POWER ALLOCATION FOR SENSING BASED SPECTRUM SHARING COGNITIVE RADIO NETWORK WITH MULTIPLE PRIMARY USERS

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ABSTRACT

In this paper, considering multiple primary users in a cognitive radio network, the problem of optimal power allocation strategy is studied to maximize the capacity of the secondary system. The primary users are each licensed to an adjacent narrowband channel and the secondary users simultaneously sense multiple narrow band channels and aggregate them for transmission. The sensing based spectrum sharing scheme where the secondary transmitter sense and then transmit is considered and the system model with multiple primary users is proposed. An algorithm using Lagrange dual decomposition method is proposed to solve the problem subject to average interference constraint and average transmit power constraint of secondary user. The optimization of power for maximizing the outage capacity is also studied and both are compared with opportunistic spectrum access scheme. Simulation results show that the proposed system maximizes the channel capacity of the secondary user of Cognitive network.

Keywords:
Cognitive radio network, optimal power allocation, outage capacity, sensing based spectrum sharing, throughput maximization, wideband detector.

1. Introduction

Fixed spectrum assignment policy is used by today's wireless networks by which the spectrum is regulated by government agencies and is assigned to license holders or services. Although the fixed spectrum assignment policy generally served well in the past, there is a dramatic increase in the access to the limited spectrum for wireless services in the recent years. Federal Communications Commission in the United States found that most of the licensed radio frequency spectrum was not efficiently utilized in temporal and geographical dimensions [1, 2]. To improve the spectral efficiency, FCC has recently allowed accessing the spectrum of licensed users called as primary users by unlicensed users called as secondary users without causing interference to primary transmission. The limited available spectrum and the inefficiency in the spectrum usage necessitate a new communication paradigm to exploit the existing wireless spectrum opportunistically [3, 4]. Dynamic spectrum access is proposed to solve these current spectrum inefficiency problems. The inefficient usage of the existing spectrum can be improved through opportunistic access to the licensed bands without interfering with The existing users. Cognitive radio techniques provide the capability to use or share the spectrum in an opportunistic manner. Dynamic spectrum access techniques allow the Cognitive radio to operate in the best available channel. More specifically, the Cognitive radio technology will enable the users to determine which portions of the spectrum is available and detect the presence of licensed users who are called primary users (spectrum sensing), select the best available channel (spectrum management), coordinate access to this channel with unlicensed users who are called secondary users (spectrum sharing), and vacate the channel when a licensed user is detected (spectrum mobility).

In order to access the licensed spectrum by the secondary users, three approaches have been developed for cognitive radio: (i) in the opportunistic spectrum access (OSA), the secondary users can access the spectrum of primary users when the primary user band is free [7], (ii) in the spectrum sharing (SS), the primary and secondary users both simultaneously access the spectrum without interference to the primary network [9], and (iii) in the sensing-based spectrum sharing (SSS), the secondary transmitter sense the channel and transmit by changing the transmit power as per the detection of the spectrum by sensing [10].

In sensing based spectrum sharing system the frame structure will have sensing and a data transmission slot. Probability of detection and probability of false alarm are two important parameters that are associated with spectrum sensing.

When the probability of detection is higher, the primary users can be protected and when the probability of false alarm is lower, the channel can be used by secondary user and high throughput for the secondary users can be achieved. Thus there could be a fundamental tradeoff between sensing capability and achievable throughput for the secondary network.

The authors in [11] consider OSA CR network and optimize the sensing time to maximize the capacity for a single frequency band and for wideband network, optimization of sensing time and power allocation for maximizing the capacity was studied in [12]. Higher throughput was obtained with the average constraint than the instantaneous transmit power constraint. A cross-layer optimization problem is used to design the sensing time and optimize the transmit power to maximize the cognitive system throughput while keeping the interference to the primary user under a threshold constraint in paper [5]. The ergodic and outage capacities of Rayleigh fading channel for spectrum sharing cognitive radio network were investigated in [8] and Sensing based spectrum sharing model is proposed in [10] and the problem of optimizing transmit power for increasing the capacity is investigated. Both the perfect sensing case and imperfect sensing case were analyzed.

In a wideband cognitive radio system, the transmitter sense and transmit simultaneously in all frequency bands .Here, the secondary users should have information of all the bands prior to every transmission.

In [14], the power allocation problem is studied for wide band SSS CR network. To detect the channel, simultaneous multiband detection is used and compared with wideband OSA. Here both the average interference power constraint and average transmit power constraint is considered.

The authors in [15] proposed a new receiver and a frame structure for a spectrum sharing network where sensing and transmission occur simultaneously which results in sensing module always being active which will consume much power. And also the capacity with outage and truncated channel inversion with fixed rate (TIFR) capacity were studied. Motivated by the previous work, to improve the rate of CR network and save the power, receiver and sensing structure is modified and analyzed in [16] for the two schemes namely WSSS scheme and WOSA scheme.

In this paper, a sensing based spectrum sharing cognitive network is proposed using multiple primary users where the secondary user share a channel allocated to multiple primary users under the constraint of average transmit power of SU and the interference power to each primary user. Here, the receiver structure and the frame structure used in [15, 16] is not used due to its complexity and the frame structure consisting of sensing slot and data transmission slot [14] is used. The secondary user simultaneously sense all the primary channels during sensing slot and aggregate them for data transmission during data transmission slot. Then, the outage capacity of the secondary user considering multiple primary is studied and also the truncated channel inversion with fixed rate (TIFR) capacity is investigated. An algorithm is proposed by which power is allocated in an efficient manner while satisfying all constraints in the system. Through extensive simulation it was justified that the proposed scheme can achieve better performance compared with the previous work.

To summarize, the key contributions of this paper are as follows. The system model and wideband joint detector is presented in Section 2. In section 3, wideband sensing based spectrum sharing cognitive radio network using multiple primary user is proposed. In Section 4, optimization problem is formulated such as to maximize the capacity of the proposed cognitive radio system considering both the average transmit and interference power constraints and by lagrangian dual decomposition method, the optimal power levels are obtained. In section 5, the algorithm for power allocation for the wideband opportunistic spectrum access using multiple primary users is proposed and compared. The outage capacity is investigated in Section 6, simulation results are discussed in Section 7. The conclusions are given in Section 8.

2. System model

The primary system is assumed to consist of a multiple primary transmitter receiver pairs as in Fig.1, each licensed to adjacent narrow band channels which are orthogonal. The M channels of the primary user are aggregated by the secondary user. The secondary system is assumed to be a multicarrier modulation based system and transmit over a wideband channel. PT/ST denotes the primary/secondary transmitter and PR/SR denotes the primary/secondary receiver. The secondary user simultaneously senses the multiple channels by using the wideband joint detector [13].The received signal is passed through the BPF to filter the corresponding band and then to the energy detector and threshold device as shown in Fig.2.

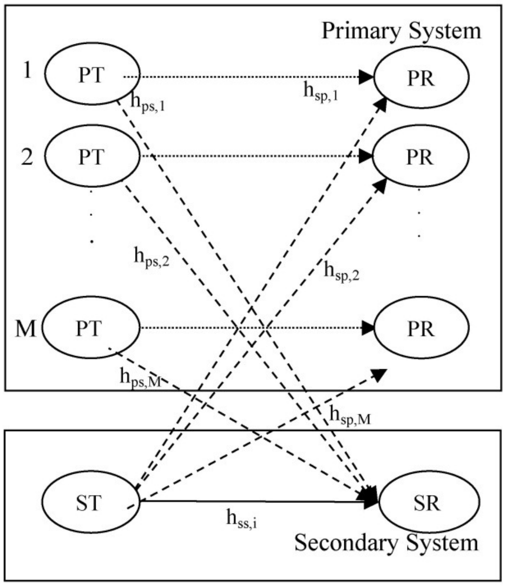

Fig. 1 System model.

The instantaneous channel power gain are assumed to be flat fading and the channel gain between SU-TX to SU-RX, SU-TX to i^{th} PU-RX, and i^{th}PU-TX to SU-RX are denoted by $h_{ss,i}, h_{sp,i}, h_{ps,i}$ respectively as shown in (Fig1). The noise is assumed to be complex Gaussian with zero mean and variance N_0. It is also assumed to be independent and identically distributed.

The status of a channel $i = 1 \ldots\ldots M$ is detected as either free or busy and the corresponding hypothesis is represented as $(H_{0,i})$ and $(H_{1,i})$ respectively for the frequency band i and $P(H_{0,i})$ and $P(H_{1,i})$ denote the probability that the frequency band is idle and active, respectively.

If the primary is busy and sensed as present, detection is perfect and if it is free and detected as present it is false alarm. The probability of detection is denoted by P_d and probability of false alarm is denoted by P_{fa}. When sensing is performed by using energy detector, $P_{d,i}$ and $P_{fa,i}$ for the i^{th}channel are given by Eq. (2).

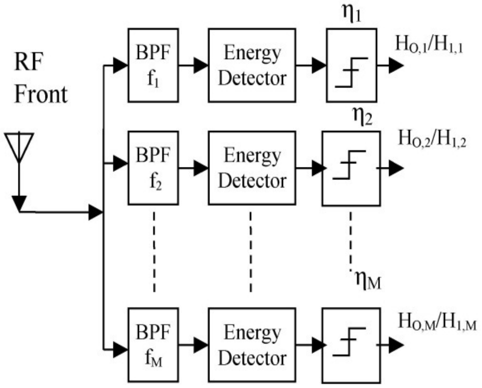

Fig. 2 Block diagram of wideband detector.

$$P_{d,i} = Q\left(\left(\frac{\eta_i}{\sigma_n{}^2} - \gamma_i - 1\right)\sqrt{\frac{\tau f_s}{2\gamma_i+1}}\right)$$
$$P_{fa,i} = \left(\sqrt{2\gamma_i + 1}Q^{-1}(P_{d,i}) + \sqrt{\tau f_s}\gamma_i\right) \quad (1)$$

respectively. τ represents the sensing time, f_s denotes the sampling frequency, γ_i is the received signal to noise ratio (SNR) in the secondary receiver on channel i and η_i is the energy threshold in the channel i. The total frame duration is denoted by T, and sensing duration by τ and $\mathrm{T} - \tau$ is for data transmission as shown in Fig.3.

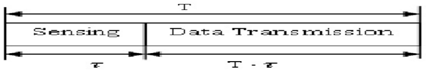

Fig. 3 Frame structure.

3. Proposed sensing based spectrum sharing network using multiple primary users.

In the proposed wideband sensing based spectrum sharing scheme using multiple primary users (WSSMPU), the secondary receiver sense the primary user's band simultaneously at the beginning of each frame and the sensed result is sent by feedback channel to secondary transmitter. Based on the sensing decision the secondary user transmit with $P_{0,i}$ or with $P_{1,i}$. $P_{0,i} \geq P_{1,i}$.

The transmission rate of CR network when primary is idle / busy is given by

$$R_{0,i} = log_2(1 + h_{ss,i}P_{0,i} \; /\sigma_n^2)$$
$$R_{1,i} = log_2(1 + h_{ss,i}P_{1,i} \; /h_{ps,i}P_{p,i} + \sigma_n^2) \quad (2)$$

respectively. The power of primary transmitter is denoted by $P_{p,i}$ for the i^{th}channel.

Due to imperfect sensing, the true state of the primary band will be falsely detected and there are four possible rate expressions for the secondary user.

$$R_{00,i} = log_2(1 + h_{ss,i}P_{0,i} \; /\sigma_n^2) \quad (3)$$

$$R_{01,i} = log_2(1 + h_{ss,i}P_{1,i} \; /\sigma_n^2) \quad (4)$$
$$R_{10,i} = log_2(1 + h_{ss,i}P_{0,i} \; /h_{ps,i}P_{p,i} + \sigma_n^2) \quad (5)$$
$$R_{11,i} = log_2(1 + h_{ss,i}P_{1,i} \; /h_{ps,i}P_{p,i} + \sigma_n^2) \quad (6)$$

Here, the first subscript represents the true status of the primary users ("0" for idle, "1" for active) and the second subscript reperesents the sensing decision. ("0" for absent, "1" for present).

Thus, the average throughput of the i^{th}channel for the wideband sensing-based spectrum sharing model is given as [15].

$$C_i = a_{0,i}\mathrm{R}_{00,\mathrm{i}} + a_{1,i}R_{01,i} + b_{0,i}R_{10,i} + b_{1,i}R_{11,i} \quad (7)$$

Where $a_{0,i},\ a_{1,i},\ b_{0,i},\ and\ b_{1,i}$ are given by

$$a_{0,i} = P(H_{0,i})(1 - P_{fa,i})$$
$$b_{0,i} = P(H_{1,i})(1 - P_{d,i}) \quad (8)$$

$$a_{1,i} = P(H_{0,i})P_{fa,i}$$
$$b_{1,i} = P(H_{1,i})P_{d,i} \quad (9)$$

The transmit power constraint is given as

$$\frac{\mathrm{T}-\tau}{\mathrm{T}} E\{\sum_{i=1}^{M}[\, a_{0,i}P_{0,i} + a_{1,i}P_{1,i} + b_{0,i}P_{0,i} + b_{1,i}P_{1,i}]\} \leq P_{av} \quad (10)$$

and the interference constraint is

$$\frac{\mathrm{T}-\tau}{\mathrm{T}} E\{b_{0,i}P_{0,i}h_{sp,i} + b_{1,i}P_{1,i}h_{sp,i}\} \leq \Gamma_i,$$
$$i = 1 \ldots\ldots M \quad (11)$$

The maximum average transmit power of the secondary user is denoted as P_{av} and the maximum interference power that can be applied to the i^{th} primary users is denoted by Γ_i. In [17], it was shown higher ergodic throughput for the cognitive radio system is achieved by considering the average interference power constraint compared to the peak interference power constraint.

4. Optimal power allocation

The optimization of power allocation and maximizing the capacity of a CR network with multiple primary users is formulated as

$$\underset{\{P_0,P_1,\tau\}}{\mathrm{Maximize}}\ C = E\left\{\sum_{\mathrm{i}=1}^{\mathrm{M}} \frac{\mathrm{T}-\tau}{\mathrm{T}} \mathrm{C}_i\right\} \quad (12)$$

Subject to the constraints given by the Eq. (10) and (11), and

$$P_{0,i} \geq 0, P_{1,i} \geq 0 \quad i = 1 \ldots\ldots M \qquad 0 \leq \tau \leq T$$

The probability of detection and probability of false alarm depend on sensing time τ and the above problem is convex optimization problem while considering the transmit powers P_0 and P_1, and when considering sensing time, it is not convex. The sensing time can be obtained using one dimensional exhaustive search since it lies within the interval $(0\!:\!T)$.

The convex optimization problem can be solved using Lagrangian dual optimization problem [14], [20]. By applying Lagrangian to the maximization problem, we get

$$L(P_0, P_1, \lambda, \mu_i) = E\{\sum_{i=1}^{M} \frac{\mathrm{T}-\tau}{\mathrm{T}}(\, a_{0,i}R_{00,i} + a_{1,i}R_{11,i} + b_{0,i}R_{10,i} + b_{1,i}R_{01,i})\} - \lambda[\frac{\mathrm{T}-\tau}{\mathrm{T}}\, E\{\, \sum_{i=1}^{M}(\, a_{0,i}P_{0,i} + a_{1,i}P_{1,i} + b_{0,i}P_{0,i} + b_{1,i}P_{1,i})\} - P_{av}] - \sum_{i=1}^{M}\mu_i\, [\frac{\mathrm{T}-\tau}{\mathrm{T}}E\{b_{0,i}P_{0,i}h_{sp,i} + b_{1,i}P_{0,i}h_{sp,i}\} - \Gamma_i]$$

(13)

Where λ, μ_i are lagrangian multipliers. The Lagrange dual function is given by

$$g(\lambda, \mu_i) = \underset{P_0\,, P_1}{\mathrm{Sup}}\ L(P_0, P_1, \lambda, \mu_i)$$

The joint optimization problem by the Eq. (13) can be decomposed into two sub problems for the transmit powers.

Sub problem 1 (SP1):

$$Maximize\ E\{\sum_{i=1}^{M}\frac{\mathrm{T}-\tau}{\mathrm{T}}[a_{0,i}\, R_{00,i} + b_{0,i}R_{10,i}]\} - \lambda E\{\sum_{i=1}^{M}\frac{\mathrm{T}-\tau}{\mathrm{T}}[a_{0,i}\, P_{0,i} + b_{0,i}P_{0,i}]\} - \sum_{i=1}^{M}\mu_i\frac{\mathrm{T}-\tau}{\mathrm{T}}E\{h_{sp,i}b_{0,i}P_{0,i}\}, \quad P_{0,i} \geq 0 \quad (14)$$

Sub problem 2 (SP2):

$$Maximize\ E\{\sum_{i=1}^{M}\frac{\mathrm{T}-\tau}{\mathrm{T}}[a_{1,i}\, R_{11,i} + b_{1,i}R_{01,i}]\} - \lambda E\{\sum_{i=1}^{M}\frac{\mathrm{T}-\tau}{\mathrm{T}}[a_{1,i}\, P_{0,i} + b_{1,i}P_{1,i}]\} - \sum_{i=1}^{M}\mu_j\frac{\mathrm{T}-\tau}{\mathrm{T}}E\{h_{sp,i}b_{1,i}P_{1,i}\}, \quad P_{1,i} \geq 0 \quad (15)$$

The above sub problems (SP1 and SP2) are convex optimization problems. Using the Lagrangian functions for the above sub problems, the optimal power levels can be obtained by applying the Karush-Kuhn-Tucker (KKT) conditions.

Let $\frac{\partial SP1}{\partial P_{0,i}} = 0$, and $\frac{\partial SP2}{\partial P_{1,i}} = 0$ and we get the following quadratic equations.

$$(P_{0,i})^2 + A_{0,i}P_{0,i} + B_{0,i} = 0 \ \text{ and }$$
$$(P_{1,i})^2 + A_{1,i}P_{0,i} + B_{1,i} = 0 \quad (16)$$

Where $\mathrm{A}_{0,\mathrm{i}}, \mathrm{B}_{0,\mathrm{i}}, \mathrm{A}_{1,\mathrm{i},} \mathrm{B}_{1,\mathrm{i}}$ are given by

$$A_{0,i} = \frac{log_2(e)(a_{0,i} + b_{0,i})}{\lambda(a_{0,i} + b_{0,i}) + \mu_i b_{0,ih_{sp,i}}} - \frac{2\sigma_n^2 + h_{ps,i}P_{p,i}}{h_{ss,i}},$$

$$B_{0,i} = \frac{1}{h_{ss,i}}\{\frac{\sigma_n^2 + h_{ps,i}P_{p,i}}{g_{ss,i}\sigma_n^{-2}} - \frac{log_2(e)(a_{0,i}(\mathrm{\sigma}_\mathrm{n}^2 + \mathrm{h}_{\mathrm{ps,i}}\mathrm{P}_{\mathrm{p,i}}) + b_{0,i}\sigma_n^2)}{\lambda(a_{0,i} + b_{0,i}) + \mu_i b_{0,ih_{sp,i}}} \quad (17)$$

$$A_{1,i} = \frac{log_2(e)(a_{1,i} + b_{1,i})}{\lambda(a_{1,i} + b_{1,i}) + \mu_i b_{1,i}h_{sp,i}} - \frac{2\sigma_n^2 + h_{ps,i}P_{p,i}}{h_{ss,i}}$$

$$B_{1,i} =$$

$$\frac{1}{h_{ss,i}}\{\frac{\sigma_n^2+h_{ps,i}P_{p,i}}{g_{ss,i}\sigma_n^{-2}}-\frac{log_2(e)(a_{1,i}(\sigma_n^2+h_{ps,i}P_{p,i})+b_{1,i}\sigma_n^2)}{\lambda(a_{1,i}+b_{1,i})+\mu_i b_{1,i}h_{sp,i}} \tag{18}$$

By solving this, the power allocation for a given λ, μ_i is given by

$$P_{0,i}=\left[\frac{A_{0,i}+\sqrt{\Delta_{0,i}}}{2}\right]^+, P_{1,i}=\left[\frac{A_{1,i}+\sqrt{\Delta_{1,i}}}{2}\right]^+ \text{Where}$$
$$\Delta_{0,i}=A^2{}_{0,i}-4B_{0,i}\Delta_{1,i}=4\,A^2{}_{1,i}-4B_{1,i} \tag{19}$$

The optimization problem is now to minimize the dual function $g(\lambda, \mu_i)$ and the Lagrangian multipliers λ and μ_i are updated using sub gradient method [18], [19].

Proposition 1: The sub gradient of the dual function $g(\lambda, \mu_i)$ is $[D, \mathbf{E}^T]$, where D is given by $D = P_{av} - E\sum_{i=1}^{M}\{[a_{0,i}+b_{0,i}]P_{0,i}+[a_{1,i}+b_{1,i}]P_{1,i}\}$ and $\boldsymbol{E}$ is a vector given by $\boldsymbol{E}_i = (\Gamma_i - \mathrm{E}[b_{0,i}P_{0,i}h_{sp,i}+b_{1,i}P_{1,i}h_{sp,i}])$ for i= 1 …..M . The value of λ_{k+1}, $\mu_{i,k+1}$ is obtained using the following expressions.

$$\lambda_{k+1} = max\left[\lambda_k + a\left\{E\left\{\sum_{i=1}^{M}[(a_{0,i}+b_{0,i})P_{0,i}] + (a_{1,i}+b_{1,i})P_{1,i}\right\} - P_{av}\right\}\right]$$

$\mu_{i,k+1} = \max[\mu_{i,k} + a\{E\{\mathrm{h}_{sp,i}b_{0,i}P_{1,i} + \mathrm{h}_{sp,i}b_{1,i}P_{1,i}\} - \Gamma_i\}]$ where a is a step size.

The proposed algorithm for power optimization for maximizing the capacity for WSSMPU is given in the following table.

Power optimization algorithm for wideband sensing based spectrum sharing cognitive radio network using multiple primary users.

```
Step 1: Initialize: $\lambda_1$, $\mu_{i,1}$, $k = 1$;
Step 2: For $\tau = 0:T$ ; Find $P_{d,i}$ and $P_{fa,i}$
Step 3: Optimization: Find $\boldsymbol{P_0}, \boldsymbol{P_1}$ using Eqs. (17) - (19);
    Update $\lambda_{k+1}, \mu_{i,k+1}$ using
    $\lambda_{k+1} = max\left[\lambda_k + a\left\{E\left\{\sum_{i=1}^{M}[(a_{0,i}+b_{0,i})P_{0,i}] + (a_{1,i}+b_{1,i})P_{1,i}\right\} - P_{av}\right\}\right]$
    $\mu_{i,k+1} = max[\mu_{i,k} + a\{E\{\mathrm{h}_{sp,i}b_{0,i}P_{1,i} + \mathrm{h}_{sp,i}b_{1,i}P_{1,i}\} - \Gamma_i\}]$
    Until $\lambda_{k+1}, \mu_{k+1}$ converge.
    Otherwise
    $k = k + 1$, go to step 3.
Step 4: Optimal Sensing time is given by
    $\tau_{opt} = \arg\ Max\ C\ (\boldsymbol{\tau}, \boldsymbol{P_0}, \boldsymbol{P_1})$
    and optimal Power levels are given by
    $[\boldsymbol{P}_{\mathbf{0},opt}, \boldsymbol{P}_{\mathbf{1},opt}] = [\boldsymbol{P_0}, \boldsymbol{P_1}]\tau = \tau_{opt}$
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5. Opportunistic spectrum access scheme using multiple primary users

In the opportunistic spectrum access scheme, the secondary user accesses the channel only when it is not used by the primary user that is when the primary is absent. In this proposed wideband opportunistic spectrum access using multiple primary users the secondary user simultaneously sense all the primary user bands and transmit in those frequency channels that are detected to be free. Therefore, high probability of detection is required to provide better service for the primary network. However, considering imperfect spectrum sensing, when the primary is present, it will detect it as absent and will access it .This will cause harmful interference to the primary user.

For the wideband opportunistic spectrum access scheme the average transmission rate of the i^{th}channel is given by

$$C_i = P(H_{0,i})(1-P_{fa,i})R_{0,i} + P(H_{1,i})(1-P_{d,i})R_{1,i} \tag{20}$$

Where $R_{0,i}$ and $R_{1,i}$ are given by Eq. (2).

The average transmit power constraint and the average interference power constraint is now given by

$$\frac{\mathrm{T}-\tau}{\mathrm{T}}E\{\textstyle\sum_{i=1}^{M}[a_{0,i}P_{0,i}+b_{0,i}P_{0,i}]\} \le P_{av}$$
$$\frac{\mathrm{T}-\tau}{\mathrm{T}}E\{b_{0,i}P_{0,i}h_{sp,i}\} \le \Gamma_i,\ i = 1\ …..M \tag{21}$$

The optimization of power to maximize the capacity of a wideband opportunistic spectrum access cognitive radio network using multiple primary users (WOSMPU) is formulated as follows.

$$Maximize\ \mathrm{C} = E\{\textstyle\sum_{i=1}^{M}\frac{T-\tau}{T}.C_i\}i = 1\ …..M \tag{22}$$

Subject to the constraints given by the Eq. (21), $P_{o,i} \ge 0, P_{1,i} \ge 0$

The problem is solved by using lagrangian decomposition method and then the Karush-Kuhn-Tucker (KKT) conditions applied to get the optimal transmit power as

$$P_{0,i} = \left[\frac{A_{0,i} + \sqrt{\Delta_{0,i}}}{2}\right]^{+} \tag{23}$$

The optimal values of λ and μ_i are found by using the ellipsoid method and the required sub gradient [24] of the dual functionh (λ, μ_i) is $[\mathrm{F}, \mathbf{G}^T]$, where F is given by $F = P_{av} - E\sum_{i=1}^{M}\{[a_{0,i} + b_{0,i}]\, P_{0,i}]\}$, **G** is a vector with $G_i = (\Gamma_i - \mathrm{E}[b_{0,i}P_{0,i}h_{sp,i}])$, $i = 1 \ldots\ldots M, \lambda \geq 0$, $\mu_i \geq 0$ and $P_{O,i}$ is the corresponding optimal power allocation in above equation for fixed λ and μ_i.

The algorithm for power optimization for maximizing the average data rate of the (WOSMPU) is given in the following table.

Optimal power allocation algorithm for wideband opportunistic based spectrum sharing for cognitive radio network using multiple primary users.

```
Step 1: Initialize: $\lambda_1$, $\mu_{i,1}$, $k = 1$;
Step 2: For $\tau = 0{:}T$ ; Find $P_{d,i}$ and $P_{fa,i}$
Step 3: Optimization: Calculate $\boldsymbol{P_0}$ using (23);
    Update $\lambda_{k+1}, \mu_{k+1}$ using
    $\lambda_{k+1} = max\left[\lambda_k + a\left\{E\left\{\sum_{i=1}^{M}[(a_{0,i} + b_{0,i})P_{0,i}] +\right\} - P_{av}\right\}\right]$
    $\mu_{i,k+1} = max[\mu_{i,k}\ a\{E\{h_{sp,i}b_{0,i}P_{1,i}\} - \Gamma_i\}]$
    If $\lambda_{k+1}$, $\mu_{k+1}$ converge, the process is end.
    Otherwise k=k+1, go to step 3.
Step 4: Optimal sensing time and optimal power are given by $\tau_{opt} = \arg\, max\mathrm{C}(\tau, \boldsymbol{P_0})$
    $[\boldsymbol{P_{0,opt}}] = [\boldsymbol{P_0}]\tau = \tau_{opt}$
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6. Outage capacity of the proposed spectrum sharing scheme.

The outage capacity [21], [22] can be an appropriate metric for the capacity of the system for slow fading channels and for voice and video transmission. Capacity with outage is defined as the maximum rate that can be transmitted over a channel with some outage probability. By this, high data ate can be achieved and decoded correctly except when the channel is in deep fading. The outage probability is defined as the probability that the transmission cannot be decoded with negligible error probability. By allowing the system to lose some data in the event of deep fades, a higher data rate can be maintained than if all data must be received correctly regardless of the fading state, as is the case for ergodic capacity.

Considering transmit power and interference power constraint, the outage capacity of the proposed SSS cognitive radio system using multiple primary users is investigated in this section. We also consider, as in [8], the truncated channel inversion with fixed rate (TIFR) technique. Here, the secondary transmitter invert the channel side information (CSI) to have a signal-to-noise ratio that is constant at the secondary receiver when the channels fading is above a certain threshold value [25]. Outage capacity is achieved with a truncated channel inversion policy for power adaptation that only compensates for fading above a certain cutoff fade depth. Non zero rate can be achieved for an outage probability target P_{out} by this adaptive transmission scheme. This will be useful for Rayleigh fading channels when the fading is severe.

6.1 Outage capacity under average transmit and interference constraints

We now consider here the outage capacity of the proposed spectrum sharing cognitive radio system and derive the power allocation problem.

In the TIFR technique, the secondary transmitter inverts the channel fading to achieve a constant rate at the secondary receiver when the channel fading is higher than a threshold. Let $\gamma_{0,i}$ and $\gamma_{1,i}$ be the threshold value when the primary is free and busy respectively. The secondary user transmit powers are given by

$$P_{0,i} = \begin{cases} \dfrac{\alpha_i}{g_{ss,i}}, & \dfrac{g_{sp,i}}{g_{ss,i}} \leq \dfrac{\gamma_{0,i}}{\sigma^2} \\ = 0 & \dfrac{g_{sp,i}}{g_{ss,i}} > \dfrac{\gamma_{0,i}}{\sigma^2} \end{cases} \tag{24}$$

$$P_{1,i} = \begin{cases} \dfrac{\alpha_i}{g_{ss,i}}, & \dfrac{g_{sp,i}}{g_{ss,i}} \leq \dfrac{\gamma_{1,i}}{\sigma^2} \\ = 0 & \dfrac{g_{sp,i}}{g_{ss,i}} > \dfrac{\gamma_{1,i}}{\sigma^2} \end{cases} \tag{25}$$

where the parameters $\gamma_{0,i}$, , $\gamma_{1,i}$, and α must be found so that the average transmit power constraint given by the Eq.(10) and the average interference power constraint given by the Eq.(11) are satisfied..

The following constraints should be satisfied by α based on the above constraints.

$$\alpha_i = \Gamma\mathrm{i}\,\{P(H_{1,i})(1 - P_{d,i})\left[\log\left(1 + \frac{\gamma_{0,i}}{\sigma^2}\right) - \frac{\gamma_{0,i}}{\gamma_{0,i} + \sigma^2}\right] + P(H_{1,i})P_{d,i}\left[\log\left(1 + \frac{\gamma_{1,i}}{\sigma^2}\right) - \frac{\gamma_{1,i}}{\gamma_{1,i} + \sigma^2}\right]$$

$$= t_1(\gamma_{0,i}, \gamma_{1,i}) \tag{26}$$

$$\alpha_i \leq \frac{P_{av}}{k_{0,i}\log\left(1+\frac{\gamma_{0,i}}{\sigma^2}\right)+k_{1,i}\log(1+\frac{\gamma_{1,i}}{\sigma^2})} = \mathrm{t}_2(\gamma_{0,\mathrm{i}},\gamma_{1,\mathrm{i}}) \tag{27}$$

The expressions for $K_{0,i}$ and $K_{1,i}$ are given by

$$K_{0,i} = P\left(H_{0,i}\right)\left(1-P_{fa,i}\right)+P(H_{1,i})P_{d,i}$$
$$K_{1,i} = P\left(H_{0,i}\right)P_{fa,i}+P(H_{1,i})P_{d,i} \tag{28}$$

The outage probability of WSSMPU CR network is given by

$$P_{out,i} = \frac{K_{0,i}\sigma^2}{\gamma_{0,\mathrm{i}}+\sigma^2}+\frac{K_{1,i}\sigma^2}{\gamma_{1,\mathrm{i}}+\sigma^2} \tag{29}$$

The channel capacity using the TIFR technique for the i^{th}channel is given as follows:

$$C_{TIFR,i} = Max\,\{\log\,(1+\frac{1}{\sigma^2}\,\{\min\{\mathrm{t}_1(\gamma_{0,\mathrm{i}},\gamma_{1,\mathrm{i}}),\mathrm{t}_2\,(\gamma_{0,\mathrm{i}},\gamma_{1,\mathrm{i}})\})(1-\frac{K_{0,i}\sigma^2}{\gamma_{0,\mathrm{i}}+\sigma^2}-\frac{K_{1,i}\sigma^2}{\gamma_{1,\mathrm{i}}+\sigma^2})\} \tag{30}$$

Therefore, the total channel capacity under the TIFR policy for WSSMPU is given by

$$C_{TIFR} = \sum_{i=1}^{M}(\,C_{TIFR,i}) \tag{31}$$

The parameters t_1 and t_2 take the following form since the target outage probability to be satisfied.

$$\overline{\mathrm{t}_1}\left(\gamma_{0,\mathrm{i}}\right) = \frac{\Gamma}{\mathrm{P}\,(\mathrm{H}_{1,\mathrm{i}})}\{\frac{\log\left(1+\frac{\gamma_{0,\mathrm{i}}}{\sigma^2}\right)-\frac{\gamma_{0,\mathrm{i}}}{\gamma_{0,\mathrm{i}}+\sigma^2}}{(1-\mathrm{P}_{\mathrm{d,i}})^{-1}}+\mathrm{P}_{\mathrm{d,i}}\,[\frac{\overline{P_{out}}}{\mathrm{K}_{1,\mathrm{i}}}+\log\left(\frac{K_{1,i}}{\overline{P_{Out}}-\frac{K_{1,i}\sigma^2}{\gamma_{0,i}+\sigma^2}}\right)-\frac{\mathrm{K}_{0,\mathrm{i}}\sigma^2}{\mathrm{K}_1(\gamma_{0,\mathrm{i}}+\sigma^2)}-1]\}^{-1} \tag{32}$$

$$\overline{\mathrm{t}_2}\left(\gamma_{0,\mathrm{i}}\right) = P_{avg}[\mathrm{K}_{0,\mathrm{i}}\log\left(1+\frac{\gamma_{0,i}}{\sigma^2}\right)+K_{1,i}\,\log\,\left(K_{1,i}\,\right)-K_{1,i}\,\log(\,\overline{P_{out}}-\frac{\mathrm{K}_{0,\mathrm{i}}\sigma^2}{\gamma_{0,\mathrm{i}}+\sigma^2}\,)\,]^{-1} \tag{33}$$

respectively. The outage capacity of the proposed WSSMPU cognitive radio system with for the i^{th} channel is given by

$$C_{out,i} = max\{\log\left(1+\frac{1}{\sigma^2}min\{\,\overline{t_1}\left(\gamma_{0,i}\right),\overline{\mathrm{t}_2}\left(\gamma_{0,\mathrm{i}}\right)\}\right)\,(1-\overline{P_{out}})\} \tag{34}$$

Where $\overline{P_{out}}$ is the target outage probability.

Therefore, the outage capacity of the proposed cognitive radio system using multiple primary users is given by

$$C_{out} = \sum_{i=1}^{M}(C_{out,i}). \tag{35}$$

7. Simulation results

In our simulation, the number of primary users is assumed as $M = 3$ and the bandwidth of narrow band channels considered here are of each $6\,MHz$ and the sampling frequency to be $6\,MHz$.The channels are assumed to be block flat fading channel and their power gains are ergodic stationary and exponentially distributed with unit mean. The SNR received from each primary user at the secondary receiver is assumed to be $-12dB, -15dB\;and -20dB$ respectively. The transmit power P_p of all the primary user is assumed to be$10dB$, and the noise variance $N_o = 1$. The frame duration T is set as $100\,ms$.

Fig.4 and Fig.5 displays the total ergodic throughput as a function of sensing time τ for the WSSMPU and the WOSMPU cognitive network for values of $P\left(H_{0,i}\right) = 0.8$ and $P\left(H_{0,i}\right) = 0.6$ for i = 1, 2, 3 and the average transmit power to be 20dB and 10dB respectively. The maximum average interference power Γi tolerable by each primary user is set to be $-10dB, -12dB\;and -15dB$.

It can be clearly seen from the (Fig.4 and 5) that the proposed wideband sensing based spectrum sharing scheme using multiple primary users (WSSMPU) is having higher ergodic throughput since the interference introduced to each primary user is different and also it is higher compared with WOSMPU, which is because in WSS scheme the secondary user transmit during both idle and busy state of primary user. The throughput decreases as sensing time increases since the data transmission time decreases.

In (Fig.6) displays the total ergodic throughput as a function of average transmit power constraint P_{av} for probabilities $P(H_{0,i}) = 0.8$ and $P(H_{0,i}) = 0.6$ for $i = 1\,,2,3$ .The maximum average interference power Γi that is tolerable by each primary user is assumed to be $(-10dB, -12dB\,, -15dB)$. Here we study the effect of average transmit power on ergodic throughput. It is found in (Fig.6) that the throughput increases as transmit power increases. For high transmit power the throughput slightly remain constant.

In WSS scheme, more transmit power is allocated when the primary is free and when the primary is active, the interference power has to be considered. Even if the transmit power is large, throughput have little effect due to the interference power constraint.

In (Fig.7) the maximum capacity versus average interference power Γ considering TIFR transmission is given for the proposed WSSMPU and WOSMPU. Here we have assumed the interference power that is tolerable by each primary user is same.

It can be clearly seen from (Fig.7) that higher capacity is achieved by the proposed spectrum sharing scheme under *CTIFR* technique compared to the conventional spectrum sharing [8] and sensing based spectrum scheme in [15], due to multiple primary users having different interference power constraint and also by the fact that the secondary users sense the state of the primary user and make a more efficient use of the available spectrum. The proposed scheme offers an efficient way to perform spectrum sensing and adapt the transmit power to protect the primary users from harmful interference.

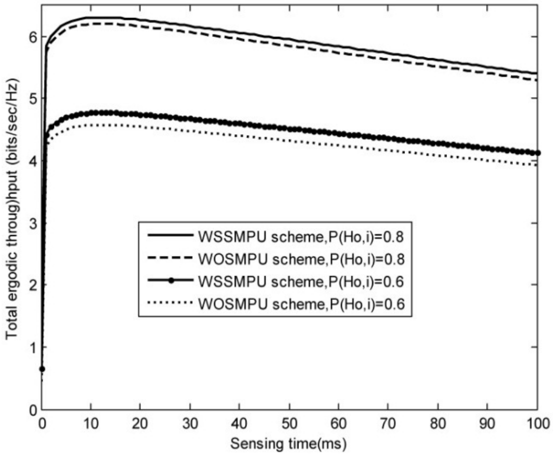

Fig. 4 Total ergodic throughput versus sensing time under $P_{av}20dB$for different values of($PH_{0,i}$).

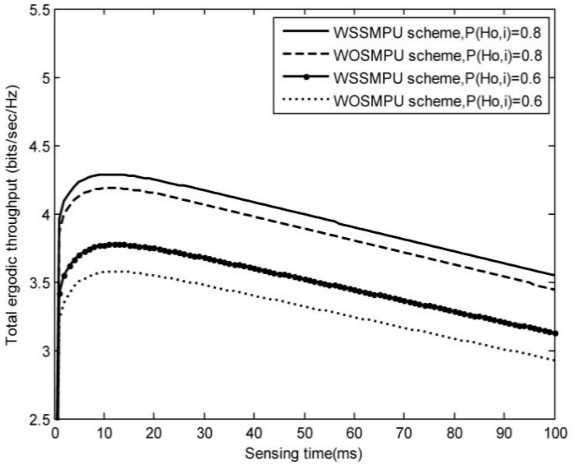

Fig. 5 Total ergodic throughput versus sensing time under $P_{av} = 10dB$ for different values of($PH_{0,i}$).

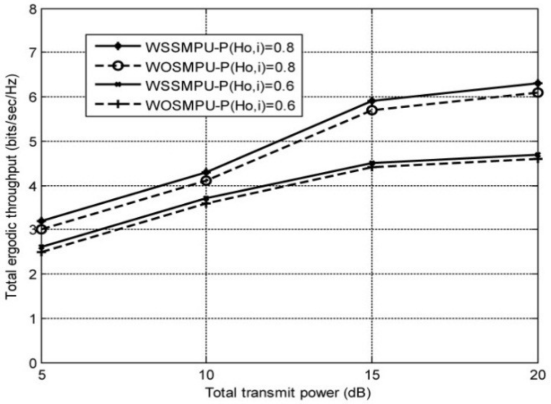

Fig. 6 Total ergodic throughput versus total transmit power P_{av} for different values of $(PH_{0,i})$.

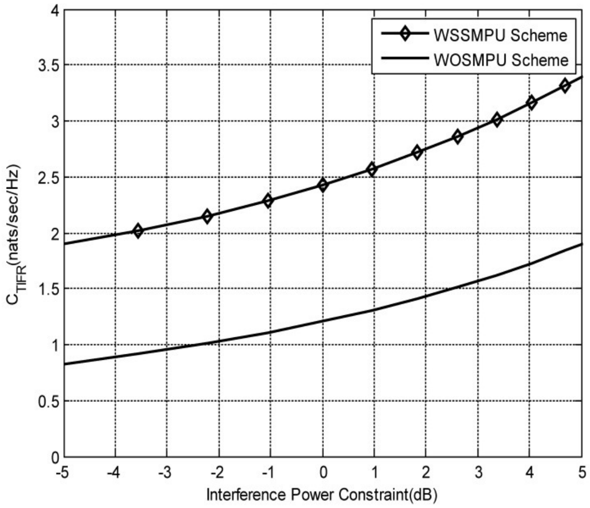

Fig. 7 Maximum capacity under the TIFR transmission policy versus the average interference constraintΓ.

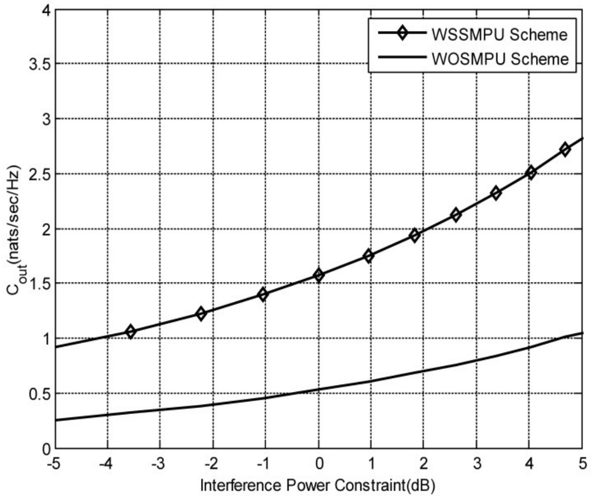

Fig. 8 Outage capacity versus the average interference constraint Γ.

The outage capacity is presented in (Fig.8) for the proposed WSSMPU and WOSMPU. Similar remarks to the CTIFR capacity can be made for the outage capacity and the outage capacity of the proposed sensing based sharing scheme using multiple primary users is higher compared to the conventional spectrum sharing scheme.

8. Conclusion

In this paper, we proposed a sensing based spectrum sharing cognitive radio system using multiple primary users that significantly improves the ergodic and outage capacity of cognitive radio networks. The optimal power to maximize the capacity of the cognitive radio system is derived and an algorithm is proposed using Lagrange dual optimization method and sub gradient projection. This is compared with Opportunistic spectrum access using multiple primary users. And also we studied the outage and TIFR capacities under average transmit and average interference power constraints. In addition, the simulation results are presented which shows that the proposed cognitive radio system with multiple primary users can significantly improve the capacity of spectrum sharing cognitive radio networks.

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