SCIENTIFIC BULLETIN
OF
THE POLITEHNICA UNIVERSITY TIMISOARA

Vol 63(77), Issue 1 MATHEMATICS –PHYSICS 2018

A NEW APPROACH OF REFRACTION FOR 3D ELECTRIC FIELD IN NONLINEAR DIELECTRICS WITH PERMANENT POLARIZATION AND RANDOM ANISOTROPY
Part III. Applications of the new refraction theorems for particular cases

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Abstract

Using a new permittivity - defined by author (in Part I) for dielectrics with permanent polarization we will demonstrate new theorems of refraction (in Part II), more general, for three-dimensional (3D) electric field lines at the separation surface of two nonlinear and anisotropic materials with permanent polarization, which have random polarization main directions. Then (in Part three), some applications of the new refraction theorems are presented, for particular cases. [1]

1 Applications of the new refraction theorems for particular cases.

1.1 Fields 3D in nonlinear and isotropic dielectrics, with permanent polarization.

For isotropic dielectrics, the components of calculation permittivity in the two materials are:

$$\varepsilon_{p1x} = \varepsilon_{p1y} = \varepsilon_{p1z} = \varepsilon_{p1}\,, \tag{1.1}$$

$$\varepsilon_{p2x} = \varepsilon_{p2y} = \varepsilon_{p2z} = \varepsilon_{p2}\,. \tag{1.2}$$

If we take into account equations (1.1) and (1.2), the theorem (31) from [10] for refraction of electric field strength lines becomes

$$\begin{aligned} &\varepsilon_{p1}(E_{1xn} + E_{1yn} + E_{1zn}) - \varepsilon_{p2}(E_{2xn} + E_{2yn} + E_{2zn}) + \\ &+(P_{p1xn} + P_{p1yn} + P_{p1zn}) - (P_{p2xn} + P_{p2yn} + P_{p2zn}) = 0\,. \end{aligned} \tag{1.3}$$

[1] Keywords and phrases: *a new permittivity, permanent polarization, random anisotropy, 3D refraction theorems*

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Considering the signification from theorem (31) from [10], the expression (3) may be written shortly in this way:

$$\varepsilon_{p1}\, E_{1n} = \varepsilon_{p2}\, E_{2n} - (P_{p1n} - P_{p2n})\,. \tag{1.4}$$

Alike, taking into account equations (1.1) and (1.2), the theorem (31) from [10], for refraction of calculation electric flux density lines, becomes:

$$\frac{1}{\varepsilon_{p1}}(D_{p1xt} + D_{p1yt} + D_{p1zt}) - \frac{1}{\varepsilon_{p2}}(D_{p2xt} + D_{p2yt} + D_{p2zt}) = 0. \tag{1.5}$$

Considering the signification from theorem (36) from [10], the expression (1.5) can be written in a more concise form:

$$\frac{D_{p1t}}{\varepsilon_{p1}} = \frac{D_{p2t}}{\varepsilon_{p2}}\,. \tag{1.6}$$

Equations (1.4) and (1.6) represent the theorems of refraction for $\overline{E}$ and $\overline{D}_p$ in 3D fields, for nonlinear and isotropic dielectrics, with permanent polarization. We can remark that, *the theorem (6) of refraction in dielectrics with permanent polarization (*for the tangent components of $\overline{D}_p$*) has a similar form* (but another content) *with the classical theorem of refraction in materials without permanent polarization.*
Also, *the theorem (6) has a more simple form than classical treatment for refraction of electric flux density lines* $\overline{D}$, in nonlinear and isotropic dielectrics with permanent polarization (see [6], eq. (31)). *This simple form occurs as a result of the introduction of new quantities* $\overline{D}_p$ *and* $\overline{\overline{\varepsilon}}_p$.

1.2 Fields 3D in nonlinear and isotropic dielectrics without permanent polarization

For dielectrics without permanent polarization ($\overline{P}_p = 0$), from eq. (1.5) from [9] we obtain $\overline{D}_p = \overline{D}$. Also, from eq. (1.8) from [9], for isotropic media we can write $\varepsilon_p = D_p/E = D/E$. So $\varepsilon_p = \varepsilon$, which means that (if the dielectric is without permanent polarization)the calculation permittivity is identical with the classical permittivity.

Particularizing eq.(1.4) and (1.6) for this case and taking into account the previous observation, we obtain

$$\frac{\varepsilon_{p1}}{\varepsilon_{p2}} = \frac{D_{p1t}}{D_{p2t}} = \frac{E_{2n}}{E_{1n}} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{D_{1t}}{D_{2t}}. \tag{1.7}$$

The dielectrics are isotropic and therefore $\overline{D}_p$ and $\overline{E}$ have the same spectrum. Since $\overline{D}_p$ and $\overline{D}$ are identical, it follows that $\overline{D}$ and $\overline{E}$ have the same spectrum.

1.3 Fields 2D in nonlinear and anisotropic/isotropic dielectrics with permanent polarization

In the case of nonlinear and anisotropic or isotropic dielectrics, for two-dimensional (2D) field, vectors $\overline{D}_p$, $\overline{E}$ and $\overline{P}_p$ have not the components after z axis, but only after x and y axes. Eq. (1.4) and (1.6) are valid in this case, but z components missing from the detailed eq. (1.3) and (1.5).

For *2D fields, in anisotropic dielectrics by orthogonal directions*, if we represent $\overline{D}_{p\lambda}$ and $\overline{E}_{\lambda}$ vectors and their normal and tangential components to the surfaceS_{12}, we obtain the representations of Figure 1 (for $\overline{D}_p$) and Figure 2 (for $\overline{E}$). These are analogous to classical representation, but $\overline{D}_p$ in place to $\overline{D}$ (see [8], Figure 2 and Figure 3).

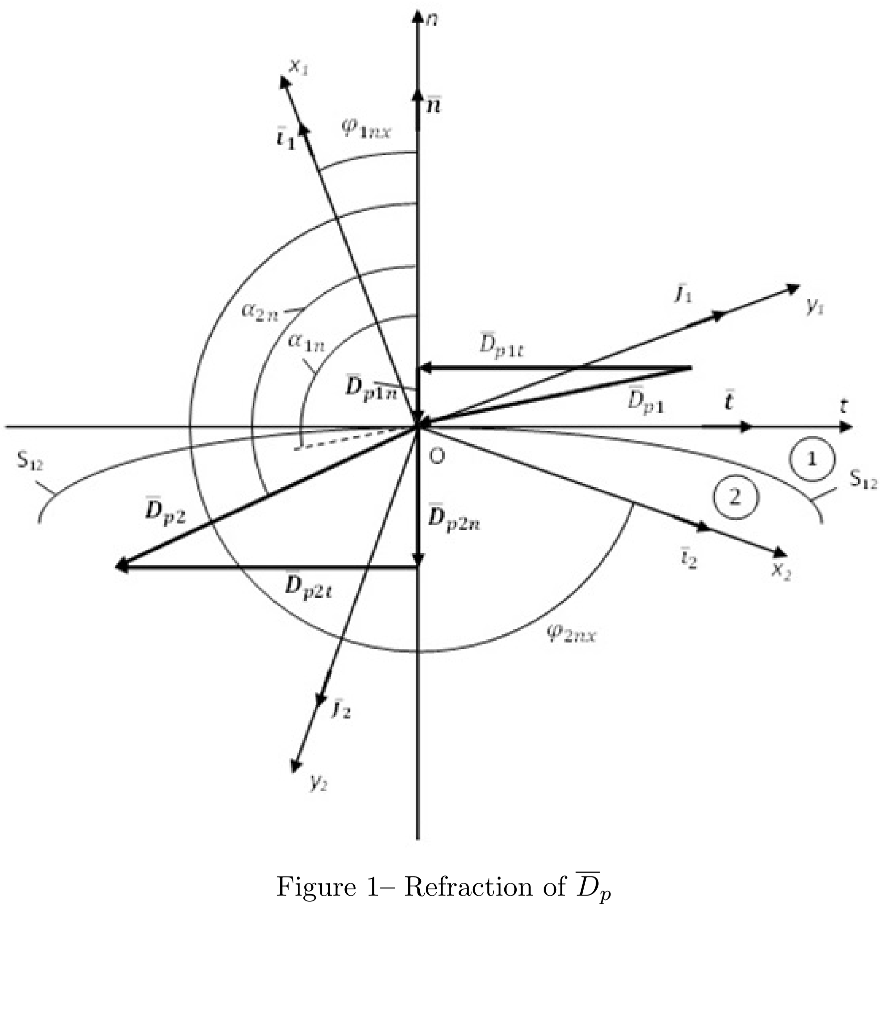

Figure 1– Refraction of $\overline{D}_p$

For *isotropic dielectrics* $\overline{D}_p$ *and* $\overline{E}$ *have the same spectrum* and would obtain similar representations to those in Figure 3 and Figure 4, but with
$\alpha_\lambda = \beta_\lambda\,,\ \ (\lambda = 1,\ 2)$.

We remark that *the classical quantities* $\overline{D}$ *and* $\overline{E}$ *have not the same spectrum* (even if the dielectrics are isotropic) *because it is permanent polarization.*

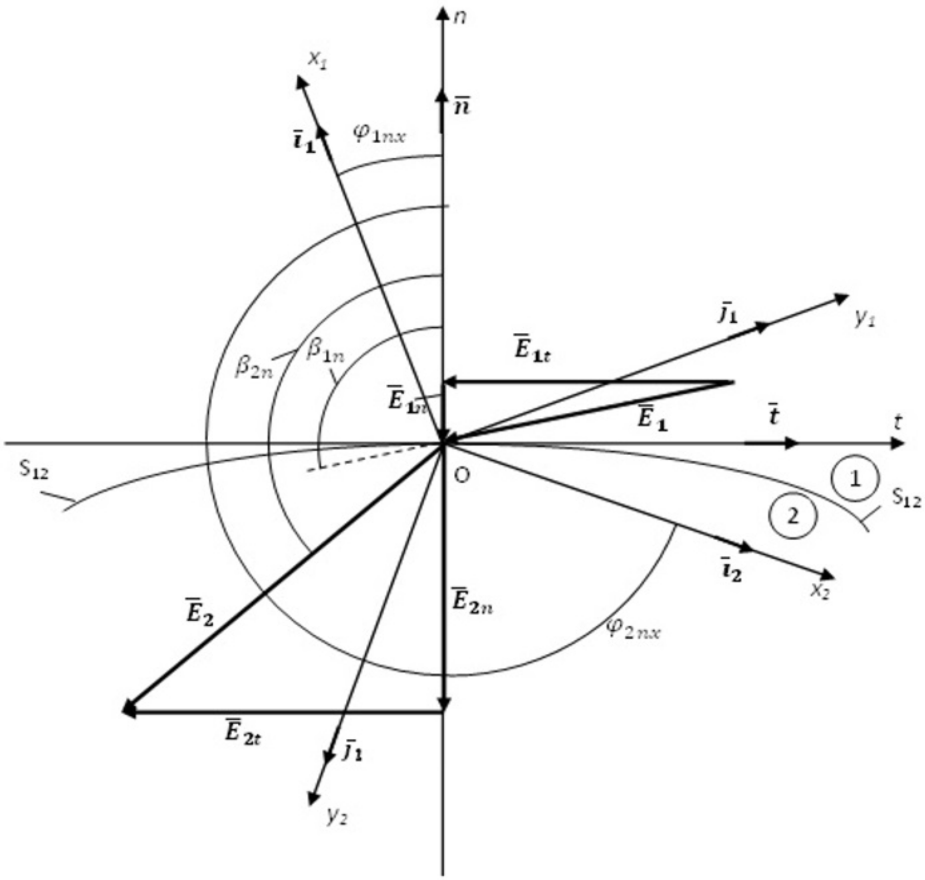

Figure 2– Refraction of $\overline{E}$

1.4 Fields 2D in nonlinear and isotropic dielectrics without permanent polarization

In this case $\overline{D}_{p\lambda} = \overline{D}_\lambda\,,\quad \varepsilon_{p\lambda} = \varepsilon_\lambda\,,\quad \alpha_{\lambda n} = \beta_{\lambda n}\,,\quad \alpha_{\lambda t} = \beta_{\lambda t}$ and $\alpha_{\lambda n} + \alpha_{\lambda t} = 90^0(\lambda = 1,\ 2\,)$. Taking into account and the classical representation for 2D field refraction in isotropic dielectrics without permanent polarization [1, 2, 3], with these specifications, eq. (1.7) can be completed, being found and classical expressions:

$$\frac{\varepsilon_{p1}}{\varepsilon_{p2}} = \frac{D_{p1t}}{D_{p2t}} = \frac{E_{2n}}{E_{1n}} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{D_{1t}}{D_{2t}} = \frac{tg\ \alpha_{1n}}{tg\ \alpha_{2n}} = \frac{tg\ \beta_{1n}}{tg\ \beta_{2n}}. \tag{1.8}$$

2 Other specifications

From the general expressions of refraction theorems for lines of vectors $\overline{E}$ and $\overline{D}_p$, or from particular forms already mentioned, can be obtained also other particular forms. Such cases are possible when the permanent polarization vectors $\overline{P}_p$ have particular orientations, when one of the dielectrics has permanent polarization and the other one does not (for example: dielectric with permanent polarization – air, dielectric with permanent polarization – ordinary dielectric without permanent polarization and so on), or when the polarization main directions are particular orientations(for example, rectangular directions) etc.

If known the electric hysteresis cycle for the isotropic dielectric, we should determine the nonlinear function$D_p(E)$. Then, it can be determined the diagram of nonlinear function $\varepsilon_{rp}(E)$ (or $\varepsilon_p(E)$), following the procedure used by the author for the permeability of permanent magnets (see [5], [7]). For an anisotropic dielectric with permanent polarization, the electric hysteresis cycles must be known by the all main directions of polarization. In this case, it can be determined (following similar procedures) the diagrams of nonlinear functions $D_{p\nu}(E)$ and $\varepsilon_{rp\nu}(E)$, where $\nu = x\ ,\ y\ ,\ z$.

It notes that similar theorems were demonstrated by the same author (see [5], [7]) for the *magnetic field lines refraction in materials with permanent magnetization* (i.e. permanent magnets). If compare the two situations, is *remarkable analogy* between the equations for electric field refraction in dielectrics with permanent polarization, respectively the equations for magnetic field refraction in permanent magnets.

3 Conclusions

Referring to the entire work (Part one, Part two, Part three) it highlights the following conclusions:

The introduction of calculation flux density $\overline{D}_p$ and of new relative permittivity $\overline{\overline{\varepsilon}}_{rp}$ for anisotropic dielectrics, nonlinear and with permanent polarization (in Part one, namely [9]) is a useful operation, because *the solution of field problem can be obtained in an advantageous way.*

In applications that refer to new forms of the theorems for 3D electric field lines refractions in nonlinear dielectrics, with permanent polarization and random anisotropy (in Part two, namely [10]), *the equations obtained are more concise, so simpler*. It is possible to make and *useful analogies* with the simpler case of the materials without permanent polarization.

As applications of new defined quantities, for anisotropic dielectrics with random polarization main directions and with permanent polarization, the author has demonstrated (in Part two, namely [10]) *new refraction theorems for 3D electric field* (eq. (31) from [10] for electric field strength $\overline{E}$, respectively eq. (36) from [10] for calculation electric flux density $\overline{D}_p$).

Starting from these general forms of the theorems, some particular forms have been deduced, in this paper (Part three). These can be useful in solving the electric field problems for nonlinear, anisotropic systems and with permanent polarization.

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