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ANALYTIC APPROXIMATE APPROACHES AND EXACT SOLUTIONS FOR THE EQUATION ON *MHD* FLOW OF A POWER-LAW VISCOUS FLUID

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Abstract

We use a recently modified version of the Optimal Homotopy Asymptotic Method (OHAM) to compute approximate analytic solutions for the equation on magnetohydrodynamic (MHD) flow of a power-law viscous fluid over a stretching sheet and to analyze the effect of the magnetic parameter on the flow. The accuracy of our approximate solutions is emphasized by a comparison with numerical results obtained by using the fourth order Runge-Kutta method. It is shows that the exact solution is computed via OHAM technique. [1]

Keywords and phrases: *optimal homotopy asymptotic method (OHAM), magnetohydrodynamic flow, viscous fluid.*

1 Introduction

The proprieties of viscoelastic materials have been intensively studied in recent years because of their industrial and technological applications such as plastic processing, cosmetics, paint flow, adhesives, accelerators, electrostatic filters, etc [1].

Andersson et al. [2] have further investigated the magnetohydrodynamic flow over a stretching sheet of an electrically conducting incompressible fluid obeying the power-law model. Cortell [8] investigated the laminar boundary layer flow induced in a quiescent visco-elastic fluid by a permeable stretched flat surface with non-linearly (quadratic) velocity. Different methods are applied to study

[1]MSC (2010): 65L60, 76A10, 76D05, 76D10, 76M25, 76M55, 80M25, 34F15

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the MHD fluid flow with power-law model, such as: homotopy analysis method (HAM) Abbas et al. [3], Hayat et al. [4], Mabood [10], homotopy perturbation method (HPM) in Raftari et al. [6], [9], homotopy perturbation sumudu transform method (HPSTM) in Sushila et al. [7]. A numerically central-difference scheme is applied in Chen [5].

The aim of the present paper is to propose some accurate, analytic approximate approaches for the equation on magnetohydrodynamic (MHD) flow of a power-law viscous fluid over a stretching sheet and to analyze the effect of the magnetic parameter on the flow using an analytical technique, namely optimal homotopy asymptotic method [11]-[14].

The validity of our procedure, which does not imply the presence of a small parameter in the equation, is based on the construction and determination of the auxiliary functions combined with a convenient way to optimally control the convergence of the solution. The efficiency of the proposed procedure is proves while an accurate solution is explicitly analytically obtained in an iterative way after only one iteration. Two ways to construct some approximate analytic solutions are presented and the exact solution is obtained.

The paper is organized as follows: in the second section the equation on magnetohydrodynamic flow of a power-law viscous fluid over a stretching sheet is presented. In the third section a briefly presentation of the Optimal Homotopy Asymptotic Method, developed in [14] and used in the last part in order to obtain the approximate analytic solutions of the nonlinear differential equation. The last section treats two analytic approaches using rational functions and exponential functions. The exact solution is obtained using exponential functions.

2 Equation of motion

The dimensionless equation on magnetohydrodynamic flow of a power-law viscous fluid over a stretching sheet can be written as [15]:

$$X'''(t) + X(t)X''(t) - (X'(t))^2 - mX'(t) = 0, \tag{1}$$

with the initial/ boundary conditions

$$X(0) = 0,\ X'(0) = 1,\ \lim_{t\to\infty} X'(t) = 0, \tag{2}$$

where $t > 0$, m is magnetic parameter and prime denotes derivative with respect to t.

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3 Basic ideas of the optimal homotopy asymptotic method

In [13] the authors compute analytical approximate solutions for equation:

$$L\Big(F(t)\Big) + N\Big(F(t)\Big) = 0, \tag{3}$$

subject to the boundary / initial conditions of the type

$$B\Big(F(t), \frac{dF(t)}{dt}\Big) = 0. \tag{4}$$

For the flow of viscous fluid determined by Eq. (1) with initial/boundary conditions (2), the corresponding operators L, N and B will be introduced in the next section.

We build the following homotopy based on OHAM [11]-[14]:

$$\begin{aligned} &\mathcal{H}\Big[L\Big(F(t,p)\Big),\ H(t,C_i),\ N\Big(F(t,p)\Big)\Big] = \\ &= L\Big(F_0(t)\Big) + p\Big[L\Big(F_1(t,C_i)\Big) - H(t,C_i)N\Big(F_0(t)\Big)\Big] \equiv 0, \end{aligned} \tag{5}$$

where $p \in [0,\ 1]$ is the embedding parameter, L is a linear operator, and

$$H(t,C_i) \neq 0$$

is an auxiliary convergence-control function. H is a function of the variable t and of the parameters C_1, C_2, ..., C_s.

The homotopy (5) satisfies the following properties:

$$\mathcal{H}\Big[L\Big(F(t,0)\Big),\ H(t,C_i),\ N\Big(F(t,0)\Big)\Big] = L\Big(F_0(t)\Big) = 0, \tag{6}$$

$$\mathcal{H}\Big[L\Big(F(t,1)\Big),\ H(t,C_i),\ N\Big(F(t,1)\Big)\Big] = L\Big(F_1(t,C_i)\Big) - H(t,C_i)N\Big(F_0(t)\Big) = 0. \tag{7}$$

Let choose the function F as:

$$F(t,p) = F_0(t) + pF_1(t,C_i), \tag{8}$$

and then from the Eq. (5) we obtain the governing equations of $F_0(t)$ and $F_1(t,C_i)$ by equating the coefficients of p^0 and p^1, respectively:

$$L\Big(F_0(t)\Big) = 0, \qquad B\Big(F_0(t), \frac{dF_0(t)}{dt}\Big) = 0, \tag{9}$$

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$$\begin{aligned} &L\Big(F_1(t,C_i)\Big) = H(t,C_i)N\Big(F_0(t)\Big), \\ &B\Big(F_1(t,C_i), \tfrac{dF_1(t,C_i)}{dt}\Big) = 0, \quad i = 1,2,...,s. \end{aligned} \tag{10}$$

One can find $F_0(t)$ by solving the linear Eq. (9). The nonlinear operator N is defined by:

$$N\left(F_0(t)\right) = \textstyle\sum_{i=1}^{n} h_i(t)g_i(t), \tag{11}$$

so one can compute $F_1(t,C_i)$, where n is a positive integer, and $h_i(t)$ and $g_i(t)$ are known functions that depend on $F_0(t)$ and on N.

Then the general solution of the nonhomogeneous linear equation (10) is obtained by summing the general solution of the corresponding homogeneous equation and a particular solution of the nonhomogeneous equation. But the computation of such a particular solution is not possible in most cases, so the computation of the function $F_1(t,C_i)$, follows the next steps:

- We consider the $F_1(t,C_i)$ of the form:

$$F_1(t,C_i) = \textstyle\sum_{i=1}^{m} H_i(t,h_j(t),C_j)g_i(t), \quad j = 1,\ ...,\ s, \tag{12}$$

or

$$\begin{aligned} &F_1(t,C_i) = \textstyle\sum_{i=1}^{m} H_i(t,g_j(t),C_j)h_i(t), \quad j = 1,\ ...,\ s, \\ &B\left(F_1(t,C_i),\ \tfrac{dF_1(t,C_i)}{dt}\right) = 0. \end{aligned} \tag{13}$$

The above expressions of $H_i(t,h_j(t),C_j)$ contain linear combinations of the functions h_j, $j = 1,\ ...,\ s$ and the parameters C_j, $j = 1,\ ...,\ s$. The summation limit m is an arbitrary positive integer number.

- With Eq. (8), the first-order analytical approximate solution of Eqs. (3) - (4) is:

$$\overline{F}(t,C_i) = F(t,1) = F_0(t) + F_1(t,C_i). \tag{14}$$

- Finally, the convergence-control parameters C_1, C_2, ..., C_s can be optimally computed using methods as: the least square method, the Galerkin method, the collocation method, the Kantorowich method, or the weighted residual method.

With these parameters known, the first-order approximate solution (14) is well-determined.

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4 Application of the OHAM to nonlinear problem

Applying our procedure to obtain approximate solutions of Eq. (1) with the initial / boundary conditions Eq. (2).

- *Case 1*

  In the first case of the nonlinear equation Eq. (1), we choose the linear operator of the form:

$$L_f(t) = f'''(t) + \tfrac{3K}{Kt+1} f''(t), \tag{15}$$

where K is an unknown positive parameter and will be determined later.

Using [13], it is easy to show that the linear operator is not unique.

The initial approximation $f_0(t)$ can be obtained from the following equation:

$$\begin{aligned} &f_0'''(t) + \tfrac{3K}{Kt+1} f_0''(t) = 0, \\ &f_0(0) = 0, \quad f_0'(0) = 1, \quad \lim_{t\to\infty} f_0'(t) = 0, \end{aligned} \tag{16}$$

with solution

$$f_0(t) = \tfrac{1}{K} - \tfrac{1}{K} \cdot \tfrac{1}{Kt+1}. \tag{17}$$

The nonlinear operator $N_f(t)$, corresponding to nonlinear differential Eq. (1), is defined by:

$$N_f\Big(t\Big) = -\tfrac{3K}{Kt+1} f'' + ff'' - (f')^2 - mf'. \tag{18}$$

For the initial approximation $f_0(t)$ given by Eq. (17), the nonlinear operator Eq. (18) becomes:

$$\begin{aligned} &N_{f_0}(t) = -\tfrac{3K}{Kt+1} f_0'' + f_0 f_0'' - (f_0')^2 - mf_0' = \\ &= -\tfrac{m}{(Kt+1)^2} - \tfrac{2}{(Kt+1)^3} + \tfrac{6K^2+1}{(Kt+1)^4}. \end{aligned} \tag{19}$$

Comparing Eqs. (19) and (11), one can write:

$$\begin{aligned} &h_1(t) = -m, \quad g_1(t) = \tfrac{1}{(Kt+1)^2}, \\ &h_2(t) = -2, \quad g_2(t) = \tfrac{1}{(Kt+1)^3}, \\ &h_3(t) = 6K^2 + 1, \quad g_3(t) = \tfrac{1}{(Kt+1)^4}. \end{aligned} \tag{20}$$

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The function $f_1(t)$ given by Eq. (12) becomes:

$$f_1(t,C_i) = H_1(t,C_i)\tfrac{1}{(Kt+1)^2} + H_2(t,C_i)\tfrac{1}{(Kt+1)^3} + H_3(t,C_i)\tfrac{1}{(Kt+1)^4}, \tag{21}$$

where we have freedom to choose a lot of possibilities for the unknown functions H_i, $i = 1,\ 2,\ 3$ as follows (see Marinca and Herisanu [13]):

$$\begin{aligned} H_1(t,C_i) &= C_1(Kt+1)^2 + C_2(Kt+1) + C_3, \\ H_2(t,C_i) &= C_4 + \tfrac{C_5}{Kt+1} + \tfrac{C_6}{(Kt+1)^2}, \\ H_3(t,C_i) &= \textstyle\sum_{i=0}^{4} \tfrac{C_{i+7}}{(Kt+1)^{i+2}} + \tfrac{C_{12}t^2}{(Kt+1)^7}. \end{aligned} \tag{22}$$

From Eq. (22) and Eq. (21) we have:

$$f_1(t,C_i) = C_1 + \textstyle\sum_{i=1}^{10} \tfrac{C_{i+1}}{(Kt+1)^i} + \tfrac{C_{12}t^2}{(Kt+1)^{11}}, \tag{23}$$

where

$$C_1 = \sum_{i=3}^{11} (iK - K - 1)C_i, \quad C_2 = -K \cdot \sum_{i=3}^{11} (i-1)C_i.$$

The first-order approximate solution given by Eq. (14) is obtained from Eqs. (17) and (23):

$$\overline{f}(t,C_i) = f_0(t) + f_1(t,C_i) = \tfrac{1}{K} - \tfrac{1}{K} \cdot \tfrac{1}{Kt+1} + C_1 + \textstyle\sum_{i=1}^{10} \tfrac{C_{i+1}}{(Kt+1)^i} + \tfrac{C_{12}t^2}{(Kt+1)^{11}}. \tag{24}$$

- *Case 2*

  In the second case of the Eq. (1), we choose the linear operator:

$$L_f(t) = f'''(t) - K^2 f'(t), \tag{25}$$

where K is an unknown positive parameter and will be determined later.
The initial approximation $f_0(t)$ can be obtained from the following problem:

$$\begin{aligned} &f_0'''(t) - K^2 f_0'(t) = 0, \\ &f_0(0) = 0, \quad f_0'(0) = 1, \quad \lim_{t\to\infty} f_0'(t) = 0, \end{aligned} \tag{26}$$

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which has the solution

$$f_0(t) = \frac{1}{K}\Big(1 - e^{-Kt}\Big). \tag{27}$$

The nonlinear operator $N_f(t)$, corresponding to nonlinear differential Eq. (1), is defined by:

$$N_f\Big(t\Big) = K^2 f' + ff'' - (f')^2 - mf'. \tag{28}$$

For the initial approximation $f_0(t)$ given by Eq. (27), the nonlinear operator from the Eq. (28) becomes:

$$N_{f_0}(t) = K^2 f_0' + f_0 f_0'' - (f_0')^2 - mf_0' = \Big(K^2 - m - 1\Big)e^{-Kt}. \tag{29}$$

For $K = \sqrt{m+1}$ with $m > -1$, it can be notice that $N_{f_0}(t)$ is identically null, so the **exact solution** is $f_0(t) = \frac{1}{K}\Big(1 - e^{-Kt}\Big)$, as in [2].

If $m \leq -1$, then comparing Eqs. (19) and (11), one can write:

$$h_1(t) = K^2 - m - 1, \quad g_1(t) = e^{-Kt}. \tag{30}$$

The function $f_1(t)$ given by Eq. (12) becomes:

$$f_1(t, C_i) = H_1(t, C_i)e^{-Kt}, \tag{31}$$

where we can choose many possibilities for the unknown functions H_1, as follows (see Marinca and Herisanu [13]):

$$H_1(t, C_i) = C_1 + C_2 t + C_3 t^2 + C_4 e^{-Kt} + C_5 te^{-Kt} + C_6 t^2 e^{-Kt}. \tag{32}$$

Substituting Eq. (32) into Eq. (31) we have:

$$f_1(t, C_i) = \left(C_1 + C_2 t + C_3 t^2 + C_4 e^{-Kt} + C_5 te^{-Kt} + C_6 t^2 e^{-Kt}\right) e^{-Kt}, \tag{33}$$

where

$$C_4 = -C_1, \quad C_5 = -C_2 - KC_1.$$

The first-order approximate solution given by Eq. (14) is obtained from Eqs. (17) and (33):

$$\begin{aligned} \overline{f}(t, C_i) = f_0(t) + f_1(t, C_i) = \frac{1}{K}\Big(1 - e^{-Kt}\Big) + \\ + \left(C_1 + C_2 t + C_3 t^2 + C_4 e^{-Kt} + C_5 te^{-Kt} + C_6 t^2 e^{-Kt}\right) e^{-Kt}. \end{aligned} \tag{34}$$

In this way, we can find other solutions as well as in [13].

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5 Numerical simulations

In this section, the advance of the OHAM technique is proved using a comparison of our approximate solutions with numerical solution obtained via the fourth-order Runge-Kutta method for different values of the physical parameter m.

The convergence-control parameters K, C_i, $i = \overline{1,\ 12}$ are optimally determined by means of the least-square method using the Mathematica 9.0 software.

Example 1: If $\bar{x}(t)$ is the approximate analytic solution obtained via Optimal Homotopy Asymptotic Method [14], then for $m = -0.75$ the convergence-control parameters are respectively:

$$C_1 = -3.5876955814,\ C_2 = 0.1096257387,\ C_3 = 6.5446309647,$$

$$C_4 = -50.6301828212,\ C_5 = 215.5576321434,\ C_6 = -486.1218585709,$$

$$C_7 = 451.5351597024,\ C_8 = 203.5025800383, C_9 = -921.6008207155,$$

$$C_{10} = 854.9624118554,\ C_{11} = -275.3589466206,\ C_{12} = 2.5850354366,$$

$$K = 0.1778530630.$$

The first-order approximate solutions proposed in [14] becomes:

$$\begin{aligned}\bar{x}_1(t) = 2.0466553957 &- \frac{275.3589466206}{(1+0.1778530630t)^{10}} + \frac{854.9624118554}{(1+0.1778530630t)^{9}} - \\ &- \frac{921.6008207155}{(1+0.1778530630t)^{8}} + \frac{203.5025800383}{(1+0.1778530630t)^{7}} + \frac{451.5351597024}{(1+0.1778530630t)^{6}} - \\ &- \frac{486.1218585709}{(1+0.1778530630t)^{5}} + \frac{215.5576321434}{(1+0.1778530630t)^{4}} - \frac{50.6301828212}{(1+0.1778530630t)^{3}} + \\ &+ \frac{6.5446309647}{(1+0.1778530630t)^{2}} - \frac{0.4372613717}{1+0.1778530630t} + \frac{2.5850354366t^2}{(1+0.1778530630t)^{11}}\end{aligned} \tag{35}$$

Example 2: For $m = 0.1$ the convergence-control parameters are respectively:

$$C_1 = -5.0017436762,\ C_2 = -0.0879856744,\ C_3 = 0.2947034928,$$

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$$C_4 = -3.2746338394,\ C_5 = 19.8793518668,\ C_6 = -69.1185526838,$$

$$C_7 = 134.3757411553,\ C_8 = -123.2607022737,\ C_9 = 0.1131221714,$$

$$C_{10} = 75.4663123965,\ C_{11} = -35.4164117623,\ C_{12} = 0.4347612458,$$

$$K = 0.1679202933.$$

and therefore, the first-order approximate solutions proposed in [14] becomes:

$$\bar{x}_1(t) = 0.9536276893 - \frac{35.4164117623}{(1+0.1679202933t)^{10}} + \frac{75.4663123965}{(1+0.1679202933t)^{9}} +$$
$$+\frac{0.1131221714}{(1+0.1679202933t)^{8}} - \frac{123.2607022737}{(1+0.1679202933t)^{7}} + \frac{134.3757411553}{(1+0.1679202933t)^{6}} -$$
$$-\frac{69.1185526838}{(1+0.1679202933t)^{5}} + \frac{19.8793518668}{(1+0.1679202933t)^{4}} - \frac{3.2746338394}{(1+0.1679202933t)^{3}} +$$
$$+\frac{0.2947034928}{(1+0.1679202933t)^{2}} - \frac{0.0125582130}{1+0.1679202933t} + \frac{0.4347612458t^2}{(1+0.1679202933t)^{11}} \tag{36}$$

Example 3: If $m = 0.3$ the convergence-control parameters are respectively:

$$C_1 = -4.3700503876,\ C_2 = -0.5275789347,\ C_3 = -0.1115077163,$$

$$C_4 = -0.0613437484,\ C_5 = 5.3669736027,\ C_6 = -30.8063902687,$$

$$C_7 = 77.8607209154,\ C_8 = -86.6239975041,\ C_9 = 9.6629351317,$$

$$C_{10} = 48.6095463618,\ C_{11} = -24.7875866848,\ C_{12} = 0.4299574779,$$

$$K = 0.1905811550.$$

and therefore:

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$$\bar{x}_1(t) = 0.8765292325 - \frac{24.7875866848}{(1+0.1905811550t)^{10}} + \frac{48.6095463618}{(1+0.1905811550t)^{9}} +$$
$$+\frac{9.6629351317}{(1+0.1905811550t)^{8}} - \frac{86.6239975041}{(1+0.1905811550t)^{7}} + \frac{77.8607209154}{(1+0.1905811550t)^{6}} -$$
$$-\frac{30.8063902687}{(1+0.1905811550t)^{5}} + \frac{5.3669736027}{(1+0.1905811550t)^{4}} - \frac{0.0613437484}{(1+0.1905811550t)^{3}} -$$
$$-\frac{0.1115077163}{(1+0.1905811550t)^{2}} + \frac{0.0141206782}{1+0.1905811550t} + \frac{0.4299574779t^2}{(1+0.1905811550t)^{11}} \tag{37}$$

Finally, Tables 1 - 3 and Figures.

1-3 emphasize the accuracy of the OHAM technique by comparing the approximate analytic solutions $\bar{x}_1$, $\bar{x}_1'$ and $\bar{x}_1''$ respectively presented above with the corresponding numerical integration values.

Table 1: The comparison between the approximate solutions $\bar{x}_1$ given by Eq. (37) and the corresponding numerical solutions for $m = 0.3$
(relative errors: $\epsilon_{x_1} = |x_{1_{numerical}} - \bar{x}_1|$)

t	$x_{1_{numerical}}$	$\bar{x}_1$ given by Eq. (37)	ϵ_{x_1}
0	-5.3138 $\cdot 10^{-21}$	2.8421 $\cdot 10^{-14}$	2.8421 $\cdot 10^{-14}$
4/5	0.5247757172	0.5247597553	1.5961 $\cdot 10^{-5}$
8/5	0.7355590457	0.7355718710	1.2825 $\cdot 10^{-5}$
12/5	0.8202230346	0.8202064876	1.6547 $\cdot 10^{-5}$
16/5	0.8542294726	0.8542096875	1.9785 $\cdot 10^{-5}$
4	0.8678886217	0.8678910130	2.3912 $\cdot 10^{-6}$
24/5	0.8733750056	0.8733925671	1.7561 $\cdot 10^{-5}$
28/5	0.8755786866	0.8755948258	1.6139 $\cdot 10^{-5}$
32/5	0.8764638249	0.8764688422	5.0173 $\cdot 10^{-6}$
36/5	0.8768193536	0.8768125402	6.8134 $\cdot 10^{-6}$
8	0.8769621561	0.8769478975	1.4258 $\cdot 10^{-5}$

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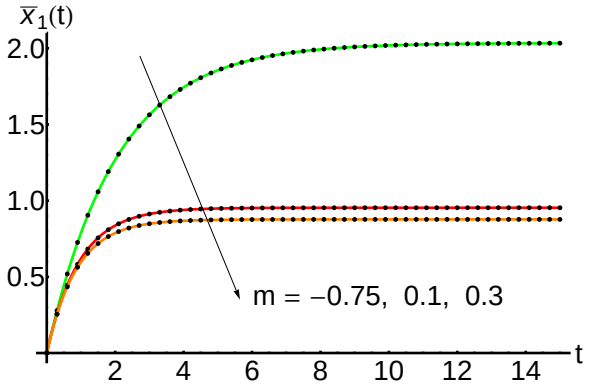


Figure 1: Comparison between the approximate solutions $\bar{x}_1$ given by Eqs. (35), (36), (37) and the corresponding numerical solutions: ———— OHAM solution, $\cdots\cdots\cdots$ numerical solution.

Table 2: The comparison between the approximate solutions $\bar{x}_1'$ from Eq. (37) and the corresponding numerical solutions for $m = 0.3$ (relative errors: $\epsilon_{x_1'} = |x'_{1_{numerical}} - \bar{x}_1'|$)

t	$x'_{1_{numerical}}$	$\bar{x}_1'$ from Eq. (37)	$\epsilon_{x_1'}$
0	1	0.9999999999	$2.7533 \cdot 10^{-14}$
4/5	0.4016636024	0.4017181931	$5.4590 \cdot 10^{-5}$
8/5	0.1613336722	0.1613171078	$1.6564 \cdot 10^{-5}$
12/5	0.0648018043	0.0647710244	$3.0779 \cdot 10^{-5}$
16/5	0.0260285476	0.0260481337	$1.9586 \cdot 10^{-5}$
4	0.0104547212	0.0104828280	$2.8106 \cdot 10^{-5}$
24/5	0.0041992804	0.0042074194	$8.1390 \cdot 10^{-6}$
28/5	0.0016866987	0.0016767703	$9.9284 \cdot 10^{-6}$
32/5	0.0006774861	0.0006615917	$1.5894 \cdot 10^{-5}$
36/5	0.0002721208	0.0002594895	$1.2631 \cdot 10^{-5}$
8	0.0001093012	0.0001035102	$5.7909 \cdot 10^{-6}$

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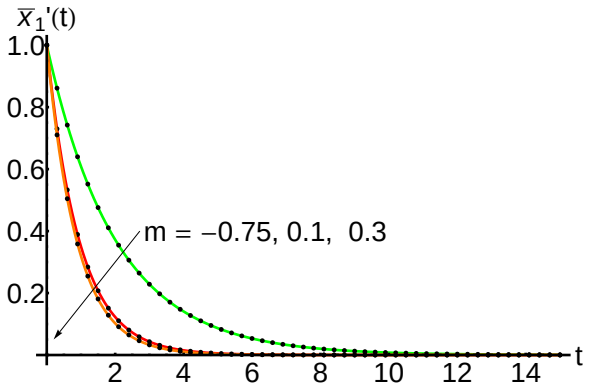


Figure 2: Comparison between the approximate solutions $\bar{x}_1'$ from Eqs. (35), (36), (37) and the corresponding numerical solutions: ———— OHAM solution, $\cdots\cdots\cdots$ numerical solution.

Table 3: The comparison between the approximate solutions $\bar{x}_1''$ from Eq. (37) and the corresponding numerical solutions for $m = 0.3$ (relative errors: $\epsilon_{x_1''} = |x''_{1_{numerical}} - \bar{x}_1''|$)

t	$x''_{1_{numerical}}$	$\bar{x}_1''$ from Eq. (37)	$\epsilon_{x_1''}$
0	-1.1401754250	-1.1401753250	$1.0000 \cdot 10^{-7}$
4/5	-0.4579636131	-0.4578267038	$1.3690 \cdot 10^{-4}$
8/5	-0.1839476837	-0.1840738336	$1.2614 \cdot 10^{-4}$
12/5	-0.0738847299	-0.0738243666	$6.0363 \cdot 10^{-5}$
16/5	-0.0296771408	-0.0296351288	$4.2011 \cdot 10^{-5}$
4	-0.0119201696	-0.0119352076	$1.5037 \cdot 10^{-5}$
24/5	-0.0047879318	-0.0048159808	$2.8049 \cdot 10^{-5}$
28/5	-0.0019231321	-0.0019383749	$1.5242 \cdot 10^{-5}$
32/5	-0.0007724403	-0.0007729498	$5.0954 \cdot 10^{-7}$
36/5	-0.0003102757	-0.0003028727	$7.4029 \cdot 10^{-6}$
8	-0.0001246235	-0.0001157465	$8.8769 \cdot 10^{-6}$

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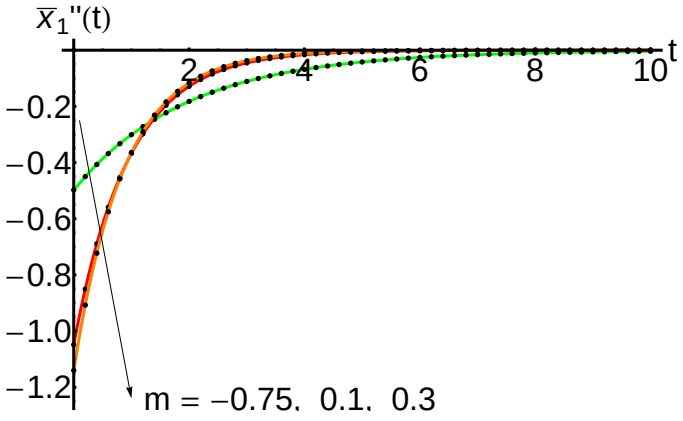


Figure 3: Comparison between the approximate solutions $\bar{x}_1''$ from Eqs. (35), (36), (37) and the corresponding numerical solutions: ———— OHAM solution, ····· numerical solution.

Conclusions

We analyze in this paper the equation of the magnetohydrodynamic flow of a power-law viscous fluid over a stretching sheet, from some geometrical point of view. We investigate the stability of the nonlinear differential problem governing this equation. First, we find a Hamilton-Poisson realization, and using specific tools, such as the energy-Casimir method.

Finally, the analytical integration of the nonlinear system (obtained via the Optimal Homotopy Asymptotic Method and presented in [14]) is done and we obtain the exact solution. Numerical integration of the controlled dynamics is obtained via the Optimal Homotopy Asymptotic Method. Numerical simulations and a comparison with Runge-Kutta 4 steps integrator are presented, too.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] J. de Vicente (editor), *Viscoelsticity - From theory to biological applications*, http://dx.doi.org/10.5772/3188 (online www.interchopen.com).

[2] H. I. Andersson, K. H. Bech, B. S. Dandapat, *Magnetohydro-dynamic flow of a power-law fluid over a stretching sheet*, Int. J. Non-Linear Mech., 27, 1992, 929–936.

[3] Z. Abbas, M. Sajid, T. Hayat, *MHD boundary-layer flow of an upper-convected Maxwell fluid in a porous channel*, Theor. Comput. Fluid Dyn., (2006) 20, 229–238.

[4] T. Hayat, Naveed Ahmed, M. Sajid, S. Asghar, *On the MHD flow of a second grade fluid in a porous channel*, Computers and Mathematics with Applications, 54 (2007) 407–414.

[5] Chien-Hsin Chen, *Effects of magnetic field and suction/injection on convection heat transfer of non-Newtonian power-law fluids past a power-law stretched sheet with surface heat flux*, International Journal of Thermal Sciences 47 (2008) 954–961.

[6] Behrouz Raftari, Ahmet Yildirim, *Computers and Mathematics with Applications*, 59 (2010) 3328–3337.

[7] Sushila, Jagdev Singh, Y.S. Shishodia, *An efficient analytical approach for MHD viscous flow over a stretching sheet via homotopy perturbation sumudu transform method*, Ain Shams Engineering Journal, (2013) 4, 549–555.

[8] Rafael Cortell, *A novel analytic solution of MHD flow for two classes of visco-elastic fluid over a sheet stretched with non-linearly (quadratic) velocity*, Meccanica (2013) 48, 2299–2310.

[9] Muhammad Usman, Zertaisha Naheed, Aqsa Nazir, Syed Tauseef Mohyud-Din, *On MHD flow of an incompressible viscous fluid*, Journal of the Egyptian Mathematical Society, (2014) 22, 214–219.

[10] Fazle Mabood, W.A. Khan, A.I.Md. Ismail, *MHD flow over exponential radiating stretching sheet using homotopy analysis method*, Journal of King Saud University Engineering Sciences, (2017) 29, 68–74.

BUPT

[11] V. Marinca, N. Herişanu, C. Bota, B. Marinca, *An optimal homotopy asymptotic method applied to the steady flow of a fourth grade fluid past a porous plate*, Applied Mathematics Letters, 22, 2009, 245–251.

[12] R.-D. Ene, V. Marinca, R. Negrea, *Planar stretching flows with partial slip*, Proceedings of the 15th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing (SYNASC 2013), 2013, 127–131.

[13] V. Marinca, R.-D. Ene, B. Marinca, R. Negrea, *Different approximations to the solution of upper-convected Maxwell fluid over a porous stretching plate*, Abstract and Applied Analysis, 2014, Article ID 139314, 13 pages.

[14] V. Marinca, N. Herişanu, *The Optimal Homotopy Asymptotic Method: Engineering Applications*, Springer Verlag, Heidelberg, 2015.

[15] R. Cortell, *A note on magnetohydrodynamic flow of a power-law fluid over a stretching sheet*, Appl. Math. Comput., 168, 2005, 557–566.

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