### REAL TIME IMPLEMENTATION OF ROBUST CONTROLLER BASED TUNING FOR DESIRED CLOSED-LOOP RESPONSE FOR SI/SO SYSTEMS

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Abstract: In this article, the IMC-PID approach is generalized to obtain the PID parameters for general models by approximating the ideal controller with a Maclaurin series in the Laplace variable. It turns out that the PID parameters so obtained provide some what better closed-loop responses than those obtained by PID controller tuned by other methods. Further all of the PID parameters depend on the desired closedloop time constant in a manner consistent with engineering intuition. The effectiveness of the PID controllers tuned by the proposed tuning method will be validated both by simulation studies and real time implementation.

Key words: IMC PID, Maclaurin series, ISE, Controller tuning.

### **1.INTRODUCTION**

Since the proportional, integral and derivative controller finds widespread use in process industries a great deal of effort has been directed at finding the best choices for the controller parameters for various process models. Among the performance criteria used for PID controller parameter tuning, the criterion to keep the response of the process close to the desired closed-loop response has gained widespread acceptance in the chemical process industries, because of it's simplicity, robustness and successful practical applications. The IMC-PID tuning method and direct synthesis method are typical tuning methods for achieving a desired loop response. Also current tuning methods yield PID parameters only for a restricted class of process models. There is no general methodology for arbitrary process other than approximating them with a first or second-order models and applying tuning rules for the approximate models. In this article, the IMC-PID approach is generalized to obtain the PID parameters for general models by approximating the ideal controller with a Maclaurin series in the Laplace variable. It turns out that the PID parameters so obtained provide somewhat better closed-loop responses.

## 2..DEVELOPMENT OF GENERAL TUNING ALGORITHM FOR PID CONTROLLER:

#### Single Degree of Freedom Controller

Consider a stable (that is no right half plane poles) process model of the form[1].

$$\mathbf{G}(\mathbf{s}) = \mathbf{P}_{\mathrm{m}}(\mathbf{s})\mathbf{P}_{\mathrm{a}}(\mathbf{s}) \tag{1}$$

Where  $P_m(s)$  is the portion of the model inverted by the controller (it must be minimum phases)  $P_A(s)$  is the portion of the model not inverted by the controller (it is usually non minimum phase that is it contains dead times and /or right half plane zeros) and  $P_A(0) = 1$ .



Fig.1.- Feedback control system

Often, the portion of the model not inverted by the controller is chosen to be all pass that is, of the form

$$P_A(s) = \prod_{i,j} \left( \frac{-\tau_i s + 1}{\tau_i s + 1} \right) \left( \frac{-\tau_i^2 s^2 - 2\tau_i \varsigma_j s + 1}{\tau_i^2 s^2 - 2\tau_i \varsigma_j s + 1} \right) \rightarrow (2)$$
  
$$\tau_i \tau_j \ge 0; 0 \le \varsigma \le 1$$

Figure 1 shows the feedback control system with IMC-PID controller  $G_c$ . Let as take  $G_D$  and  $q_r = 1$ . Since this choice gives the best least–squares response. The requirement that  $P_A(0)=1$  is necessary for the controlled variable to track its set point. The aim is to choose the controller Gc of Figure 1 to give the desired closed- loop response, C/R given by

$$\frac{C}{R} = \frac{P_A(S)}{(\lambda s + 1)^r} \to (3)$$

The term  $1/(\lambda s+i)^r$  functions as a filter with an adjustable time constant  $\lambda$  and an order 'r' chosen so that the controller  $G_c$  is realizable.

The ideal controller  $G_c$  that yields the desired loop response given by Eq 2 perfectly is given by

$$G_{c}(s) = \frac{q}{1 - Gq} = \frac{P_{m}^{-1}}{(\lambda s + 1)^{r} - P_{A}(s)} \to (4)$$

where 'q' is the IMC controller

$$P_m^{-1}(s) / (\lambda s + 1)^r \to (5)$$

The controller  $G_c$  can be approximated to obtain a PID controller by first noting that it can be expressed as

$$G_{\mathcal{C}} = \frac{f(s)}{s} \to (6)$$

whereas  $G_c$  has a pole at the origin because  $P_A(o)$  is one, f(s) will not have such a pole because the derivative of  $((\lambda s + 1)^r - P_A(s))/s$  at the origin is never zero for r greater than Zero.

Expanding G<sub>c</sub> (s) in a maclaurin series in s gives

$$G_{\mathcal{C}}(s) = \frac{1}{2} \left( f(0) + f'(0)s + \frac{f''(0)}{2}s^2 + \dots \right) \to (7)$$

It should be noted that the resulting controller has the proportional term, integral term

and derivative term, in addition to an infinite number of higher-order derivative terms. Since the controller given by Eq.7 is equivalent of the ideal controller given by Eq.4, the desired closed-loop response can be perfectly achieved if all terms in Eq.7 are implemented. In practice, however, it is impossible to implement controller given by Eq. 7 because of the infinite number of high-order derivative terms. Infact, in an actual control situation low and middle frequencies are much more important than high frequencies, and only the first three terms in Eq. 7 are often sufficient to achieve the desired closedloop performance. The controller given by Eq. 7 can be approximated to the PID controller by using only the first three terms (1/s, 1, s) in Eq. 7 and truncating all other high-order terms  $(S^2, S^3 \dots)$ . The first three terms of the above expansion can be preted as the standard PID controller given by

$$G_{\mathcal{C}}(s) = K_{\mathcal{C}}(1 + \frac{1}{T_{i}s} + T_{D}s) \rightarrow (8)$$

where  $K_c = f'(0) \rightarrow (9)$ 

$$T_{i} = \frac{f'(0)}{f(0)} \to (10)$$
$$T_{D} = \frac{f''(0)}{2f'(0)} \to (11)$$

In order to evaluate the PID controller Parameters given by the above Eqs. we let

$$D(s) = \left(\left(\lambda s + 1\right)^r - P_A(s)\right) / s \to (12)$$

Then, by Maclaurin series expansion we get  $D(0) = r\lambda - P_A(0) \rightarrow (13)$ 

$$D(0) = [r(r-1)\lambda^2 - P_A(0)]/2 \to (14)$$
  
$$D(0) = [r(r-1)(r-2)\lambda^3 - P_A(0)]/3 \to (15)$$

Using Eq. 12 the function f(s) and its first and second derivatives, all evaluated at the origin, are given by

$$f(0) = \frac{1}{K_P D(0)} \to (16)$$

$$f'(0) = \frac{[P'_m(0)D(0) + K_P D'(0)]}{[K_P D(0)]^2} \to (17)$$

$$f''(0) = f'(0) \to (18)$$

$$\left( \left( \frac{[P''_m(0)D(0) + 2P'_m(0) + K_P D''(0)2f(0)/f(0)]}{P'_m(0)D(0)} \right) + \frac{2f'(0)}{f(0)} \right) \to (19)$$

Where

$$K_P = P_m(0) = G(0) \to (20)$$

The above formulas can be used to obtain the controller gain, and integral and derivative time constants as analytical functions of the process model parameters and the closed-loop time constant  $\lambda$ .

# **3.DERIVATION OF THE PARAMETERS OF APPROXIMATED IMC-PID CONTROLLER:**

#### **One degree of Freedom Controllers:**

$$G(s) = \frac{Ke^{-\theta s}}{(\tau s + 1)} \rightarrow (21)$$
$$G_I = \frac{1}{G_{mm}} \rightarrow (22)$$
$$G_{mm} = \frac{K}{\tau s + 1} = P_m(s) \rightarrow (23)$$

$$G_{ma} = e^{-\theta s} = P_A(s) \to (24)$$

$$\frac{C}{R} = \frac{e^{-\theta s}}{\lambda s + 1} = \frac{P_A(s)}{\lambda s + 1} \rightarrow (25)$$

$$K_c = \frac{T_i}{K(\lambda + \theta)} \rightarrow (26)$$

$$T_i = \tau + \frac{\theta^2}{2(\lambda + \theta)} \rightarrow (27)$$

$$T_D = \frac{\theta^2}{2(\lambda + \theta)} \left(1 - \frac{\theta}{3T_i}\right) \rightarrow (28)$$

### 4. IMC-PID CONTROLLER

The example Process I chosen for simulation study is

$$G(s) = \frac{e^{-3s}}{(10s+1)} \rightarrow (29)$$
  
This G(s) is divided into

$$G_{mm} = \frac{1}{(10s+1)} \rightarrow (29)$$
$$G_{ma} = e^{-3s} \rightarrow (30)$$

where  $G_{mm}$  is the portion of the model inverted by the controller (it must be minimum phase) $G_{ma}$  is the portion not inverted by the controller (it is usually non minimum phase). The controller parameters derived for the proposed method of tuning IMC-PID controller as shown in Table 1. Referring the Table 1the controller parameters for the first-order process with delay are given as below :

$$K_C = \frac{T_i}{K(\lambda + \theta)} \to (31)$$
$$T_i = \tau + \frac{\theta^2}{2(\lambda + \theta)} \to (32)$$

$$T_D = \frac{\theta^2}{6(\lambda + \theta)} \left[ 3 - \frac{\theta}{T_i} \right] \to (33)$$

where k = Gain of the process $\tau = Time \text{ constant of the process}$ 

 $\theta$  = Dead time of the process

 $\lambda$  = Time constant of the desired closed-loop response

For the example process I the values of controller

parameters are calculated as

 $K_c = 2.3$ 

$$T_i = 11$$

$$T_d = 0.909$$
 for  $\lambda = 1.5$ 

The unit step response for  $\lambda = 1.5$  is shown in Figure 3.

 $\lambda_{adjusted} = 3.48$ 

 $\lambda$  is adjusted for desired closed-loop response with  $\lambda = 3.48$ .

The PID parameters are

 $K_c = 0.64$  $T_i = 4$ 

$$\begin{split} &\Gamma_i = 4.17 \\ &T_d = 0.527 \ \ for \ \lambda_{adjusted} = 3.48. \end{split}$$

The unit step response for  $\lambda_{adjusted} = 3.48$  is shown in Figure 3



Fig 2 Comparison of the ISE generated by various tuning rules.

This proposed method of tuning is compared with Rivera method. The responses are shown in Fig.3.

The Integral Square Error (ISE) is calculated for the proposed method of tuning IMC-PID controller and is compared with Rivera method of tuning IMC-PID controller. The graph is plotted for various values of  $\theta/\tau$  and is shown in Figure 2. From this figure, it is seen that the proposed tuning rule gives the smallest ISE among all tuning rules over entire range of  $\theta/\tau$ . The difference in the values of the ISE becomes more significant as the dead time effect dominates. However it is observed that the magnitude of the ISE obtained is not in good agreement with the expected magnitude of the ISE.



It is seen from all the above figures that the proposed method of tuning gives the response closer to desired response than other tuning methods.

#### **Process II:**

The example process II chosen for simulation study is

$$G(s) = \frac{e^{-1s}}{12s+1} \to (34)$$

This is again the first-order process with dead time similar as that of example process I. To compare the results of real time implementation of proposed tuning method with the simulation results, this process is taken for simulation study. eferring the Table 1 the controller parameters for the first-order process with delay are given as below :

$$K_{C} = \frac{\tau_{1}}{K(\lambda + \theta)} \rightarrow (35)$$
$$T_{i} = \tau + \frac{\theta^{2}}{2(\lambda + \theta)} \rightarrow (36)$$
$$T_{D} = \frac{\theta^{2}}{6(\lambda + \theta)} \left[ 3 - \frac{\theta}{\tau_{1}} \right] \rightarrow (37)$$

where k = Gain of the process

 $\tau = Time \ constant \ of \ the \ process \ \ Time \ of \ the \ process \ \ Time \ of \ the \ process \ \ Time \ of \ the \ process \ \ Time \ of \ the \ t$ 

 $\lambda =$  Time constant of the desired closed – loop response

Process Model	Tuning Method	Kc	Ti	T <sub>d</sub>	T <sub>f</sub>
$G = \frac{Ke^{-\theta s}}{\tau s + 1}$	Rivera et al.	$\frac{1}{K}\frac{2\tau+\theta}{2(\lambda+\theta)}$	$ au + \frac{ heta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$	
	Rivera et al. (with Filter)	$\frac{2\tau + \theta}{2K(\lambda + \theta)}$	$ au + \frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$	$\frac{\lambda\theta}{2(\lambda+\theta)}$
	Proposed	$\frac{T_i}{K(\lambda + \theta)}$	$\tau + \frac{\theta^2}{2(\lambda + \theta)}$	$\frac{\theta^2}{6(\lambda+\theta)} \left[ 3 - \frac{\theta}{T_i} \right]$	
$G = \frac{Ke^{-\theta s}}{\tau s + 1}$	Smith	$\frac{T_i}{K(\lambda + \theta)}$	τ		
	Rivera et al. <i>Improved</i> <i>IMC-PI</i>	$\frac{2\tau+\theta}{2K(\lambda+\theta)}$	$ au + \frac{\theta}{2}$		
	Proposed	$\frac{T_i}{K(\lambda + \theta)}$	$\tau + \frac{\theta^2}{2(\lambda + \theta)}$		
$G = \frac{Ke^{-\epsilon k}}{(\tau^2 s^2 + 2\zeta \tau s + 1)}$	Smith	$\frac{\tau_1 + \tau_2}{(K(\lambda + \theta))}$	$ au_1 +  au_2$	$\frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$	
	Proposed	$\frac{T_i}{K(2\lambda+\theta)}$	$2\zeta\tau - \frac{2\lambda^2 - \theta^2}{2(2\lambda + \theta)}$	$T_i - 2\zeta\tau + \frac{\tau^2 - \frac{\theta^2}{6(2\lambda + \theta)}}{T_i}$	

Table NO. 1- Various tuning rules to give the desiredClosed loop response

For the example process II the values of controller parameters are calculated as

$$K_c = 2.3$$
  
T<sub>1</sub> - 11

$$T_d = 0.909$$
 for  $\lambda = 1.5$ 

The unit step response for  $\lambda = 1.5$  is shown in Figure 4



Time in sample Fig 4-Time in samples Responses to a unit step

change in set point :  $G(s) = e^{-1s}/(12s+1)$ ;

 $\lambda = 1.5$ ;  $\lambda_{adjusted} = 4.0$ . (PID Controller)  $\lambda$  is adjusted to obtain desired closed-loop response. The PID parameters are

$$K_c = 0.41$$
  
 $T_i = 4.1$ 

$$I_d = 0.0918$$
 for  $\lambda = 4.0$ 

The unit step response for  $\lambda_{adjusted} = 4.0$  is shown in Figure 4. The proposed method of tuning is compared with Rivera method. The responses are shown in Figure 4. It is seen from all the above figure that the proposed method of tuning gives the response closer to desired response than other tuning methods.

#### 5. REAL TIME IMPLEMENTATION : DESCRIPTION OF THE PROCESS

The circuit diagram of the process taken for study is shown in Figure 5. This process is modeled using process reaction curve method. The open-loop response of the process is shown in Figure 6. From this response, the process model is determined as  $G(s) = e^{-1s}/12s+1$  and hence this is used in simulations study as process II.

#### 6. **IMPLEMENTATION** OF THE **APPROXIMATED IMC-PID CONTROLLER**

The controller parameters are calculated for the modeled process by proposed method and they are

$$\begin{array}{ll} K_c = 4.88; & \tau_I = 0.081 \\ \tau_D = 0.194 & \mbox{ with } \lambda = 1.5. \end{array}$$



Fig.5: Operational amplifier circuit for first order process with delay  $G(s) = e^{-3s}/10s+1$ 



*Fig 6* : Open loop response of Process  $G(s) = e^{-3s}/10s + 1$ 







Fig.8. The unit step response of the Process  $G(s)=e^{-1s}/12s+1$  using Rivera Method of tuning



Fig. 9.The unit step response of the Process  $G(s) = e^{-1s}/12s+1$  using Rivera Method of tuning

The unit step response with these parameters is shown in Figure 7

The desired closed – loop response is shown in the Figure 8. The proposed method is compared with Rivera method of tuning IMC – PID. These responses are compared with the simulated responses. They are found to be in good agreement with each other. It is also found that the response of the proposed method of tuning is closer to the desired response than the other tuning methods. Figure.9 respectively . The settling time of the process with the proposed method of tuning is found to be smaller than the other tuning methods.

#### 7.CONCLUSION

A brief introduction to IMC-PID controller and approximated IMC-PID controller were presented. The proposed method of tuning IMC-PID controller was studied in detail and the controller parameters were derived. Two different example processes were taken for study. This method was simulated using MATLAB software. From the results obtained it was found that the proposed method was better than the other methods of tuning. Finally the operational amplifier based process (first order with dead time) was simulated and the proposed method of tuning was implemented along with the other methods of tuning of IMC-PID controller

. It process were in good agreement with each other. It was also found that the response of the proposed method of tuning was closer to desired response than the other methods was found from the results that the responses of both the software simulated and hardware simulated

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