# Shape Representation and Retrieval Using Centroid Radii and Turning Angle 

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#### Abstract

Among all issues related to Content Based Image Retrieval systems, retrieving images based on shapes is an important one. In the paper a contourbased approach to shape representation and similarity measure is presented. The shape representation is based on centroid radii and turning angle. The proposed algorithm is invariant to translation, scale and rotation. The effectiveness of algorithm in the content-based retrieval of shapes is illustrated using a database of synthetic shapes. The results of the experiments show the competitiveness of the algorithm.


Keywords: shape representation, shape similarity measure, image retrieval, distance histograms, turning angle

## I. INTRODUCTION

Interest in the potential of digital images has increased enormously over the last few years. The internet collection of images has become very large. However, the process of locating a desired image in a large and dynamic collection emerges as a challenging problem. Hence, the image database query problem is becoming widely recognized and the search for solutions is an increasingly active area.
Problems with traditional methods of image indexing have led to the rise of interest in techniques for retrieving images on the basis of automatically derived features such as color. texture and shape. Shape of objects contained in an image is an important image feature. For retrieval based on shapes. images must be segmented into individual objects using certain methods, possibly a semiautomatic method [12]. After that, the basic issue of shape-based image retrieval is shape representation and similarity measurement between shape representations.
Shape matching has been approached in a number of ways like Fourier descriptors [2]. curvature scale space [5]. tuming angle [4], centroid radii [14], distance histograms [11], Zernike moments [ 3 ], geometric moments [1] and grid descriptions [7].

In the studies which have conducted to this paper, the distance histogram method [11] was used. In distance histogram method, shapes are represented using the distances from the centroid to its boundary (radii). The shapes will be compared by computing their histograms using the radii. The method is invariant to translation, scaling and rotation and does not care about the starting point for obtaining the radii. On the other hand, the method has a drawback: in some cases, for different shapes, their distance histograms will be the same. This it is happening because the method uses only the radii and does not care about the spatial information.
The method described in this paper is a contour based method and represents a shape by its radii and direction of the edges to eliminate the drawback of the distance histograms method.
The rest of the paper is organized as following: section Il presents the distance histogram method; section III describes a new technique for shape representation and similarity based on turning angle and centroid radii; section IV shows the results of the retrieval experiments; section V concludes the paper.

## II. THE DISTANCE HISTOGRAM METHOD

This method, proposed in [11], is based on a histogram computed from the distances between the centroid of the shape and its boundary. Accordingly to this method, the main steps for representing a shape are:

1. Calculate the centroid of a shape, knowing its polygon which approximates the boundary of the shape;
The calculation of the ( $x, y$ ) centroid coordinates of a shape is based on Green's theorem in plane. First, it is necessary to calculate the area of the polygon. Given a polygon and its vertices ( $x_{1}, y_{i}$ ), $i=0,1, \ldots, n$, $x_{1}=x_{n}$ and $y_{n}=y_{n}$, the area of a polygon in plane is obtained using the formula [9]:

[^0]$A=\frac{1}{2} \sum_{i=0}^{n-1} x_{i} y_{i+1}-x_{i+1} y_{i}$
The area computed by (1) is a signed value, where a negative sign indicates that the vertices are in clockwise order and a positive sign indicates that the vertices are in counter clockwise order.
The centroid coordinates are:
\[

$$
\begin{equation*}
\bar{x}=\frac{\mu_{x}}{A}, \bar{y}=\frac{\mu_{y}}{A} \tag{2}
\end{equation*}
$$

\]

where
$\mu_{x}=\frac{1}{6} \sum_{i=0}^{n-1}\left(x_{i+1}+x_{i}\right)\left(x_{i} y_{i+1}-x_{i+1} y_{i}\right)$
and
$\mu_{y}=\frac{1}{6} \sum_{i=0}^{n-1}\left(y_{i+1}+y_{i}\right)\left(x_{i} y_{i+1}-x_{i+1} y_{i}\right)$
2. Select a set of sample points in the boundary of the polygon, and calculate the distances between the sample points and the centroid of the shape;
The number of sample points is variable. It can be changed for different situations. But the sample points are not uniformly distributed around the boundary. Each edge has assigned a number of sample points proportionally to its length. For example, if the length of an edge is $\mathrm{L}_{\mathrm{i}}$, the perimeter of the shape is L , and the total number of sample points is N , then the number of sample points in that edge $N_{i}$ will be:

$$
\begin{equation*}
N_{i}=\frac{L_{i}}{L} N \tag{5}
\end{equation*}
$$

These points are evenly spread on the edge.
The sample points and the centroid of the shape are used to calculate the distances. For example, given a sample point $\mathrm{s}_{\mathrm{i}}=\left(\mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{i}}\right)$ and the centroid $\mathrm{c}=\left(\mathrm{x}_{\mathrm{c}}\right.$, $y_{c}$ ), the distance between them is:

$$
\begin{equation*}
d\left(s_{i}, c\right)=\sqrt{\left(x_{i}-x_{c}\right)^{2}+\left(y_{c}-y_{i}\right)^{2}} \tag{6}
\end{equation*}
$$

3. Construct a distanco histogr mbusua war wer distances obtained at step 2;
the range of all distances $\left[0, D_{\text {max }}\right]$ is separated into several ranges, i. e. R ranges. Then, the ranges of distances can be represented by: $\left[0, D_{\text {max }} / R\right]$, $\left(\mathrm{D}_{\max } / \mathrm{R}, 2 \mathrm{D}_{\max } / \mathrm{R}\right],\left(2 \mathrm{D}_{\max } / \mathrm{R}, 3 \mathrm{D}_{\max } / \mathrm{R}\right], \ldots,((\mathrm{R}-\mathrm{a})$ $\left.\mathrm{D}_{\text {max }} / \mathrm{R}, \mathrm{D}_{\text {max }}\right]$ and the distance histogram can be represented as: $D:\left(d_{0}, d_{1}, d_{2}, \ldots, d_{\text {R. }}\right)$, where di,
$i \in[0, R-1]$ is the number of distances belong to this distance range.
4. Normalize the distance histogram;

Two similar shapes at different scales will have different values of distances calculated at step 2 . To make this representation invariant to scale these distances must be normalized for scaling. The process of distance normalization consists of dividing the value of all distances by the value of the maximum distance. After that the value of all distances will be in $[0,1]$. And because the sample points are chosen based on the length of the edge, evenly spread on it. two similar shapes with different sizes will generate the same normalized distances. Therefore the method is invariant to scale after normalization.
This approach is invariant to translation because the distance set will not change after translating the shape. The sample points are chosen in each edge proportionally with the edge's length and they are spread evenly on it. The location of the sample points will not change after rotating the shape. Therefore this method is invariant to rotation because the distance set will not change after rotating.
After representing the shapes by distance histograms, the similarity among them can be calculated by Euclidean distance between their distance histograms. For example, for two shapes with the distance histograms $\mathrm{Dl}:\left(\mathrm{dl}_{0}, \mathrm{dl}_{1}, \mathrm{dl}_{2}, \ldots\right.$, $\left.\mathrm{d} 1_{R-1}\right)$ and $\mathrm{D} 2:\left(\mathrm{d} 2_{i}, \mathrm{~d} 2_{1}, \mathrm{~d} 2_{2}, \ldots, \mathrm{~d} 2_{\mathrm{R} \cdot \mathrm{I}}\right)$ the distance between them is:

$$
\begin{equation*}
d(D 1, D 2)=\sqrt{\sum_{i=0}^{R-1}\left(d 1_{i}-d 2_{i}\right)^{2}} \tag{7}
\end{equation*}
$$

## III. A NEW SHAPE REPRESENTATION

For the two different shapes in Fig. 1, applying the distance histogram method, their histograms will be similar as it is shown in Fig. 2. Therefore, using the distance histogram method the two shapes will be similar, although they look different. This is because the distance histogram method discards spatial information.


Fig. 1. Two different shapes with similar histograms
To consider spatial representation, the method proposed in this paper represents a shape using the radii and directions of the edges. The directions of the edges are represented by the turning angles of each edge. Turning angle [4] is defined as the angle
formed with a reference axis by the counterclockwise tangent to the boundary of a shape which goes from a boundary point of a shape to the next one. In Fig. 3 the edge directions of the shape are represented by their turning angles, where the reference axis is considered to be the x axis.
Therefore, a shape will be represented by its edge directions, each edge direction having associated a hist of corresponding radii.


Fig. 2. The histograms of shapes from Fig. I


Fig 3. A shape and turning angles of each edge
The main steps of the proposed method are:

1. determine the edges' directions of the shape;
2. calculate the centroid of the shape like in distance histogram method - equation (2);
3. select a set of sample points in the boundary of the polygon and calculate the distances between the sample points and the centroid of the shape, like in distance histogram method equation (5);
4. construct a list of edges' directions such that each entry in this list will have associated all the radii corresponding to that direction.
This representation is invariant to translation, but it is not invariant to scale. For making it invariant to scale, like in distance histogram method, all the distances between the centroid and the chosen sample points will be nomalized. The normalization of the radii consists of dividing the value of all distances by the value of the maximum distance. The representation is not invariant to rotation For a shape rotated with an angle $\alpha$, all the edges' directions will increase or decrease with the same angle $\alpha$.
To see if two shapes are similar using this method, first the lists of directions must be compared, and if these directions are similar the radii associated with them must be also compared.

Representing the shapes as above, one shape can have more edge directions that the other. But it cannot say that the shapes are different because the shapes can be affected by noise. To make this supposition, first it is necessary a preprocessing stage to approximate the boundary of a shape. The boundary approximation is a process that eliminates insignificant shape features and reduces the number of data points.
This preprocessing stage is used to reduce the influence of noise and to simplify the shapes by removing irrelevant features and keeping only which are relevant. The boundary of the shape is analyzed in a number of evolution steps [8]. On every evolution step, a pair of consecutive line segments $s_{1}, s_{2}$ is substituted with a single line segment joining the endpoints of $s_{1}$ and $s_{2}$. The key property of this evolution is the order of the substitution. The substitution is done according to a relevance measure K given by:

$$
\begin{equation*}
K\left(s_{1}, s_{2}\right)=\frac{\beta\left(s_{1}, s_{2}\right) l\left(s_{1}\right) l\left(s_{2}\right)}{l\left(s_{1}\right)+l\left(s_{2}\right)} \tag{8}
\end{equation*}
$$

where $\beta\left(s_{1}, s_{2}\right)_{\text {is the turn angle at the common }}$ vertex of segments $s_{1}, s_{2}$, and 1 is the length function normalized with respect to the total length of a polygonal curve. The evolution algorithm ....um_.. t...t v..tic... which are surrou ..ded by segments with high value of $K\left(s_{1}, s_{2}\right)$ are important while those with a low value are not. The segments correspond to noise are small segment pairs which result in small values of the relevance measure K . Thus these segment pairs are removed in an early stage of the evolution process. The vertices of the simplified contour are also vertices of the original contour.
After that, the boundary of the shape will be without noise. Now for two shapes having different numbers of edges' directions it can be said that the shapes are not similar and this method does not compute the distance between them.
The similarity between two shapes will be calculated in two steps:
Compare the edges' directions. For two shapes A and $B$ with lists of edges' directions ( $\mathrm{dA}_{1}, \mathrm{dA}_{2}, \ldots$, $\mathrm{d} \mathrm{A}_{\mathrm{n}}$ ) and ( $\mathrm{dB}_{1} . \mathrm{dB}_{2}, \ldots, \mathrm{~dB}_{\mathrm{n}}$ ) they may be similar if their directions correspond. This means that the directions are the same, or one list of directions is rotated with an angle $\alpha$. For testing this, the differences between each pair of angles need to be computed. If all these $n$ differences are approximately the same (less than a predefined threshold) the shapes may be similar. But when constructing the lists of edges' directions it is not known which edge is the first one. Therefore to see if the directions of the two shapes are the same, the differences between angles of one list of directions and circularly shift of the other list are computed. Doing this, the method will be invariant to rotation
and does not care about the starting point. If an equality of all these differences is found, then the shapes may be similar, otherwise the shapes are not similar. If an equality between the two lists of directions exists, suppose the order of these lists of directions is $\left(\mathrm{dA}^{\prime}, \mathrm{dA}^{\prime}, \ldots . \mathrm{dA}^{\prime}{ }_{n}\right)$ and ( $\mathrm{dB}^{\prime}{ }_{1}$, $\mathrm{dB}^{\prime}, \ldots, \mathrm{dB}_{n}{ }_{n}$ ).
Compare the radii associated with the lists of edges' directions obtained in step l. For example, it must compare the radii corresponding to direction $\mathrm{dA}^{\prime}$, with the radii corresponding to direction $\mathrm{dB}^{\prime}$, and so on. In fact it must be compared the lists of radii. To compare these values it is sufficient to compare the standard deviation of the radii associated with each direction. The distance between the two shapes will be the Euclidean distance between the standard deviations of radii corresponding to each direction. Consider that the standard deviations of radii corresponding to each direction are (stdA' ${ }_{1}$, stdA' ${ }_{2}$, $\left.\ldots, \operatorname{stdA}_{n}\right)$ and ( $\operatorname{stdB}^{\prime}, \operatorname{stdB}^{\prime}, \ldots, s^{\prime}, \ldots$ ' $_{n}$ ). Then the distance between the two shapes will be:

$$
\begin{equation*}
d(A, B)=\sqrt{\sum_{i=1}^{n}\left(\operatorname{stdA}_{i}^{\prime}-s t d B_{i}^{\prime}\right)^{2}} \tag{9}
\end{equation*}
$$

## IV. RETRIEVAL EXPERIMENTS

To test the retrieval performance of the proposed method compared with the distance histogram method, a retrieval framework has been implemented on a database with synthetic shapes. The performance has been evaluated using precision and recall [6].
Precision is defined as the ratio of the number of similar shapes retrieved to the total number of shapes retrieved. Recall is defined as the ratio of the number of similar shapes retrieved to the total number of similar shapes in the whole database. Precision indicates accuracy of the retrieval and recall indicates the robustness of the retrieval performance.
The database used consists of approximate 3,000 polygons. The average precision and recall of the shapes used as queries is given in Fig. 4.


Fig. 4 Average retrieval performance of the two methods
The method proposed in this paper outperforms the distance histogram method by precision and recall. This method by the preprocessing stage for boundary approximation has a supplementary step compared to distance histogram method. But this
stage eliminates the noise and reduces the number of edges, making easier to obtain the sample points.

## V. CONCLUSIONS

This paper presents a new shape representation and retrieval method based on the shapes' contour which has a better retrieval performance compared to the distance histogram method. The method is invariant to translation, scale and rotation. The distance histogram method discards spatial information to obtain rotation invariant. In the proposed method, the radii together with the edges' directions associated with them are used for shape representation.
In the presented method a preprocessing step for reducing noise is necessary. However this step reduces the number of edges for a shape and computing the sample points from the shapes boundary is easier.

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