chebyshev and Gold sequences (N=63, $K \in \{12, \ldots, 30\}$, E_b/N_0=18dB). The common faded power ratio is $\gamma^2 = 0.1$.

V. CONCLUSIONS

A family of optimal spreading sequences for the A-DS-CDMA system with the SGA approximation hypothesis was introduced for minimizing the average BER. The generation method for the optimal Chebyshev maps and their correlation properties were also presented.

The BER performance of the A-DS-CDMA system was estimated assuming a frequency non-selective fading channel with AWGN noise. An asynchronous DS-CDMA system using optimal sequences offers a capacity increase of about 15% than when white sequences or Gold codes are used.

The sensitive dependency of chaotic maps on the initial condition offers both a greater number of available sequences and security increase.

The simulation results show that optimal Chebyshev sequences are better than Gold sequences in terms of the average BER per user, which is consistent with the analytical result presented Section II and depicted in Fig. 1.

There are some differences between the simulation and analytical results given the fact that Gold sequences are not perfectly white, Chebyshev sequences are in fact pseudo-optimal, and the SGA approximation is not quite valid for a small number of users.

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