

# ENVIRONMENTALLY SUSTAINABLE ECONOMIC DISPATCH USING GREY WOLVES OPTIMIZATION

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**Abstract:** *This paper delineates a computational framework to ascertain optimum thermal generation schedule using newfangled grey wolves optimization (GWO) technique corresponding to environmentally sustainable, economic operation. This scheduling problem is devised as a bi-objective optimization and linear interpolated price penalty model is developed based on simple analytical geometry equations which blends two non-commensurable objectives perfectly. In order to obtain high-quality solutions within lesser executing time, the algorithm parameters are nicely replaced with system parameters that carry out global and local search process in the feasible region collaboratively. Further, an appropriate constraint handling mechanism is suitably incorporated in the algorithm that intern produces a stable convergence characteristic. The effectiveness of the proposed approach is illustrated on six unit thermal systems with due consideration of transmission line loss and valve point loading effect. The desired GWO technique reports a new feasible solution for quadratic and non-convex thermal operating model which is compared with the solution that has evolved earlier and the comparison shows that the GWO technique has outstripped other algorithms effectively.*

**Key words:** *Nonlinear and non-convex operating model, bi-objective optimization, economic-environmental impacts, gray wolves optimization, interpolated price penalty factor.*

## 1. Introduction

**Progressive economic dispatch:** The world's largest economy and fastest growing energy market mainly rely on the electricity from fossil fuel based thermal power plants. The twenty third issues of the Indian energy statistics report reveal that India holds fourth largest place on the world's energy market, its electricity generation from utilities and non-utilities altogether during 2014-15 were 71.01%, 13.04%, 1.82% and 14.11% from thermal, hydro, nuclear and non-utilities respectively. The thermal power plant shares more proportion on the total generation and the emission released during production causes inevitably dominance impact of environmental. With the increased concern over environmental protection, the power industries are forced to modify their operation strategies for the generation of electrical energy not only at minimum energy cost, but also at minimum

pollution level to meet the requirements of the increasing demand [1-2]. Further, it has been recognized that the energy utilization improvement and environmental impact assessment are an essential step to achieve sustainable development of a country. With its rapid economic up growth, the rising energy consumption as well as environmental pollution has been impelling the researchers to derive a strategic balance between economic development, energy consumption and environmental sustainability [3].

Economic dispatch (ED) is a cognitive process that optimizes the power generation intent to minimize the total operating cost and to meet load demands over a schedule period while satisfying the various equality and inequality constraint. Due to the apprehension over environmental pollution and clean air amendment forces the utilities to serve electricity, cheapest possible price with cleanliness environment. Hence, the ED now becomes an environmentally constrained economic dispatch problem (ECED).

**State-of-the art, literatures:** The literature survey basically focused on economic load dispatch (ELD) and combined economic-emission dispatch (CEED) in thermal power systems. This review covers three methodologies based classification such as classical, meta-heuristic and hybrid optimization techniques.

**Classical optimization techniques:** Over the past decades, a number of conventional approaches were applied for solving the ED problem. In which, direct Newton-Raphson [4], branch-and-bound [5] and interior point methods [6] have been addressed solution for ELD problem. Likewise, Lagrangian relaxation [7] and back propagation neural network (BPNN) [8] methods were found solution to CEED. Despite, classical methods have found an accurate solution; it uses a single path search method based on the deterministic transition rule, while searching the optimal solution in the search space. Hence, these methods have taken the more computational time and have occupied more memory space.

**Meta-heuristic optimization techniques:** These are attaining more popular because of its derivative free mechanism, population based, local optima avoidance and capable of dealing with difficult non-linear

constraints. Recently, the pattern search method [9] and chaotic bat algorithm [10] have dealt ELD problem successfully. Whereas, non-dominated sorting genetic algorithm – II (NSGA-II) [10], particle swarm optimization (PSO) [12], differential evolution (DE) [14] and multi-objective differential evolution (MODE) [13] have optimized both fuel cost and emission simultaneously. In fact, the convergence rate of opposition-based harmony search (HS) [15], tribe-modified differential evolution (Tribe-MDE) [16] and self-organizing hierarchical particle swarm optimization technique with time-varying acceleration coefficients (SOHPSO TVAC) [17] are fine-tuned while optimizing CEED problem by modifying the operator.

**Hybrid optimization methods:** In the scenario of optimization process a strategic balance between global and local search are derived by combining either two heuristic algorithms or one heuristic algorithm with a classical method. The hybrid PSO with the sequential quadratic programming (PSO-SQP) [18] technique, modified sub-gradient and harmony search (MSG-HS) algorithm [19] and hybrid shuffled DE (SDE) algorithm [20] have obtained good quality solution for ELD problem. Moreover, the hybrid genetic algorithm [22], modified neo-fuzzy neuron (NFN) [23], GA with active power optimization based on Newton's second order approach [24] and differential evolution and biogeography-based optimization (DE-BBO) algorithm [25] have determined compromised generation schedule in CEED case.

**Research gap and motivation:** However, the reported optimization techniques had found optimum solution; it is not an end global solution to ELD problem due to the common shortcomings of algorithm complexity, premature convergence due to imbalance between exploration and exploitation, and large computational time. To overcome this drawback, a new emerging optimization tool, i.e., grey wolves optimization (GWO) technique is preferred with suitable constraint handling strategy, which balances intensification and diversification through encircling, hunting and attacking processes. Then, superior convergence characteristics and performance of the GWO technique than other swarm intelligence techniques while solving economic load dispatch problem with only fuel cost as objective function [26] - [27] and the unit commitment problem [28] have been successfully analyzed.

**Highlights of this work:** As far as the state of the art, literature, there has been no attempt to demonstrate the emission constrained economic operation of thermal power system with valve point loading using GWO. Therefore, ascertaining the preeminent generation schedule for compromised fuel cost and emission release with less computational time is still a research work. This motivates the authors to contribute in this research field in the following aspects:

- The six unit thermal power system's data are suitably incorporated into the coded GWO technique.
- A linear interpolation model is proposed to blend the fuel cost and emission releases.
- A benchmark emission constrained economic generation schedule is derived using GWO technique; it seems to be the first attempt.

**Paper organization:** The paper is organized into six sections, the next section describes the mathematical formulation of the emission constrained economic dispatch problem, whereas, section 3 deals GWO technique as an optimization tool is briefed. Section 4 deals application of GWO's technique for finding an optimal generation schedule. The numerical simulation results are presented and have compared in section 5. Finally, the conclusion is presented in the last section.

## 1. Emission Constrained Economic Dispatch Model

**Objective Function:** As stated earlier the ECED problem is formulated as a bi-objective framework, and is described mathematically as follows:

$$\text{Minimize } \sum_{i=1}^N \{F(P_{gi}), E(P_{gi})\} \quad (1)$$

Where,  $P_{gi}$  is active power generation (MW) of  $i^{\text{th}}$  unit,  $F$  and  $E$  are total fuel cost (FC) of generation in the system (Rs./hr) and emission release (kg/hr) respectively and  $N$  is the number of thermal units.

**Cost function:** Revenue analysis of utility relies linear, quadratic and cubic cost functions. If a cost function is said to be an economically meaningful and legitimate that it should be satisfied the restriction imposed by the parameter and variable. In case of quadratic cost function there are three restrictions to be satisfied where as the cubic cost function need to satisfy additionally one inequality restriction. Moreover, one of the quandaries of cubic cost function should be hypothesized. Therefore, economists are chosen quadratic cost function either maximize profit maximum or minimize operating cost. Particularly, the total fuel cost of thermal plant is expressed sum of multiple quadratic cost function in terms of real power generation and is mathematically defined as follows:

$$F(P_{gi}) = \sum_{i=1}^N (a_i P_{gi}^2 + b_i P_{gi} + c_i) \quad (2)$$

Where,  $a_i$ ,  $b_i$  and  $c_i$  are the fuel cost coefficients of the  $i^{\text{th}}$  generating. The significant effect of valve point loading on total FC can be pragmatically designated as the superposition of quadratic and sinusoidal function. The total generation cost with valve point loading is given by:

$$F(P_{gi}) = \sum_{i=1}^N (a_i P_{gi}^2 + b_i P_{gi} + c_i) + \left| d_i \sin \left\{ e_i (P_{gi}^{\text{min}} - P_{gi}) \right\} \right| \quad (3)$$

Where,  $d_i$  and  $e_i$  are the coefficients of the effect of the valve point loading of  $i^{th}$  generating unit.

**Emission function:** The emission generated by each generating unit may be approximated as a quadratic function of the power output of the generator. The total amount of emission released is given by:

$$E(P_{gi}) = \alpha_i + \beta_i P_{gi} + \gamma_i P_{gi}^2 \quad (4)$$

Where,  $\alpha_i$ ,  $\beta_i$  and  $\gamma_i$  are the emission coefficients of the  $i^{th}$  unit.

**Handing Bi-Objective:** The bi-objective problem of emission constrained economic dispatch (ECED) can be converted into the single objective optimization problem by introducing a normalized price penalty factor. The price penalty factor is defined as the ratio between the average full load fuel cost and average emission of the corresponding generator as its maximum output.

**Computation of modified price penalty factor**

**Step 1:** The computation of  $h_{max}$  :

$$h_{max} = \frac{F(P_{gi}^{max})/P_{gi}^{max}}{E(P_{gi}^{max})/P_{gi}^{max}} \quad (5)$$

**Step 2:** According to  $h_{max}$  the thermal units were ranked in ascending order.

**Step 3:** Then full-load capacity of each unit was added one at a time starting from the lowest  $h_{max}$  until  $\sum P_{gi}^{max} \geq P_D$  have been discerned.

**Step 4:** In this procedure  $h_{max}$  related to last unit was considered as a price penalty factor to trade-off two conflict objectives.

**Computation of normalized price penalty factor:**

While performing **step 3** sum of the maximum capacity of thermal units often greater than demand, it may lead approximate value. In order to determine the no-inferior solution an accurate model is necessary which is not explored in the literature. This drawback can be rectified by incorporating a simple mathematical technique with the usual procedure.

Let,  $P_{g1}$  is the maximum capacity of a unit at that moment by adding the same causes sum total exceeds the load demand  $P_D$  and its corresponding price penalty factor is  $h_1$ . The maximum capacity  $P_{go}$  is the predecessor and the associated price penalty is  $h_o$ . Then the normalized price penalty factor ( $h_t$ ) can be determined using (6).

$$h_t = h_o + \left( \frac{h_1 - h_o}{P_{g1} - P_{go}} \right) * (P_D - P_{go}) \quad (6)$$

Now, the objective function has detailed in (1) can be defined by introducing  $h_t$ , then the objective function of ECED problem is defined as,

$$\text{Minimize } \{F(P_{gi}) + h_t * E(P_{gi})\} \quad (7)$$

**Equality Constraint:** The algebraic sum of the total generated power of all generating units, power

demand by the load and the total transmission loss ( $P_L$ ) must be equal to zero.

$$\sum_{i=1}^N P_{gi} - P_D - P_L = 0 \quad (8)$$

The Kron's loss formula is given by:

$$P_L = \sum_{m=1}^N \sum_{n=1}^N P_{gm} B_{mn} P_{gn} + \sum_{m=1}^N B_{m,0} P_{gm} + B_{00} \quad (9)$$

Where,  $B_{mn}$ ,  $B_{m0}$  and  $B_{00}$  are the elements of loss coefficient matrix.

**Inequality Constraint:** The power output of each generating unit must be greater than or equal to the minimum power permitted and also be less than or equal to maximum power permitted on that specified unit. Thus the inequality constraint is expressed as:

$$P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max} \quad i = 1, 2, \dots, N \quad (10)$$

Where,  $P_{gi}^{min}$ ,  $P_{gi}^{max}$  are the minimum and maximum power generation limit of the  $i^{th}$  unit.

### 3. Overview of GWO Technique

It is a population based metaheuristic algorithm and developed by Mirjalili et al., in 2014 which is inspired from the leadership hierarchy and the hunting mechanism of gray wolves in nature. Generally, the populations of grey wolves have average crowd size of 5-12 and the cluster organizes compactly through the hierarchy. The most dominant member is called alpha; the immediate successive ranked wolves are beta, delta and omega in which beta supports in decision making whereas delta lead its lowest rank.

Hunting is a fascinating behavior of the grey wolves, by imitating this mechanism a grey wolf's optimization (GWO) technique is developed, in which a specific number of grey wolves in a group have randomly allowed to search a prey in a multidimensional space. The position of the wolves is considered as the variables to be optimized and the distance between prey and grey wolves determine the fitness value of the objective function. According to the instruction and feedback mechanism the individual grey wolf adjusts its position and moves to the best position, and the best feasible solution in the course of the iteration is saved.

The mathematical formulation of the GWO is carried through the following segments to determine the best feasible solution for any optimization problem.

- i. Encircling
- ii. Hunting
- iii. Attacking

The encircling behavior of the grey wolves is mathematically represented as follows.

$$\vec{D} = \left| \vec{C} \cdot \vec{X}_p(t) - \vec{X}(t) \right| \quad (11)$$

$$\vec{X}(t+1) = \vec{X}_p(t) - \vec{A} \cdot \vec{D} \quad (12)$$

Where,  $t$  indicates the current iteration,  $\vec{A}$  and  $\vec{C}$

are coefficient vectors,  $\bar{X}_p$  is the position vector of the prey, and  $\bar{X}$  indicates the position vector of a grey wolf. The vectors  $\bar{A}$  and  $\bar{C}$  are calculated as follows:

$$\bar{A} = 2\bar{a} \cdot \bar{r}_1 - \bar{a} \quad (13)$$

$$\bar{C} = 2 \cdot \bar{r}_2 \quad (14)$$

Where,  $\bar{r}_1$  and  $\bar{r}_2$  are random vectors between 0 and 1 and  $\bar{a}$  is set to decrease from 2 to 0 over the course of iterations.

In the course of iterations the position of alpha seems to be considered as first best candidate solution and another two are beta and delta wolves' position, and then the other search agents (omega wolves) update their positions according to the position of three best search agents. It can be modeled mathematically as follows:

$$\bar{X}(t+1) = \frac{\bar{X}_1(t) + \bar{X}_2(t) + \bar{X}_3(t)}{3} \quad (15)$$

Where,

$$\bar{X}_1 = \bar{X}_\alpha - \bar{A}_1 \cdot (\bar{D}_\alpha); \bar{D}_\alpha = |\bar{C}_1 \cdot \bar{X}_\alpha - \bar{X}| \quad (16)$$

$$\bar{X}_2 = \bar{X}_\beta - \bar{A}_2 \cdot (\bar{D}_\beta); \bar{D}_\beta = |\bar{C}_2 \cdot \bar{X}_\beta - \bar{X}| \quad (17)$$

$$\bar{X}_3 = \bar{X}_\delta - \bar{A}_3 \cdot (\bar{D}_\delta); \bar{D}_\delta = |\bar{C}_3 \cdot \bar{X}_\delta - \bar{X}| \quad (18)$$

Over the course of iterations the value of  $\bar{a}$  has decreased linearly from 2 to 0, thus  $\bar{A}$  also decreased by  $\bar{a}$ . As  $\bar{A}$  is fluctuating randomly in between the range [-a, a] the candidate solution is converged towards prey if  $|\bar{A}| < 1$  that means forcing the wolves to attack the prey otherwise forces the wolves to search another best candidate solution (alpha) and this process repeats till the termination criterion is fulfilled.

#### 4. Application of GWO Technique for ECED

**Step 1-Initialization and structure of candidate solution:** The active thermal power generation is a control variable that representing the position of the wolves to be evolved. This is randomly engendered within the operational limits based on (19).

$$P_{g,i} = rand * (P_{gi}^{\max} - P_{gi}^{\min}) + P_{gi}^{\min} \quad (19)$$

Then, the initial population matrix is created as follows:

$$X = \begin{bmatrix} P_{g1}^1 & P_{g2}^1 & \dots & P_{gi}^1 & P_{gN}^1 \\ P_{g1}^2 & P_{g2}^2 & \dots & P_{gi}^2 & P_{gN}^2 \\ \dots & \dots & \dots & \dots & \dots \\ P_{g1}^{SP} & P_{g2}^{SP} & \dots & P_{gi}^{SP} & P_{gN}^{SP} \end{bmatrix} \quad (20)$$

Then, the initial position of the candidate solution  $X^0$  is initialized as follows.

$$X^0 = \begin{bmatrix} P_{g1}^1 & \dots & P_{g1}^{SP} & P_{g2}^1 & \dots & P_{g2}^{SP} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & P_{gi}^1 & \dots & P_{gi}^{SP} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & P_{gN}^1 & \dots & P_{gN}^{SP} & \dots \end{bmatrix} \quad (21)$$

**Step 2-Estimation of augmented objective function:** From the initial position of the population the objective function is calculated. In order to handle equality constraint violation an augmented objective function (AOF) is derived using (22), which is the aggregate of the objective function considered and absolute value in violation of power balance constraint with a high valued scalar multiplier. Further, this mechanism converts the primal constrained problem into an unconstrained problem and guides the search process towards the desirable solution

$$AOF = \left( objective + 1000 * \left| \sum_{i=1}^N P_{gi} - (P_D + P_L) \right| \right) \quad (22)$$

**Step 3-Evaluation of Fitness and the best position:** The fitness value of all individuals of the current candidate solution matrix ( $X^0$ ) is calculated using (23). The fitness of  $i^{th}$  individual represents its (wolf's) distance from the prey. Sort the population from minimum to maximum, an individual having the minimum fitness is imitated as the alpha; second and third minimum is beta and delta respectively.

$$Fitness = AOF \quad (23)$$

**Step 4-Modifying agent position for optimal solution:** The position of the  $i^{th}$  agent should be updated in accordance with (15). The position of each search agent represents a potential solution comprised of an active power generation of ELD problem. The new position of each agent may violate allowable ranges and it is limited to the respective range.

**Step 5-Fitness re-estimation:** With the new position of each control variable, the AOF is calculated as described in the *steps 2* and followed *step 3* is performed to identify a global best solution.

**Step 6-Modification of thermal generation schedule:** The N-1 thermal generations are retained at the optimum value and one thermal generation, i.e.,  $d^{th}$  is modified to satisfy the power balance equation based on solution repair strategy. It can be solved using standard algebraic method and the positive root is chosen as the generation of the slack thermal unit that satisfies the equality constraint (8), perfectly.

$$B_{dd} P_d^2 + \left( 2 \sum_{m=1}^{(N-1)} B_{d,m} P_m - 1 \right) P_d + \left( \sum_{m=1}^{(N-1)} \sum_n P_m B_{mn} P_n + \sum_{m=1}^{(N-1)} B_{m,0} P_m - \sum_{m=1}^{(N-1)} P_m + B_{00} + P_D \right) = 0 \quad (24)$$

**Step 7-Inequality constraints handling mechanism:** The decision variables of thermal plant output power are kept in the valid range by handling appropriately. Generally, it can be checked whether the operating limits of the active power of all generating units are violated or not. If any power generation is less than the minimum level, it is made equal to the minimum value. Similarly, if it is greater than the maximum level, it is assigned its maximum value.

**Step 8-Stopping criterion:** If  $iter < \max Cycle$ , go to step 2. Otherwise, the GWO terminates.

## 5. Simulation Results and Discussion

**Description of test system:** The environmentally constrained economic operation of thermal plants is mathematically formulated as an optimization problem. A standard test system consists of six units is considered to investigate the performance of the GWO. The coefficients of cost and emission characteristics, generator operating limits are referred from [17]. The operation is performed for 500MW, 700MW and 900MW static load. Initially, ECED is carried for nonlinear quadratic operating model further; the complexity of the dispatch model is increased by considering non-convex operating model. In the both model network line losses are considered and the loss coefficients are taken from the reference [11]. The GWO algorithm is coded in MATLAB 7.9 platform and is executed on Intel core i5 processor 2.30 GHz and 4 GB RAM personal computer.

**Optimum Operation For Economic Schedule:** The applicability of the GWO method for finding the optimum operation of thermal power system has been explored by analyzing two operating models, one is nonlinear quadratic operating cost model (module-I) whereas, the second is a non-convex operating model (module-II). In both the model fuel cost is considered as objective function and the algorithm is executed 500 repetitive iterations with same control parameters. Simulated optimum generation schedule of modules-I and II corresponding to the minimum fuel cost per hour are presented in Table 1 and Table 2 respectively. These tables mainly consist of generation schedule in MW and computational time in second for 500MW, 700MW and 900MW. Additionally, the emission release (kg/hr) associates with this generation schedule also presented for better understanding. It is perceived from both the tables that the generation of each unit is within its operating limits and the fuel cost is increased while considering valve point loading.

Table 1 Economic generation schedule for quadratic cost model

Unit (MW)	Demand (MW)		
	500	700	900
P1(MW)	10.0000	10	80.62866
P2(MW)	17.1430	15.28374	122.7474
P3(MW)	118.198	147.7319	205.3744
P4(MW)	55.2233	129.4321	57.41022
P5(MW)	183.7044	227.8039	248.9675
P6(MW)	125.2857	188.7093	212.2364
Total Generation (MW)	509.5542	718.961	927.3646
Losses (MW)	9.54982	18.95229	27.90137
CPU Time(s)	8.8	10.90	11.18
Iterations	500	500	500
Fuel Cost (Rs/h)	27592.91	37013.08	48956.81
Emission release (Kg/h)	296.2263	506.5107	775.9482

Table 2 Economic generation schedule for non-convex cost model

Unit (MW)	Demand (MW)		
	500	700	900
P1(MW)	10.0000	79.5697	61.9150
P2(MW)	10.0000	111.8795	144.7654
P3(MW)	108.7219	113.5731	190.5014
P4(MW)	55.8396	56.9169	133.5023
P5(MW)	130.0000	142.7639	190.6761
P6(MW)	195.5236	212.2509	204.924
Total Generation (MW)	510.0851	716.9539	926.2842
Losses (MW)	10.084	16.913	26.264
CPU Time(s)	10.91	10.73	10.77
Iterations	500	500	500
Fuel Cost(Rs/h)	27767.4390	38983.0200	49309.9200
Emission in (Kg/h)	301.4472	473.1433	717.4716

### Optimum Operation For Minimum Emission:

The potentiality of the GWO in minimizing pollutant emission release that has supplemented with the thermal power generation is examined for the stated modules in section 5.2 with same MW load. The algorithm is converged at acceptable pollutant emission level (kg/hr).

Table 3 Generation schedule of quadratic cost model for minimum emission

Unit (MW)	Demand (MW)		
	500	700	900
P1(MW)	10	20.71155	112.6034
P2(MW)	29.18051	107.8707	145.1237
P3(MW)	76.09383	135.4213	148.3091
P4(MW)	106.6669	124.8187	118.4906
P5(MW)	157.1404	166.8462	200.3894
P6(MW)	132.2234	160.1151	202.0595
Total Generation (MW)	509.305	715.7836	926.9757
Losses (MW)	9.305	15.783	26.975
CPU Time(s)	11.58	11.49	12.36
Iterations	500	500	500
Emission release (Kg/h)	272.4754	455.8172	688.0668
Fuel Cost(Rs/h)	28044.2100	38147.59	49957.45

Table 4 Generation schedule of non convex cost model for minimum emission

Unit (MW)	Demand (MW)		
	500	700	900
P1(MW)	34.79092	84.70206	10
P2(MW)	80.67597	110.8959	141.8252
P3(MW)	74.91931	116.0293	196.0193
P4(MW)	62.69852	80.90215	156.0095
P5(MW)	130.2411	159.4576	141.8264
P6(MW)	125.0057	164.259	282.2158
Total Generation (MW)	508.3315	716.246	927.8962
Losses (MW)	8.331	16.246	27.896
CPU Time(s)	11.09	11.14	12.36
Iterations	500	500	500
Emission release (Kg/h)	274.0989	445.0791	808.2655
Fuel Cost (Rs/h)	28461.63	38942.16	49406.43

Consequent optimum generation schedule and computational time are tabulated for nonlinear quadratic model in Table 3 and non-convex operating model in Table 4. From the test results it is understood that the GWO has curtailed the emission release in greater extent and the stated objective, i.e., minimum pollutant emission is achieved without violating the power generation limits to meet stipulated load demand.

**Emission Constrained Economic Dispatch:** Simulation results that were discussed in sections 5.2 and 5.3 reveals the conflict nature of fuel cost and emission release, i.e., while minimizing fuel cost alone the corresponding emission release increases in unacceptable value whereas, the fuel cost would be more while minimizing emission release separately.

Table 5 Generation schedule of quadratic cost model for compromised optimum solution

Unit (MW)	Demand (MW)		
	500	700	900
P1(MW)	10	20.71155	102.3468
P2(MW)	29.30387	107.8707	139.6853
P3(MW)	95.78365	135.4213	91.4198
P4(MW)	64.88712	124.8187	123.6492
P5(MW)	159.5717	166.8462	251.5216
P6(MW)	149.872	160.1151	219.7210
Total Generation (MW)	509.4183	715.7836	928.3437
Losses (MW)	9.4175	16.22349	28.4896
CPU Time(s)	11.06	12.12	11.55
Iterations	500	500	500
Fuel Cost (Rs/h)	27596.5500	38147.5900	49527.7400
Emission release (Kg/h)	277.4680	455.8172	716.5007

Therefore, a linear interpolated price penalty factor approach is structured with GWO to blend these objectives. The normalized price penalty factors that have computed corresponding to the load 500MW, 700MW and 900MW for module-I are 43.7327, 44.3088 and 44.9258, and module-II are 43.8077, 44.3732 and 44.9780 respectively. These factors are accurately combined the bi-objectives and help provided for optimizing simultaneously. Moreover, optimum generation schedules, CPU time, losses and total generation related to the compromised fuel cost and emission release are given in Table 5 for module-I and Table 6 for module-II. It demonstrates that the proposed method not only satisfy active power balance constraint obviously, but also within the lower and upper operating limits.

Table 6 Generation schedule of non-convex cost model for compromised optimum solution

Unit (MW)	Demand (MW)		
	500	700	900
P1(MW)	10	49.8193	110.4963
P2(MW)	46.32485	85.4388	144.3008
P3(MW)	54.46726	48.6947	149.3766
P4(MW)	94.67492	71.9836	48.2942
P5(MW)	160.1301	296.5524	165.7847
P6(MW)	143.7522	166.1397	311.1522
Total Generation (MW)	509.3493	718.6284	929.4047
Losses (MW)	9.3620	18.9926	29.4362
CPU Time(s)	11.06	11.4	13.87
Iterations	500	500	500
Fuel Cost(Rs/h)	27742.5600	38145.2900	50512.0300
Emission release (Kg/h)	280.1733	561.2595	805.6692

Table 7 Comparison of feasible solution of GWO with SOHPSO for quadratic cost model

Demand (MW)	Objective	SOHPSO [17]			Proposed Method		
		Pure ELD	Pure ED	ECED	Pure ELD	Pure ED	ECED
500	FC (Rs./hr)	28,079.97	28,379.90	28,379.90	27592.90	28044.21	27596.55
	ER (kg/hr)	310.13	276.55	276.55	296.23	273.4	277.468
700	FC (Rs./hr)	38,208.00	39,444.32	38,818.92	37013.08	38481.19	38147.59
	ER (kg/hr)	536.77	462.90	468.35	506.5107	443.12	455.8172
900	FC (Rs./hr)	49,306.94	50,971.84	50,127.87	48956.81	49957.45	49527.74
	ER (kg/hr)	845.11	750.77	759.59	775.9482	688.07	716.50

Table 8 Comparison of feasible solution of GWO for quadratic and non-convex cost models

Demand (MW)	Objective	GWO's solution for Quadratic cost model			GWO's solution for Non-convex cost model		
		Pure ELD	Pure ED	ECED	Pure ELD	Pure ED	ECED
500	FC (Rs./hr)	27592.90	28044.21	27596.55	27767.43	28461.63	27742.56
	ER (kg/hr)	296.23	273.4	277.468	301.44	274.09	280.17
700	FC (Rs./hr)	37013.08	38481.19	38145.29	38983.02	38942.16	38147.59
	ER (kg/hr)	506.51	443.12	455.8172	473.14	445.07	561.25
900	FC (Rs./hr)	48956.81	49957.45	49527.74	49309.92	49406.43	50512.03
	ER (kg/hr)	775.94	688.07	716.50	717.47	808.26	805.66

**Feasible solution:** In order to reveal the superiority of the GWO technique in solving emission, economic, and emission constrained economic operation of

nonlinear thermal model, the simulated results are compared with SOHPSO [17] in Table 7. From the comparison, it is noticed that the proposed technique

has saved fuel cost 487.07 Rs./hr, 1194.92 Rs./hr and 350.13 Rs./hr for 500MW, 700MW and 900MW respectively, emission release has reduced 3.15kg/hr, 25.23 kg/hr and 62.7kg/hr for 500MW, 700MW and 900MW respectively than SOHPSO [17]. During the compromised operation, fuel cost saving and emission release reduction are 487.07Rs./hr & 0.0kg./hr for 500MW, 671.33 Rs/hr & 50.6935 kg/hr for 700MW and 600.03Rs/hr& 43.09kg/hr than SOHPSO [17].

In case of non-convex operating model, most of the researches have followed different load schemes. For uniqueness, the performance of GWO in solving non-convex and nonlinear operating model for all operations with same load demand has been compared in Table 8. The inclusion of valve point loading of thermal plant leads to multiple minima's in the search space. Thus, the fuel cost is raised 174.53 Rs/hr, 1969.94Rs/hr and 449.62Rs/hr for 500MW, 700MW and 900MW respectively, in the case of emission dispatch the pollutant emission has increased 0.69 kg/hr, 1.95kg/hr and 120.19kg/hr for 500MW, 700MW and 900MW respectively than without valve point loading. Similarly, in the case of compromised dispatch the fuel cost 146.01 Rs/hr, 1.7 Rs/hr and 984.29 Rs/hr, and emission release 2.7kg/hr, 105.432 kg/hr and 89.16 kg/hr have been increased than without valve point loading for 500MW, 700MW and 900MW respectively.

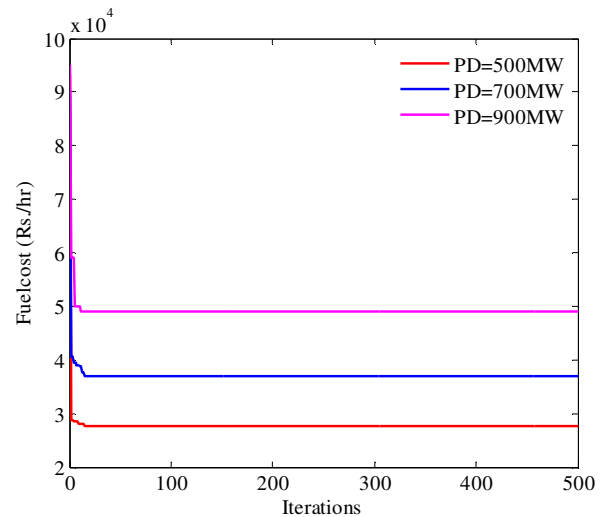
Table 9 Comparison of compromised feasible solution of quadratic cost model

Demand (MW)	Methods	FC (Rs/hr)	ER (kg/hr)
500	NR [17]	28550.15	312.51
	FCGA [17]	28231.06	304.90
	NSGA [17]	28291.11	284.36
	BBO [17]	28318.50	279.30
	SOHPSO [17]	28379.90	276.55
	GWO	27596.55	277.47
	700	NR [17]	39070.74
FCGA [17]		38408.82	527.46
NSGA [17]		38671.81	484.93
BBO [17]		38828.26	476.40
SOHPSO [17]		38818.92	468.35
GWO		38147.59	455.82
900		NR [17]	50807.24
	FCGA [17]	49674.28	850.29
	NSGA [17]	50126.05	784.69
	BBO [17]	50297.27	765.08
	SOHPSO [17]	50127.87	759.59
	GWO	49527.74	716.50

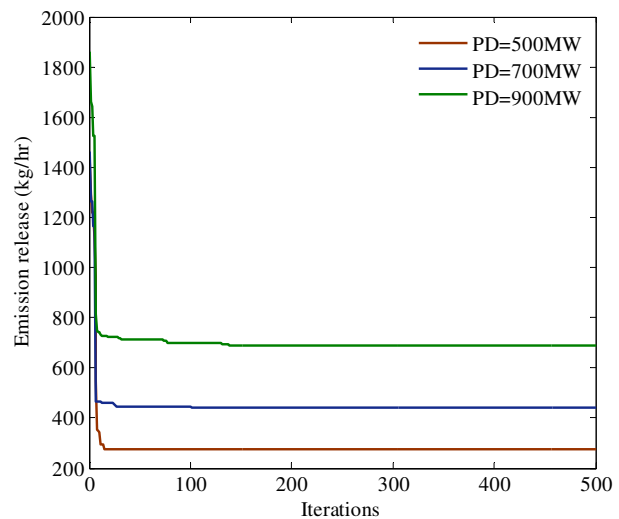
To show the diversity in comparison of emission constrained economic operation of nonlinear thermal unit, the compromised fuel cost and emission release have been compared with NR [17], FCGA [17], NSGA [17], BBO [17] and SOHPSO [17] in Table 9 for 500MW, 700MW and 900MW. The GWO has obtained better compromised fuel cost and emission reduction than other contestant algorithms, but there is no significant pollutant emission reduction as

compared with SOHPSO [17] for 500MW. If SOHPSO is trying to minimize fuel cost further the corresponding emission release will be certainly greater than what the GWO technique has obtained.

**Solution Quality:** The GWO algorithm is used for determining the optimum operation setting of thermal power system, the optimum values and feasible solution for different case studies have been presented in the previous sections. Generally, the solution quality can be explored by comparative analysis; therefore the feasible solutions for all operations over twenty trials are statically analyzed and the test results are presented in Table 10. It is noticed that the GWO technique is the best in minimizing fuel cost, emission release and compromised solution for 700MW load. Moreover, lower standard deviations of GWO shows the average and worst values are very close to its best value and also is ranked first in optimizing consider objective function.



Figures 1 Convergence characteristics of GWO for economic generation schedule (Module-I)



Figures 2 Convergence characteristics of GWO for minimum emission (Module-I)

Table 10 Statistical comparison of feasible solution for the demand 700 MW

Attribute	Methods	ELD		ECED	
		FC (Rs/hr)	ER (kg/hr)	FC (Rs/hr)	ER (kg/hr)
Best	SOHPSO [17]	38208.00	39444.32	38818.92	468.35
	GWO	37013.08	38481.19	38147.59	455.81
Average	SOHPSO [17]	38208.56	39444.75	38819.15	469.25
	GWO	37013.50	38483.38	38147.59	456.22
Worst	SOHPSO [17]	38210.00	39448.54	38820.84	470.00
	GWO	37015.01	38484.30	38149.45	457.22
Std.	SOHPSO [17]	1.41	2.98	1.36	1.17
Dev.	GWO	1.36	2.07	1.32	1.00

Another event that has decided the quality of the solution is convergence behavior. Figures 1 and 2 show GWO's rapid and steady convergence characteristics over 500 iterations with several initial random wolf's, while minimizing fuel cost and emission release respectively for 500MW, 700MW and 900MW. It is observed that the accelerating rate at the beginning is very high which shows the convergence speed of the GWO technique to produce globally best solution in a reliable manner.

From Table 10 it is observed that the GWO technique has determined best fuel cost and minimum emission release, and also can be stated that both are conflicting nature. Thus, the trade-off between them is achieved using linear interpolated normalized price penalty factor approach. Figure 3 - 5 shows the optimal fronts that have obtained by the GWO technique for twenty independent trials for 500MW, 700MW, 900MW respectively, where the optimal fronts for both cases, i.e., nonlinear and non-convex cost model are compared. The trade-off curve confirms that GWO technique is nicely compromised fuel cost and emission release.

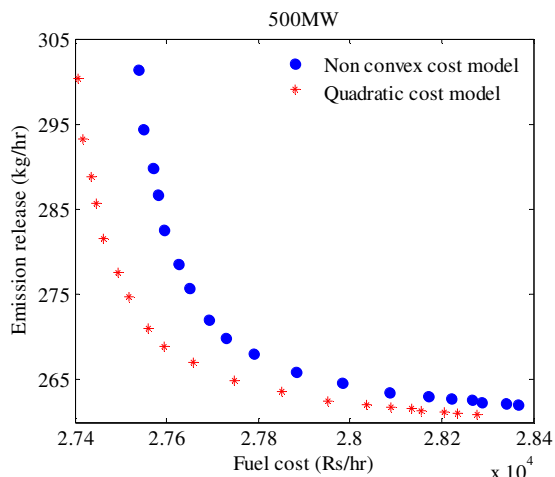


Figure 3 Comparison of optimal trade-off solution obtained by GWO for 500MW

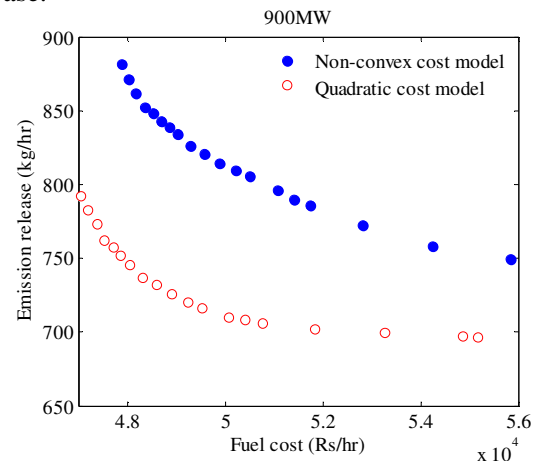


Figure 5 Comparison of optimal trade-off solution obtained by GWO for 900MW

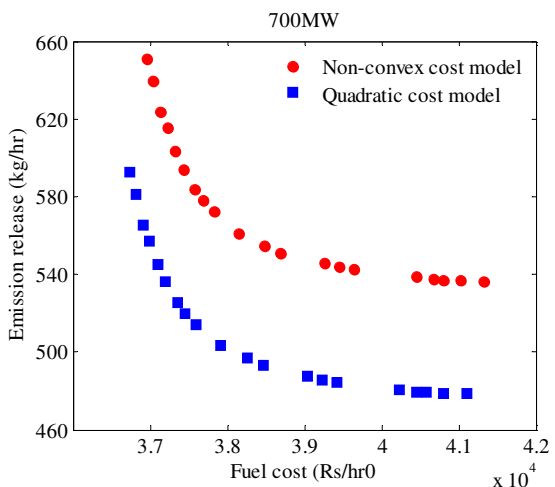


Figure 4 Comparison of optimal trade-off solution obtained by GWO for 700MW

## 6. Conclusion

Economic and environmentally sustainable operation of thermal power system offers tough challenges to the researchers; hence the ELD problem is formulated as a bi - objective framework. Initially, fuel cost and emission release are optimized separately using a GWO technique whereas, the interpolated price penalty approach has been employed and optimized the objective functions simultaneously. The optimum generation schedule that has been obtained by GWO technique perfectly met the specified load demand. The simulated results have been compared with earlier research work. Therefore, it is concluded that the proposed algorithm can be robust and effective alternative for solving bi-objective economic load



dispatch problem without and with valve point loading effect. Further, provides solution to serve electricity in affordable price with the cleanliness environment to the society. Finally, the numerical results would be useful for regulatory bodies, policy makers and power system planners.

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