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Wireless Channel Estimation Techniques using Pilot Carries in OFDM Systems

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Abstract – The objective of this study is to improve channel estimation accuracy in OFDM systems, which have high data rate and channel capacity and adequate performance in frequency selective fading channels. The first contribution of the paper lies in highlighting the appropriateness of the time-frequency representations for the analysis of OFDM communication systems. We show that the delay and Doppler spreading are the most important characteristics of a communication channel and we indicate how can be used the ambiguity function to estimate these characteristics. Next, we prove that all the on air wireless communication channels are identifiable. The channel estimation based on block-type and comb-type pilot arrangement is studied through different algorithms such as LS, MMSE, LMMSE. Experimental results highlight the merits of the method proposed.

Keywords: OFDM, channel estimation, pilots carries, block-type, comb-type.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has recently been applied widely in on air wireless communication systems due to its high data rate transmission capability with high bandwidth efficiency and its robustness to multi-path delay. OFDM divides the high-rate stream into parallel lower rate data and hence increases the symbol duration, thus helping to eliminate Inter Symbol Interference (ISI). OFDM is considered as an efficient modulation technique for broadband access in a very dispersive environment.

In on air wireless communication systems, the propagation of radio waves is mainly affected by three basic radio propagation mechanisms namely diffraction, reflection, and scattering. Diffraction occurs when the radio path between the transmitter and receiver is obstructed by a surface with sharp irregularities or small openings, appearing as a bending of waves around the small obstacles and openings. Reflection occurs when a radio wave propagating in one medium impinges upon another medium with different electromagnetic properties, for example, surface of the earth, buildings and walls. Scattering is the physical phenomenon in which the radiation of an electromagnetic wave is forced to deviate from a straight path by one or more localized obstacles, with small dimensions compared to the wavelength [3]. In addition to the additive noise which is the most common source of transmitted signal distortion, the on air wireless channel is significantly degraded by another phenomenon called fading. Fading is referred to as the variation of the signal amplitude over time and frequency.

There is one major property that decides the feature of the channel: the time invariance. It implies a channel that has a frequency response constant in time. That is, the frequency response of the channel is the same at all time. This kind of channel is usually relatively easy to estimate since it requires a single estimation, which will hold through the transmission. This is not true for the time-variant channels. The channel’s frequency response is not constant but depends on the time instant. If the frequency response is calculated at time instance $t = t_0$ the result may be very different if the calculation is done at the time instance $t = t_1$.

The impact of environment is called fading and can be spilt in two different types: fast fading and slow fading. These are relative terms and depend on the symbol period. However, the fast fading is often neglected since it is difficult to accomplish for because of the fast variations. Slow fading is assumed when the frequency response of the channel can be considered constant during at least one symbol period. What mainly affects the rate of the fading is the mobility of the receiver relative to the transmitter. If the receiver is moving with some velocity compared to the transmitter, the phase shift will change. This is known as Doppler shift:

$$\nu = \frac{v f_c}{c} \qquad (1)$$

where: v is the relative velocity between the transmitter and the receiver, f_c is the carrier frequency and c is the propagation velocity of the transmitted signal (speed of light for the on air communication systems). The Doppler rate is a measure that gives information about how fast the channel is varying compared to the data rate. If the relative Doppler rate is high, on the order of

10 the channel varies a lot between successive symbols.

Slow fading has been said to be the outcome of the signal reflections on small obstacles in the near surroundings. When modelling the channel, there is one parameter which is closely related to these reflections and that is the channel delay spread τ_{max}. The value of τ_{max} decides the difference in time between the arrival of signals on the longer path and on the shorter path that reach the receiver. Similarly is defined the Doppler spreading ν_{max}.

1) Channel Spread and Underspread Property: The best characterization of the communication channels is done by the Doppler and delay spreading functions. These functions can be estimated by computing the ambiguity function of the signal received for a specific sounding signal, $S_{\mathrm{H}}(\tau, v)$, whose variables are the delay and the Doppler frequency.

For spreading functions with finite support, a formal definition of the underspread property can be obtained by circumscribing the support region of $S_{\mathrm{H}}(\tau, v)$ with a rectangle that is centred on the origin of the reference system (delay-Doppler plane) and whose side lengths equal twice the channel's maximum delay τ_{max} (delay spreading) and maximum Doppler frequency ν_{max} (Doppler spreading) respectively. The centre of the rectangle is immaterial for the definition of the underspread property and is chosen to be the origin for simplicity of exposition. The area of this rectangle $d_{\mathrm{H}} = 4\tau_{max}v_{max}$ measures the channel's overall Time-Frequency (T-F) dispersion and is referred to as the channel spread. A channel is then said to be underspread if $d_{\mathrm{H}} \leq 1$ and overspread if $d_{\mathrm{H}} > 1$. For spreading functions that do not have finite support, the channel spread can be quantified in terms of moments. For multipath propagation, we have $d_{\mathrm{H}} \sim 1/c^2$ [4]. Hence, the spread of radio channels (where *c* equals the speed of light) is typically much smaller than that of underwater acoustic channels (where *c* equals the speed of sound). In fact, radio channels have d_{H} of the order of 10^{-6} to 10^{-3} and thus are highly underspread, whereas underwater acoustic channels can even be overspread.

The goal of channel/system identification is to determine a channel/system frequency response H from the output signal $y(t) = (\mathrm{H}x)(t)$ given knowledge of the sounding (or probing) signal $x(t)$. Let us consider a T-F dispersive channel H with spreading function $S_{\mathrm{H}}(\tau, v)$ supported in $[-\tau_{max}, \tau_{max}] \times [-v_{max}, v_{max}]$. In a practical scenario with finite input signal bandwidth *B* and finite output signal observation time *D*, the input-output relation is discretized, resulting in an input-output relation of the form $y=Xs$. The system identification problem thus amounts to reconstructing **s** from $y=Xs$, i.e., solving a linear system of equations. Clearly, for the existence of a unique solution **s**, it is necessary that the number $|S|$ of unknowns be smaller than or equal to the number *N* of equations, which corresponds to the discrete underspread condition $|S| \leq N$, due to $|S| = \lceil 4\tau_{max}v_{max}BD \rceil$ and $N = \lceil BD \rceil$. This is equivalent to $\lceil 4\tau_{max}v_{max}BD \rceil \leq \lceil BD \rceil$ and hence, effectively, to $d_{\mathrm{H}} = 4\tau_{max}v_{max} \leq 1$ which implies that only underspread systems are identifiable.

The major challenge faced in Multiple Input Multiple Output (MIMO)-OFDM systems is how to obtain the channel state information accurately and promptly for coherent detection of information symbols. We can obtain and track the channel state information by using a channel estimator at the receiver. Channel estimation methods can be divided into two groups. The first method is called *block-type* channel estimation, where pilot tones are inserted in all of the OFDM sub-carriers, as training signals for channel estimation [1]. The second channel estimation method uses pilot tones inserted between data sub-carriers in each of the OFDM blocks and is called *comb-type* channel estimation [1].

Both channel estimation methods have their own advantages and disadvantages. They can use both time and frequency correlations of the channel but, for low complexity estimators, generally frequency correlation is utilized. In this paper, we discuss low complexity channel estimation methods for OFDM systems. The block type pilot channel estimation has been developed under the assumption of slow fading channel; this assumes that the channel transfer function is not changing very rapidly it can be constant over transmission of few OFDM symbols. The comb-type pilot channel estimation has been introduced in case where the channel changes even in one OFDM block. Since we can estimate the frequency response of the channel at the frequencies of pilot inserted sub-carriers, we can obtain the whole channel frequency response by using an interpolation method [2].

The structure of the paper is the following. In section II is presented the architecture of the OFDM communication system. In the third section are introduced two channel estimation techniques. The aim of section IV is the presentation of the simulation results. The conclusions are presented in the last section.

II. SYSTEM DESCRIPTION

In OFDM system, a very high rate data stream is divided into multiple parallel low rate data streams. Each smaller data stream is then mapped to individual data sub-carrier and modulated using varying levels of Phase Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM) modulation schemes depending on the signal quality. The advantage of such an OFDM scheme is that it needs less bandwidth than Frequency Division Multiplexing (FDM) to carry the same amount of information which translates to higher spectral efficiency. To make efficient use of available bandwidth, the sub-carriers are very tightly spaced. Due to the orthogonality among the sub-carriers, there is virtually no inter-carrier interference (ICI) among adjacent sub-carriers. The effect of ISI is suppressed by virtue of a longer symbol period of the parallel OFDM sub-carriers than a single carrier system and the use of a cyclic prefix (CP). In an OFDM system, the signal to be transmitted is defined in the frequency domain. A serial to parallel (S/P) converter first collects the serial data symbols into a data block S_k of dimension M.

$$S_k = \left[S_k[0], S_k[1], \ldots, S_k\left[M-1^T\right]\right] . \qquad (2)$$

This vector of data symbols is then passed through an Inverse Fast Fourier Transform (IFFT) block which results in a set of N complex time domain samples x_k given as: $x_k = \left[x_k[0], x_k[1], \ldots, x_k\left[N-1^T\right]\right]$.

Next, a guard period is created at the beginning of each OFDM symbol to eliminate the impact of ISI caused by multipath propagation. The guard period is obtained by adding a Cyclic Prefix (CP) at the beginning of the symbol x_k. In order to demodulate the OFDM signal at the receiver end, the reverse operations are performed. The serial to parallel converter collects the data symbols first. Then the CP is removed such that only an ISI free block of samples is passed to the DFT.

In OFDM transmission scheme, the sub-carriers are closely spaced to each other without causing interference, thus removing the required guard bands between adjacent sub-carriers. This is possible because the sub-carriers or the frequencies are orthogonal i.e. the peak of one sub-carrier coincides with the null of the adjacent sub-carriers. The time-frequency representation of the OFDM signal is shown in figure 1.

1) Wireless Channel model

Assigning the right model for the channel in any communication system is crucial. If the wrong assumptions are made, the risk of having bad performance increases immensely. Not only are the channels and the assumed channel model important, but also the mobility of the system.

Describing the channel in order to find a good model for it, demands some kind of restriction about how the energy of the channel is distributed. The channel model used in OFDM is a dispersive channel that is a frequency selective channel that attenuates distinct frequencies. A type of distortion that appears in the system is Additive White Gaussian Noise (AWGN). AWGN is always present no matter what channel model is assumed. It is due to random fluctuation of the electrons at the receiver and is sometimes referred to as thermal noise as the movements of the electrons increases with temperature.

The problem with a fading channel is that it destroys the orthogonality of the OFDM subcarriers.

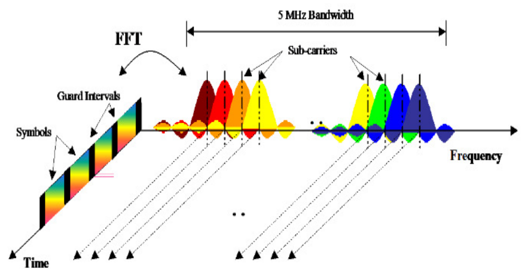

Fig. 1 Time-Frequency Representation of an OFDM Signal [3] .

This is an effect of the Doppler shift that broadens the signal spectrum of the subcarriers and therefore they overlap causing intercarrier interference (ICI). This make OFDM very sensitive to frequency offset. There is another problem that is associated with the propagation paths and their different lengths. Copies of the sent signal might be included in the next time slot as an effect of combination of transmission paths. This kind of distortion, when the previously sent signal is interfering with the current sent signal, is called ISI. This problem can be solved by inserting a guard interval in between successive OFDM symbols.

III. CHANNEL ESTIMATION

To be able to estimate the original transmitted OFDM symbol, we need accurate channel state information [1]. Channel state information can be obtained by using transmitted data and pilot tones. We can obtain and track the channel state information by using a channel estimator at the receiver. Channel estimation methods can be divided into two groups. The first method is called *block-type* channel estimation where pilot tones are inserted in all of the OFDM sub-carriers as training signals for channel estimation. After we get the initial state of the channel, a decision-directed algorithm must be used in order to track the channel variations. The second channel estimation method uses pilot tones inserted between data sub-carriers in each of the OFDM blocks and is called *comb-type* channel estimation [3].

Both channel estimation methods have their own advantages and disadvantages. They can use both time and frequency correlations of the channel but, for low complexity estimators, generally, frequency correlation is utilized. Channel estimation methods also show some difference between systems with a single-transmit antenna and systems with transmitter diversity. While the complexity of the estimators is low for systems with a single-transmit antenna, the estimators for systems with multiple transmitters are very complex. This high complexity stems from the fact that signals transmitted from multiple antennas interfere with each other at the receivers.

In this chapter, we discuss both block-type and comb-type low complexity channel estimation methods for OFDM systems.

The signal model used for the pilot based channel estimation is given by the following equation:

$$Y = XH + W = XFh + W \ , \qquad (3)$$

where:
Y is the received output signal, X is the transmitted input signal, W is the AWGN noise, h is the channel impulse response, H is the channel frequency response and F is the Fourier Transform Matrix.

1) Block Type Pilot Arrangement

In Block type pilot arrangement, OFDM symbols with pilots at all subcarriers, referred to as pilot symbols are transmitted periodically for channel estimation. A frequency-domain interpolation is performed using the pilots to estimate the channel. In order to keep track of the time varying channel characteristics, the period of the pilot symbols in time or the pilot symbol period S_t must be less than the channel's coherence time. As the coherence time is equivalent to the inverse of the Doppler frequency ν, the pilot symbol period must satisfy the following inequality:

$$S_t \le \frac{1}{\nu} \ . \qquad (4)$$

Block type pilot arrangement gives better performance for frequency selective channels as each pilot symbol contains known pilot signals at all of the subcarriers. However, this type of pilot arrangement is suitable only for slow fading channels.

A. Least Square Channel Estimation

The least square (LS) channel estimation is a simple estimation technique with very low complexity. It is widely used because of its simplicity.

The LS estimation channel frequency response $\hat{H}_{LS}$ is obtained by minimizing the following without noise cost function J:

$$\begin{aligned} J &= \left\| Y - X\hat{H} \right\|^2 = \left(Y - X\hat{H} \right)^H \left(Y - X\hat{H} \right) = \\ &= Y^H Y - Y^H X\hat{H} - \hat{H}X^H Y + \hat{H}^H X^H X\hat{H}. \end{aligned} \qquad (5)$$

$\hat{H}$ is the channel frequency response of the pilot OFDM symbol and $(\cdot)^H$ is the conjugate transpose operation. The LS estimated channel is obtained by setting the derivative of J with respect to $\hat{H}$ to zero:

$$\frac{\partial J}{\partial \hat{H}} = 0, \qquad (6)$$

$$\hat{H}_{LS} = \left(X^H X \right)^{-1} X^H Y = X^{-1} Y. \qquad (7)$$

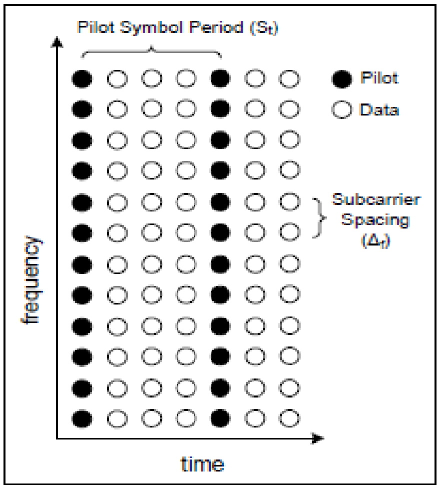

Fig. 2 Block Type Pilot Arrangement [6].

B. Minimum Mean Square Error Channel Estimation.

The minimum mean square error (MMSE) channel estimation is an estimation technique with very high computational complexity. It requires prior knowledge of the second order channel statistics.

The MMSE estimation channel frequency response $\hat{H}_{MMSE}$ is obtained by minimizing the mean square error between the actual channel H and the raw estimated channel $\hat{H}$.

$$e = H - \hat{H} \ . \qquad (8)$$

Since the channel and the AWGN noise can be assumed to be uncorrelated, the MMSE estimated channel can be expressed as,

$$\hat{H}_{MMSE} = R_{HY} R_{YY}^{-1} Y \ , \qquad (9)$$

where:
R_{HY} is the cross covariance matrix between H and Y, R_{HH}, R_{YY} is the auto covariance matrix of H and Y respectively, σ_N^2 is the noise variance:

$$\sigma_N^2 = E\left\{ \left| W(k) \right|^2 \right\}.$$

In the above equation,

$$\begin{aligned} R_{HY} &= E\left\{ HY^H \right\} = E\left\{ H\left(HX + W \right)^H \right\} = \\ &= E\left\{ HH^H X^H + HW^H \right\} = \\ &= E\left\{ HH^H \right\} X^H + 0 = R_{HH} X^H . \end{aligned} \qquad (10)$$

$$\begin{aligned} R_{YY} &= E\left\{ YY^H \right\} = E\left\{ \left(HX + W \right)\left(HX + W \right)^H \right\} = \\ &= E\left\{ HXH^H X^H + HXW + WH^H X^H + WW^H \right\} = \end{aligned}$$

$$= XE\{HH^H\}X^H + 0 + 0 + E\{WW^H\} =$$
$$= XR_{HH}X^H + \sigma_N^2 I_N. \qquad (11)$$

Assuming that the channel statistics, namely the auto-covariance of the channel and the noise variance are known at the receiver, the MMSE estimated channel can be expressed as:

$$\hat{H}_{MMSE} = R_{HH}\left(R_{HH} + \sigma_N^2\left(XX^H\right)^{-1}\right)^{-1}\hat{H}_{LS}. \quad (12)$$

2) Comb Type Pilot Arrangement

In Comb type pilot arrangement, every OFDM symbol has N_p pilot tones which are periodically inserted into the input signal X with pilot subcarrier spacing S_f. A frequency-domain interpolation along the frequency axis is performed using the pilots to estimate the channel. In order to keep track of the frequency-selective channel characteristics, the pilot subcarrier spacing must be such that it is less than the coherence bandwidth.

Comb type pilot arrangement gives better performance for fast fading channels as each OFDM symbol contains known pilot signals at some of the subcarriers. However, this type of pilot arrangement is not suitable for frequency-selective channels. As the receiver knows the pilot locations for each OFDM symbol, it estimates the channel conditions at the pilot subcarriers which are then interpolated over the total subcarrier length N to get the overall channel frequency response at each OFDM symbol.

The Least Square (LS) channel estimation at the pilot subcarriers for Comb type pilot arrangement is given by the following equation:

$$\hat{H}_{LS}^{p}(k) = \frac{Y_p}{X_p} = \frac{Y(kS_f)}{X(kS_f)} \qquad (13)$$
$$k = 0,1,\ldots,N_p - 1,$$

Where X_p is the transmitted signal with pilot symbols at location kS_f, Y_p is the receiver signal with pilot symbols at location kS_f and $S_f = \frac{N}{N_p}$. Since LS estimate is susceptible to noise and ICI, the MMSE includes the matrix inversion at each iteration, the simplified linear MMSE estimator is suggested in [5], where the inverse is needed to be calculated only once.

A. Linear Minimum Mean Square Error: LMMSE

The goal of the use of LMMSE channel estimator is to minimize the estimation's MSE. The LMMSE of the channel frequency response is given by:

$$\hat{H}_{MMSE}^{p} = R_{HH}\left(R_{HH} + \sigma_N^2\left(XX^H\right)^{-1}\right)\hat{H}_{LS}^{p}. \quad (14)$$

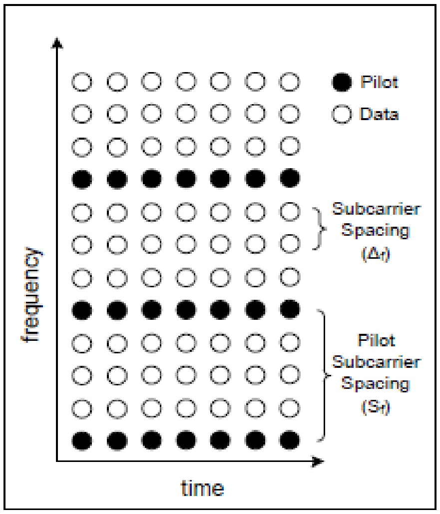

Fig. 3 Comb Type Pilot Arrangement [6].

The complexity of this estimator can be reduced by averaging the transmitted data. Therefore, we can replace the term $\left(XX^H\right)^{-1}$ in the equation (14) with its expectation: $E\left[\left(XX^H\right)^{-1}\right]$.

IV. SIMULATION RESULTS

To evaluate the performance of a digital communication system, in a mobile radio channel, by means of computer simulations, we need a simulation method that can be implemented in a computer program and that reflects the relevant statistical properties of the channels discussed above. We assume to have perfect synchronization since the aim is to observe channel estimation performance. Our goal is to emphasis and to minimize the mean square error and the symbol error rate (SER) for each estimator. The SER is defined as the ratio of the number of symbols containing errors divided by the total number of symbols transmitted. We analyze in the following the case of the Block Type Pilot Arrangement. Pilots are sent in all the sub-carriers of the first symbol of each block. We use the LS and the MMSE algorithms. Our simulations are made in MATLAB. The simulations are carried out for different signal-to-noise ratios (SNR). We compare the results of different simulations made for two path fading channels. For the SER evaluation, we have considered decision-directed estimation, without error propagation. The results are based on the simulation parameters in Table 1. Fig. 4 shows the mean square error versus SNR for the MMSE and LS estimators. The MMSE channel estimator performs better than the LS when the performance metric is the mean square error and the SER. Both estimators give lower mean square error for higher range of SNR. So, for high SNR, the LS estimation could be preferred because it is simple and adequate. However, for low SNR, a compromise between estimator' computational complexity and performance is required. It is important to establish how many pilots are needed to achieve acceptable performance. One way to do this is to look at the SER and see how it varies as a function of the SNR. The SER for different SNR indicates that the difference in quality between a low number of pilots and a high number of pilots becomes bigger as the average SNR is increased.

Table 1

Parameter	Specification
FFT size	64
No. of active Carriers	64
Guard Interval	10
Guard Type	Cyclic Prefix
Signal Constellation	BPSK
Pilot Type	Block Type

From fig.5, it can be observed that the SER performance of the MMSE estimation is much better than for the LS estimation. Compared with LS-based techniques, MMSE-based techniques yield better performance because they additionally exploit and require prior knowledge of the channel correlation and SNR.

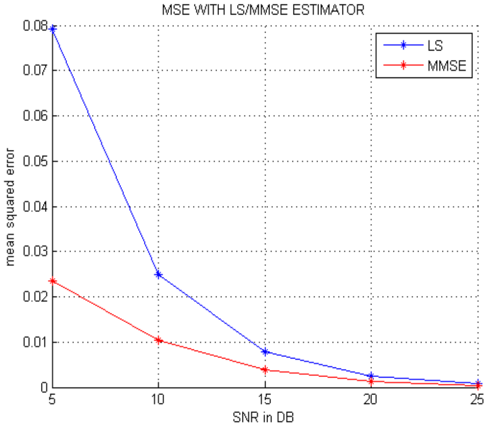

Fig. 4 Comparison of the performances of the LS and the MMSE channel estimators

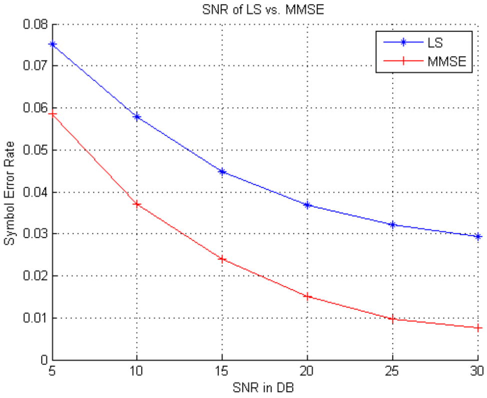

Fig. 5 LS and MMSE estimator performance comparison based on characteristics of SER vs. SNR.

V. CONCLUSION

This paper deals with a key problem for any wireless communication system, the channel estimation. This problem is analyzed in the case of OFDM based communication systems. The first contribution of the paper lies in highlighting the appropriateness of the time-frequency representations for the analysis of OFDM communication systems. We show that the delay and Doppler spreading (τ_{max} and ν_{max}) are the most important characteristics of a communication channel. Next we show that the ambiguity function of the received signal obtained for a particular sounding signal can be used to evaluate the delay and Doppler spreading of a communication channel. The notion of underspread channel is defined next using these channel characteristics, and we prove that any wireless communication channel is underspread hence identifiable. Next we describe the channel estimation technique based on pilot aided block type training symbols using LS and MMSE algorithms. Due to these channel estimation techniques, the information contained in the transmitted signal can be recovered at the receiver side. The results of different simulations are compared to conclude that LS algorithm gives less complexity because it consists of only one multiplication and one inverse operation, being recommended in the case of transmissions with high SNR and the MMSE algorithm provides comparatively better results, even in the case of smaller SNR, but requires a larger computational complexity. To reduce the number of matrix multiplications required by the MMSE algorithm, it is necessary to use the Modified MMSE algorithm.

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