

- west-east (W-E direction) is limited at $0 \dots 1500 \text{ m}$, and south-north (S-N direction) is limited at $0 \dots 1100 \text{ m}$;
- transmitter (T_x) location is: $(0 \text{ m}, 100 \text{ m})$, and receiver (R_x) location is: $(1200 \text{ m}, 1000 \text{ m})$ (Fig.1.a);
- transmitter and receiver heights are $h_{T_x} = h_{R_x} = 50 \text{ m}$ (Fig.1.b); according to the proposed map, additional altitudes are $h_{T_{xmap}} = 598 \text{ m}$, $h_{R_{xmap}} = 616 \text{ m}$.

III. TWO DIMENSIONAL CASE

Starting from the initial data and based on formulas presented in [1], the following considerations can be made:

- according to the previously presented locations for transmitter and receiver, $A = 1200 \text{ m}$ and $B = 900 \text{ m}$ values represent T_x - R_x distances in W-E, respectively S-N directions;
- $a_i \in \{0, 100, 200, \dots, 1200\} \text{ m}$ is the relative coordinate for the W-E direction and b_i is the resulting relative coordinate for S-N direction, given by:

$$b_i = b_0 + a_i \cdot B / A, \quad (1)$$

as defined in Fig.1.a. ($a_0 = 0 \text{ m}$ and $b_0 = 100 \text{ m}$ represent transmitter coordinates);

- for these values the path length between T_x - R_x , D , and the distance between two successive samples, d , are:

$$D = \sqrt{A^2 + B^2} = 1500 \text{ m}, \quad (2)$$

$$d(a_i) = \frac{D \cdot a_i}{a \cdot 12} = a_i \cdot 125 \text{ m}, \quad (3)$$

where $a = a_i - a_{i-1} = 100 \text{ m}$ and 12 value means 12 intervals ($d = 125 \text{ m}$, a constant value meaning interval between values).

- next, the ground heights corresponding to these new positions are computed; this task is accomplished using 1D interpolation in S-N direction (linear interpolation represents the simplest, but not an accurate solution, meaning a weighted mean value calculation between vertical adjacent height values from the initial map); the result is presented in Fig.2.;
- T_x - R_x path height values (represented also in Fig.2.) are obtained using the formula:

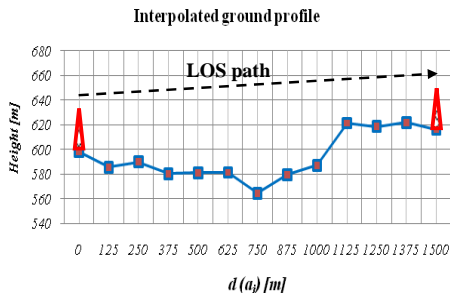


Fig. 2. T_x - R_x altitudes and LOS trajectory

$$h_{LOS}(a_i) = d(a_i) \cdot \frac{h_{R_{xmap}} + h_{R_x} - h_{T_{xmap}} - h_{T_x}}{D}; \quad (4)$$

- the first Fresnel ellipse quota values in the vertical plane, and also the real values related to the map heights are obtained using the next formulas (resulting altitudes are represented in Fig.3.) [2], [3]:

$$H_e(a_i) = \sqrt{\frac{d(a_i) \cdot [D - d(a_i)] \cdot \lambda}{D}}, \quad (5)$$

$$h_e(a_i) = h_{T_x} + h_{T_{xmap}} + h_{LOS}(a_i) \pm H_e(a_i). \quad (6)$$

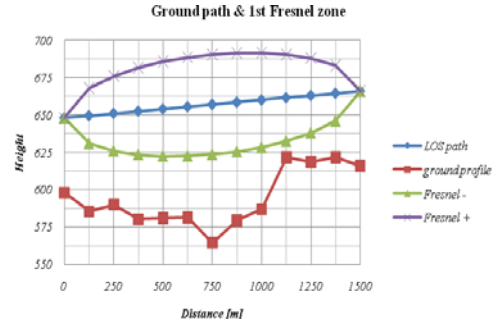


Fig. 3. T_x - R_x altitudes, LOS trajectory and 1st Fresnel ellipse

IV. THREE DIMENSIONAL CASE

The previous section presents the situation in vertical plane. The relation between ground profile and LOS path is well defined in this plane, but relation between the first one and the 1st Fresnel zone is not.

The goal is to extend the previous evaluation to the 3D comparison between 1st Fresnel ellipsoid (instead of the 1st ellipse) and ground height values between transmitter and receiver, as presented in Fig.4. In this picture are represented:

- four ellipses, represented with: red (in horizontal plane), blue (in vertical plane), light green and dark green in intermediate positions;
- eleven representations (dotted black lines) of H_e cross sections for the first horizontal Fresnel ellipse, in horizontal plane (a_i locations);

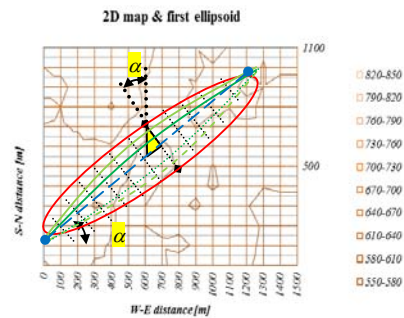


Fig. 4. First ellipses examples and parameters

- angle α between Tx-Rx path and W-E direction, meaning also the angle between one cross section (x direction) and the S-N direction:

$$\operatorname{tg}(\alpha) = B / A . \quad (7)$$

In Fig.5. is represented a cross section (x direction oriented) of the first ellipsoid for an a_i value, marked in the figure), meaning a circle with a radius $H_e(a_i)$ given by equation (5) and centered on the LOS path point corresponding to a_i .

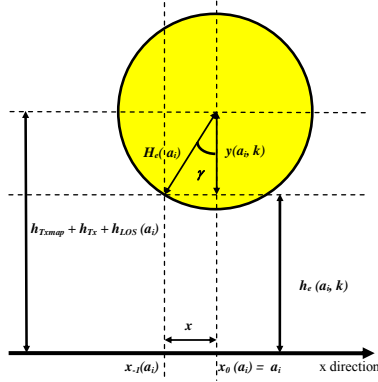


Fig. 5. Vertical cross section of the first ellipsoid for an a_i position in Tx-Rx direction

The 3D case is presented in Fig.6. (rotated with an α angle relative to the W-E direction).

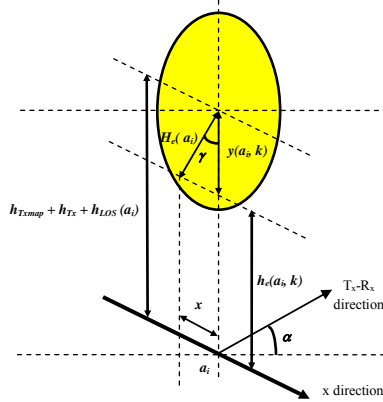


Fig. 6. 3D representation of the vertical cross section

The purpose is to establish if this section intersects the ground profile. For this, some parameters must be taken into account, for a given a_i value and given points of comparison values, presented in the following part.

In order to simplify calculi, the following parameters values will be determined only for integer a_i positions (meaning $a_k = a_0, a_{i\pm 1}, a_{i\pm 2}, \dots$ values). According to Fig.4., this means that the relative value of x parameter is:

$$\operatorname{tg}(\alpha) = \frac{d}{x} \Rightarrow x = \frac{d}{\operatorname{tg}(\alpha)} = \frac{A}{B} \cdot d , \quad (8)$$

a constant value for given initial parameters.

This means that a set of $x_k(a_i)$ values (spaced with x value) can be taken into account, satisfying the condition:

$$\begin{aligned} a_i - H_e(a_i) &= x_{\min}(a_i) = x_k(a_i) = \\ &= x_{\max}(a_i) = a_i + H_e(a_i) \end{aligned} \quad (9)$$

In order to establish the first Fresnel ellipsoid clearance, the value $h_e(a_i, k)$ is compared with the corresponding altitude of the ground profile. Its value is calculated using Fig.5. and the following relationships:

$$h_e(a_i, k) = h_{Txmap} + h_{Tx} + h_{LOS}(a_i) - y(a_i, k) , \quad (10)$$

$$y(a_i, k) = \sqrt{H_e^2(a_i) - x_k^2(a_i)} , \quad (11)$$

so:

$$\begin{aligned} h_e(a_i, k) &= h_{Txmap} + h_{Tx} + h_{LOS}(a_i) - \\ &\quad - \sqrt{H_e^2(a_i) - x_k^2(a_i)} \end{aligned} \quad (12)$$

γ angle value can be obtained using Fig.5.:

$$\begin{aligned} \sin(\gamma) &= \frac{x_k(a_i)}{H_e(a_i)} \Rightarrow \\ \Rightarrow \gamma &= \arcsin \left[\frac{x_k(a_i)}{H_e(a_i)} \right] \end{aligned} \quad (13)$$

and depends in fact, under these conditions, on a_i, k (and also a , and x , respectively) values.

After $h_e(a_i, k)$ value calculation, a comparison with 1D interpolated values for ground profile, in x direction, can be made, in order to establish the maintenance of the 1st Fresnel ellipsoid clearance.

Thus, for a given a_i, b_i values used for interpolation can be calculated, according to Fig.7. (example given for b_{i-1}):

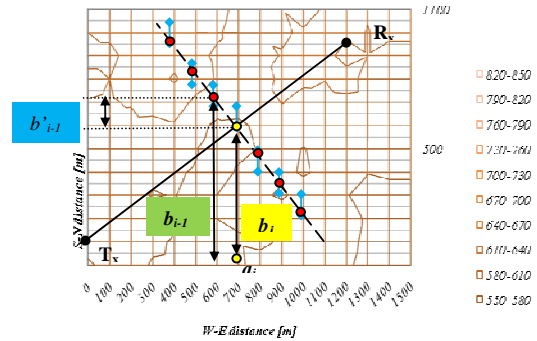


Fig.7. Representation of b_i values example

$$\begin{aligned} b_{i-1} &= b_i + b'_{i-1} = b_i + \sqrt{x^2 - a^2} \\ &= b_0 + a_i \cdot \frac{B}{A} + \sqrt{x^2 - a^2} \end{aligned} \quad (14)$$

Ground altitude interpolation is described in [1].

V. A NUMERICAL EXAMPLE

For the previous assumed values: $h_{T_{xmap}} = 598 \text{ m}$, $h_{T_x} = 50 \text{ m}$ and for $a_i = 700 \text{ m}$ (as an example), the following values are obtained:

$$x = \frac{d \cdot A}{B} = 16.667 \text{ m},$$

$$H_e(a_i) = \sqrt{\frac{d(a_i) \cdot [D - d(a_i)] \cdot \lambda}{D}} = 34.861 \text{ m}.$$

By comparing the previous results, it can be noticed that $k = -1, 0, 1$; this means $x_k \in \{-16.667 \text{ m}, 0 \text{ m}, 16.667 \text{ m}\}$. Also:

$$h_{LOS}(a_i) = d(a_i) \cdot \frac{h_{R_{xmap}} + h_{R_x} - h_{T_{xmap}} - h_{T_x}}{D} = 10.5 \text{ m}$$

$$h_e(a_i, k) = h_{T_{xmap}} + h_{T_x} + h_{LOS}(a_i) - \sqrt{H_e^2(a_i) - x_k^2(a_i)}$$

For $k = -1$, $h_e(a_i, -1) = 627.881 \text{ m}$, and corresponds to coordinates $(600 \text{ m}, 759 \text{ m})$. The same value, $h_e(a_i, 1) = 627.881 \text{ m}$ is obtained also for $k = 1$, (the opposite side), meaning coordinates $(800 \text{ m}, 491 \text{ m})$. For $k = 0$, the minimum value is $h_e(a_i, 0) = 623.639 \text{ m}$, for $(700 \text{ m}, 625 \text{ m})$ position.

Calculated values for the Fresnel ellipsoid and corresponding interpolated ground altitudes for x direction are presented in Table II and in Fig. 8.

For the chosen values of parameters, it results that the first Fresnel ellipsoid do not cross the ground path and its clearance is accomplished.

Table 1

	Values for a = 700 m						
a_i [m]	400	500	600	700	800	900	1000
x [m]	-	-	-16,7	0	16,7	-	-
h_e [m]	-	-	628	624	628	-	-
h_{map} [m]	624	619	600	580	583	582	592

This means that the formula used for free space path loss can be used [2]:

$$a_p = \left(\frac{4 \cdot \pi \cdot D}{\lambda} \right)^2 \text{ or} \quad (15)$$

$$a_p [dB] = 20 \cdot \log_{10} \left(\frac{4 \cdot \pi \cdot D}{\lambda} \right), \quad (16)$$

where: D is the path length between transmitter and receiver ($D = 1500 \text{ m}$ in the example) and $\lambda = c/f$ represents the radio wavelength ($\lambda = 3 \text{ m}$).

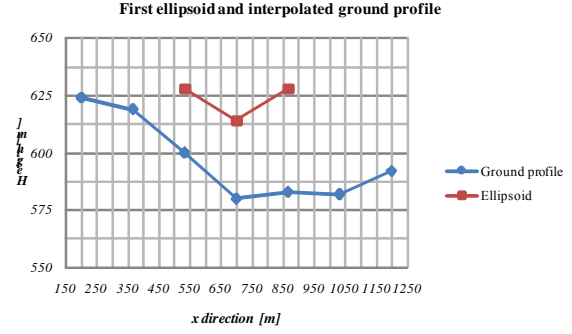


Fig.8. Final results for $a_i = 700 \text{ m}$

VI. CONCLUSIONS

A network planning for a radio system involves a proper calculation of the path loss. In order to use the free space equation ([2], [3]), line-of-sight path clearance and also an adequate 1st Fresnel zone clearance over the transmitter-receiver path must be checked.

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In [3] is specified that, for most applications, a 60 % of the 1st Fresnel zone clearance is sufficient. This paper deals with a 100 % clearance, but not only for the vertical Fresnel ellipse, extending the study to the 3D situation.

Only 1D linear interpolation was used for all presented calculi, due to simplicity. 2D interpolation may lead to accurate results.

The paper presents only the proposed method and the mathematical support. Related work was not found in the literature. Method implementation and improvements (for example to take into account the shape of the diffraction edge, or to take into account higher level Fresnel ellipsoids) represent a further target.

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