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THE DETERMINATION OF THE STATISTICAL DISTRIBUTION FUNCTION IN THE TESTING OF PRODUCT LIFE

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Abstract

In this paper I consider that a series type system is a system that functions only if all its components are functioning. In case that the distribution functions for all the components of a series type system are known, then the distribution function of the whole system is given by relation (9). Particular cases are analysed when the components are exponential distributions of the Weibull or Rayleigh type. [1]

The theory of durability has found large application in the statistical control of products, in the security of the technical systems, as well as in demographic problems such as those referring to the medium duration of life etc.

The above mentioned theory has been dealt with in several works. One must mention in this regard the basic works written by B. Epstein and M. Sobel. Control plans have been elaborated, being based on different rules of statistical distribution such as: exponential [2], Rayleigh [3], Weibull [4], gamma [5].

It follows that one of the outstanding problems of the theory of durability is the determination of the function of distribution. Although there are sufficient possibilities to determine a rule of distribution, the former are insufficient and their approximation to reality is rather unsatisfactory. For this reason we suggest new functions of distribution in this domain, even if some of them are combined with those existent.

Since $f(t)$ is the function of distribution in time, the function of distribution will be:

$$F(t) = \int_0^t f(u)du. \tag{1}$$

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The function $P(t)$ called function of survival (in demography), respectively function of security or fiability (in technics) is defined:

$$P(t) = 1 - F(t). \tag{2}$$

The mortality rate, respectively the rate of destruction is defined:

$$\lambda(t) = f(t)/(1 - F(t)). \tag{3}$$

The following relations are to be observed between these three functions:

$$f(t) = \lambda(t)P(t) \tag{4}$$

$$P(t) = e^{-\int_0^t \lambda(u)du} \tag{5}$$

The testing of the durability of a complex system cannot be carried out without a previous analysis of the durability of the component parts of the system. The question that arises is to determine the same function when the function of distribution of the component parts is known.

A system usually functions only if each component is functioning i. e. a series type connection. In these conditions the probability of their functioning is given by the rule of the multiplication of the probability of independent cases. In case that $f_i(t)$, $(i = 1, 2, ..., n)$ are the rules of the distribution of the elements of the systems, then $\lambda_i(t)$ and $P_i(t)$ are the rates, respectively the corresponding security functions.

Since $P_i(t)$ is the probability that the component i should not be out of order up to the moment t, the probability of the functioning of the whole system is:

$$P_s(t) = P_1(t) \cdot P_2(t) \cdot ... \cdot P_n(t) \tag{6}$$

Because of relation (5), relation (6) becomes:

$$P(t) = e^{-\int_0^t \sum_{i=1}^n \lambda_i(u)du} \tag{7}$$

We note: $\lambda_s = \sum_{i=1}^n \lambda_i$ and relation (7) becomes:

$$P_s(t) = e^{-\int_0^t \lambda_s du} \tag{8}$$

Also:

$$f_s(t) = \lambda_s \cdot P_s = \sum_{i=1}^n \lambda_i \cdot \prod_{i=1}^n P_i = \sum_{i=1}^n \lambda_i \cdot \frac{\prod_{i=1}^n f_i}{\prod_{i=1}^n \lambda_i}$$

and

$$f_s(t) = \frac{\sum_{i=1}^n \lambda_i}{\prod_{i=1}^n \lambda_i} \cdot \prod_{i=1}^n f_i \tag{9}$$

If the functions of distributions for the components of the systems are $f_i(t)$ the function of distribution of the whole system is given by relation (9).

We may conclude that in a series type system the rates are additive.

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We consider that the functions of all the components of the system follow Weibull's rule:

$$f_i(t) = k \cdot a_i \cdot t^{k-1} \cdot e^{-a_i \cdot t^k}$$

with the rate:

$$\lambda_i(t) = k \cdot a_i \cdot t^{k-1}$$

Calculating the functions of distribution of the system with relation (9) it results that:

$$f_s(t) = \frac{k \cdot \sum_{i=1}^{n} a_i \cdot t^{k-1}}{k^n \cdot (t^{k-1})^n \cdot \prod_{i=1}^{n} a_i} \cdot k^n \cdot (t^{k-1})^n \cdot \prod_{i=1}^{n} a_i \cdot e^{-\sum_{i=1}^{n} a_i t^k}$$

so

$$f_s(t) = k \cdot \sum_{i=1}^{n} a_i \cdot t^{k-1} \cdot e^{-\sum_{i=1}^{n} a_i t^k} \tag{10}$$

which is a Weibull distribution too.

Thus, if in a series type system each component follows a rule of Weibull distribution, the system itself functions according to a rule of Weibull distribution.

Special cases: let be $k = 1$, Weibull's rule is the exponential distribution and the theorem becomes: if in a series type system each component follows a rule of exponential distribution, the system also follows a rule of exponential distribution.

Let be $k = 2$, Weibuul's rule is the Rayleigh distribution and the theorem becomes: if in a series type system each component follows a Rayleigh distribution rule, the system also follows a Rayleigh distribution rule.

Thus one can explain the occurrence of systems with distributions of the exponential or Rayleigh type.

Let us suppose that each component of the system follows another rule of distribution, but each of them of the Weibull type, i. e.:

$$f_i(t) = k_i \cdot a_i \cdot t^{k_i - 1} \cdot e^{-a_i \cdot t^{k_i}}$$

Using relation (9) we get the distribution of system:

$$f_s(t) = \sum_{i=1}^{n} k_i \cdot a_i \cdot t^{k_i - 1} \cdot e^{-\sum_{i=1}^{n} a_i \cdot t^{k_i}} \tag{11}$$

which is no longer a Weibull distribution.

For example, a system consisting of two subsystems, one of them with an exponential distribution rule and the other with a Rayleigh type rule, will have a distribution rule got from the relation (11) for $k_1 = 1$ and $k_2 = 2$ as follows:

$$f_s(t) = (a_1 + 2a_2 \cdot t) \cdot e^{-(a_1 \cdot t + a_2 \cdot t^2)} \tag{12}$$

If only the experimental data of the system are known, to determine its distribution function, it is necessary to calculate and to represent graphically the rate

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of distribution from the experimental data in the histogram. Then the function of this curve is to be determined, preferably a sum of other functions as simple as possible, if possible a sum of exponentials, admitting that the system consisted of Weibuul components.

Provided that the rate is a sum of exponentials i. e.:

$$\lambda_s(t) = \sum_{i=1}^{n} c_i \cdot t^{b_i}$$

it may be formulated as follows:

$$\lambda_s(t) = \sum_{i=1}^{n} k_i a_i \cdot t^{k_i - 1}$$

and the function of distribution is given by relation (11).

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