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In memory of Professor Borislav D. CRSTICI on the Centenary of his birth.

CONTRIBUTIONS TO THE STUDY OF THE TURNING CURVE OF A WHEELED TRACTOR

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Abstract

This paper presents an analytical method for determining the turning curve of a wheeled tractor using certain simplifying assumptions. The study addresses the challenges associated with the turning curve, which is significantly influenced by the rotation of the front wheels relative to the longitudinal axis. By formulating the problem analytically, the paper derives equations governing the tractor's movement, providing an intrinsic description of the desired curve. The analytical approach reveals that the turning trajectory consists of two arcs of Cornu's spiral and a circular arc. Comparisons between the graphical method and the proposed analytical solution demonstrate the efficacy and simplicity of the latter. The main parameters of the turning curve are computed directly, facilitating practical applications in agricultural operations.
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1 Introduction

In various agricultural tasks involving the tractor (plowing, sowing, etc.), part of the distance traveled consists of turning curves. The empty moves made during

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turns at the ends of the plots are significant: according to Academician Svirșcevski and Șandru, on a plowed area of 100 hectares, the tractor covers a distance of 570 km, of which 53 km are traveled during turning. On average, the length of these empty moves at the plot ends represents 5-15% of the total distance traveled by the assembly. Thus, it is essential to understand this turning curve to minimize unnecessary moves.

In some specialized treatises (e.g., *Exploitation of the Machinery and Tractor Park* by A. Șandru, I. Niță, and C. Ștefan), the tractor's turning curve is approximated by a circle with a minimum radius. However, in reality, the transition from straight-line travel to circular motion cannot happen instantaneously, as would be assumed for a circular path, but instead takes time, estimated by some authors to be 3-5 seconds or even more under certain conditions. As a result, during turning, the tractor describes a certain curve. Difficulties in accurately determining the curve arise because the turning radius is variable, and the turning center is unstable.

The determination of the turning trajectory of a wheeled tractor has been the subject of research by several scientists. For instance, Academician B.S. Svirșcevski, in his work *Exploitation of the Machinery Park* (p.191), addresses this issue, presenting a construction method for this trajectory developed by Grigoriev and Cuvicinski. Svirșcevski emphasizes the problem's analytical complexity in his book and ultimately provides only the graphical construction method mentioned.

2 Background and Analytical Method

In this paper, using certain simplifying assumptions, an analytical method for determining the turning curve of a wheeled tractor is presented.

It is known that the tractor's turning curve is influenced by the rotation of the front wheels relative to the longitudinal axis. Let's consider a tractor position where the plane of the front wheels is rotated relative to the longitudinal axis by specific angles α' and α'' (see Fig. 1). Given that the tractor moves without slipping, the turning center is determined by the extensions of the front and rear wheels.

The plane XX' perpendicular to OB represents the plane of a third front wheel that could replace the two wheels to allow the tractor to follow the same path. The plane XX' makes an angle α with the longitudinal axis, located between α' and α''. Practically, α' and α'' can be considered equal to α.

Point A, whose trajectory is to be determined, is called the assembly center. From triangle ΔOAB and the notations in Fig. 1, the instantaneous curvature radius is written as:

$$R = \frac{L}{\tan \alpha}. \quad (1)$$

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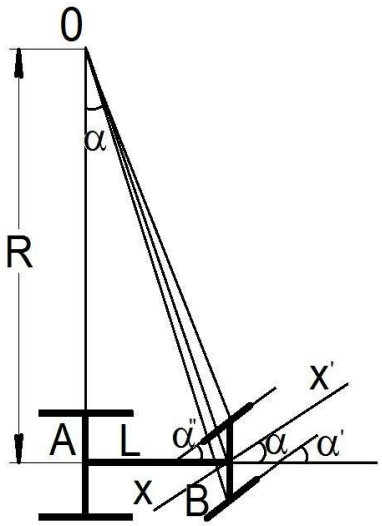


Figure 1: Turning curve geometry

Given that α does not take large values, we can replace $\tan\alpha$ with α, and equation (1) becomes:

$$R = \frac{L}{\alpha}. \quad (2)$$

3 Uniform Rotation of the Steering Wheel

Assuming the steering wheel rotates uniformly:

$$\alpha = \omega\tau, \quad (3)$$

where ω is the angular rotation speed of plane XX' (proportional to the angular rotation speed of the steering wheel), and τ represents time. We also have:

$$s = v\tau, \quad (4)$$

where v is the tractor's speed. Considering equations (3) and (4), equation (2) becomes:

$$R = \frac{Lv}{\omega s}. \quad (5)$$

Equation (5) is, in fact, the intrinsic equation of the desired curve.

$$\frac{1}{R} = \frac{d\varphi}{ds}.$$

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4 Curvature Definition and Integration

Starting from the definition of curvature:

$$\varphi = \frac{\omega}{2Lv}s^2, \quad (6)$$

where φ is the angle between the OX axis and the tangent at a point on the curve. Using the direction cosines definition:

$$\frac{dx}{ds} = \cos\varphi, \quad \frac{dy}{ds} = \sin\varphi,$$

by integrating, we derive the equation of the curve in parametric form:

$$x = \int_0^s \cos\left(\frac{\omega}{2Lv}s^2\right) ds, \quad 0 < s < s_1 \quad (7)$$

$$y = \int_0^s \sin\left(\frac{\omega}{2Lv}s^2\right) ds.$$

or, after substitution:

$$s = kt, \quad (8)$$

where:

$$k = \sqrt{\frac{\pi Lv}{\omega}}. \quad (9)$$

5 Fresnel Integrals and Cornu's Spiral

The relationships in (7) become:

$$x = k\int_0^t \cos\left(\frac{\pi}{2}t^2\right) dt \quad 0 < t < t_1. \quad (10)$$

$$y = k\int_0^t \sin\left(\frac{\pi}{2}t^2\right) dt$$

These expressions highlight the Fresnel integrals, which are present and have been tabulated. These equations are those of Cornu's spiral.

In fact, equations (10) represent only a portion of the turning curve, specifically the section during steering wheel rotation, up to the point where parameter s reaches $s_1 = v\tau_1\lambda r_1$. After the steering wheel reaches its maximum rotation, the tractor (more precisely, point A) describes an arc of a circle, after which, by reversing the steering wheel rotation, it follows a return trajectory.

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6 Symmetry of the Turning Curve

During the return phase, the curvature radius is still given by equation (2), except that angle α decreases over time,

$$\alpha = \alpha_{\max} - \omega\tau,$$

so the intrinsic equation becomes:

$$R' = \frac{Lv}{\omega\left(\frac{\alpha_{\max} v}{\omega} - s\right)}.$$

Switching to Cartesian coordinates as in the initial part of the curve, ignoring translations and rotations, we obtain:

$$x' = \int_0^{s'} \cos\left(\frac{\omega}{2Lv} s'^2\right) dc', \quad y' = \int_0^{s'} \sin\left(\frac{\omega}{2Lv} s'^2\right) dc'. \quad (11)$$

Comparing equations (7) and (11), we find that the two curves differ only by a symmetry. Thus, to trace the tractor's turning curve (point A) by 180°, which consists of two arcs of Cornu's spiral and a circular arc, the OM_1 portion is traced through points using relationships (10) (Fig. 2), with coordinates of point $M_1(x_1, y_1)$ corresponding to parameter $s_1 = v\tau_1$. The tangent at M_1, which actually makes an angle φ_1 with the OX axis as given by equation (6), helps locate the curvature center O_1 using the curvature radius at point M_1 from equation (1). The parallel to the OX axis passing through O_1 is the symmetry axis of the turning curve.

In Fig. 2, the coordinate axes are chosen such that the constants resulting from the transition from intrinsic equation (5) to Cartesian coordinates (7) are zero.

In the case of a 90° turn, we get the situation in Fig. 3, where the symmetry axis is parallel to the secondary bisector passing through point O_1 (Fig. 3).

7 Comparison with Graphical Method

To see the correspondence of these theoretical considerations, we can compare the turning curve of a tractor drawn according to the graphical trajectory of Grigoriev and Cuvicinski with the trajectory built analytically under the same conditions, following the above-described method. In Fig. 4, the curve is drawn using the graphical method, while the points marked with circles represent the analytical coordinates calculated using equations (10). It is observed that, practically, both methods yield the same curve.

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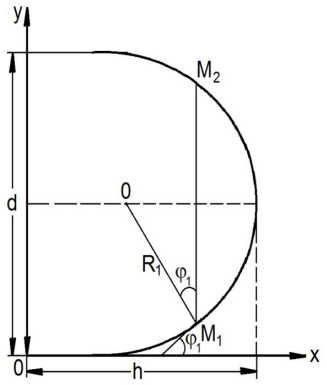


Figure 2: Calculated trajectory

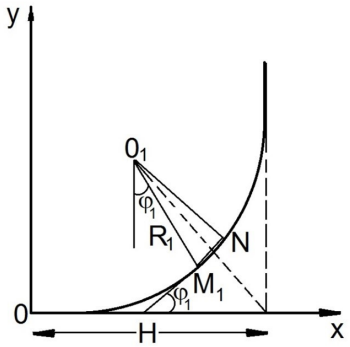


Figure 3: Symmetry of the Curve

Compared to the graphical method, the analytical method, specifically the construction based on equations (10), is simpler, as the values of the integrals can be found in tables.

The construction and calculation of coordinates were performed under the following initial conditions:

$$\tau = 5\,\mathrm{s}, \quad L = 2.445\,\mathrm{m}, \quad v = 1.2\,\mathrm{m/s}, \quad \omega = 6\,\mathrm{rad/s}.$$

The analytical method also allows for the direct calculation of the main parameters of the turning curve d and h (Fig. 2) and H (Fig. 3), without needing graphical construction. Indeed, from Fig. 2:

$$d = 2(y_1 + R_1 \cos\varphi_1),$$

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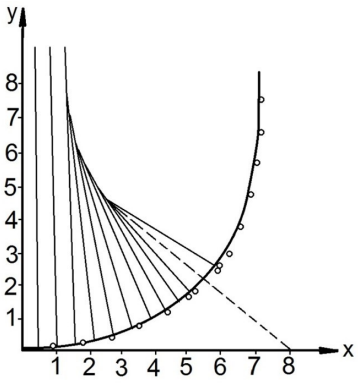


Figure 4: Calculated Trajectory and Graphical Method

$$h = x_1 + R_1 - R_1 \sin\varphi_1,$$

or considering relations (10):

$$d = 2k \int_0^{t_1} \sin\left(\frac{\pi}{2}t^2\right)\, dt + 2R_1 \cos\varphi_1, \quad (12)$$

$$h = k \int_0^{t_1} \cos\left(\frac{\pi}{2}t^2\right)\, dt + R_1(1 - \sin\varphi_1), \quad (13)$$

where:

$$k = \sqrt{\frac{\pi L v}{\omega}}, \quad s_1 = v\tau_1, \quad t_1 = \frac{s_1}{k}, \quad R_1 = \frac{L}{\tan\alpha_{\max}}, \quad \varphi_1 = \frac{\omega}{2Lv}s_1^2.$$

In the 90° turning case, Fig. 3 yields:

$$M_1N = 2R_1 \sin\left(\frac{\pi}{4} - \varphi_1\right),$$

and so:

$$H = x_1 + 2R_1 \sin\left(\frac{\pi}{4} - \varphi_1\right)\cos\frac{\pi}{4} + y_1,$$

or considering relations (10):

$$H = k \int_0^{t_1} \cos\left(\frac{\pi}{2}t^2\right)\, dt + k \int_0^{t_1} \sin\left(\frac{\pi}{2}t^2\right)\, dt + \sqrt{2}R_1 \sin\left(\frac{\pi}{4} - \varphi_1\right), \quad (14)$$

where k, R_1, t_1, and φ_1 are calculated similarly as for relations (12) and (13).

In conclusion, the turning curve of a wheeled tractor can be represented using equations (10), while the main parameters d, h for a 180° turn, and H for a 90° turn, can be calculated using equations (12), (13), and (14).

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In the particular case $\tau = 0$ where the tractor enters and exits the turn with the steering wheel rotated and fixed, the primary parameters given by formulas (12), (13), and (14) take the values $d = 2R_1$, $h = R_1$, and $H = R_1$, which is evident.

For the numerical case considered in curve construction:

$$\tau = 5\,\text{s}, \quad L = 2.445\,\text{m}, \quad v = 1.2\,\text{m/s}, \quad \omega = 6\,\text{rad/s},$$

the intermediate constants are:

$$k = 9.38, \quad s_1 = 6, \quad t_1 = 0.64, \quad R_1 = 4.23,$$

while the main parameters derived from equations (12), (13), and (14) are:

$$d = 9.27\,\text{m}, \quad h = 7.45\,\text{m}, \quad H = 7.85\,\text{m}.$$

Conclusions

The study of the turning curve of a wheeled tractor reveals the complex dynamics involved in transitioning from straight-line motion to a curved path. Using simplifying assumptions, an analytical approach was developed that accurately models the tractor's turning trajectory. The results demonstrate that the turning path can be described by two arcs of Cornu's spiral and a circular segment, providing a more realistic representation than simple circular approximations.

The proposed analytical method has several advantages over traditional graphical methods, including simplified calculations and direct determination of key parameters such as the curvature radius and angular relationships. These findings have practical implications for optimizing tractor operations in agricultural settings, reducing unnecessary movement, and enhancing efficiency.

Furthermore, comparisons with graphical methods indicate a high level of agreement, validating the analytical model. The equations derived, particularly those involving intrinsic and Cartesian coordinates, offer a robust foundation for further research and practical applications in machinery design and agricultural field management.

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