Control of oscillations of a joint driven by elastic tendons by way of the Speed Gradient method

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Abstract – In recent years, there is an increased interest in robotic manipulators with elastic tendon transmission between motor and joint. Such systems allow temporary energy storage and retrieval, and may make periodic trajectories more efficient as long as they can make use of the passive dynamics of the system. This paper studies a simplified model of one revolute joint and two antagonist tendons. The unforced system behaviour is modelled first, then a torque compensation and full state feedback controller for the motors is implemented. The controller is augmented by the speed gradient method to make better use of the underlying oscillatory dynamics.
Keywords: elastic tendon robots; full state feedback; speed gradient method; oscillation control

I. INTRODUCTION

The development of humanoid and walker robots has revealed a shortcoming of usual robotic actuation: it is "stiff", less agile and energy efficient than an animal. For example, if it were to jump, a robot with joints driven by motors in the typical fashion would need to expend the same amount of energy for every jump. A kangaroo on the other hand can recuperate some of the energy by way of the elasticity of its tendons, so jumping again wouldn't require as much effort as the first.

As a result, but for other reasons as well, in recent years robots that mimic the biological muscle and tendon systems have been developped. [3] is a comprehensive look into the design for a controller for such a robotic system, in which the tendons have a nonlinearly varying elasticity.

It remains an open problem to design controllers that can best make use of the system dynamics to sustain a desired level of oscillation, while keeping the control effort low. This paper presents a foray into the topic, by applying a technique for oscillation control (the speed gradient method of [1, 2]) for the first time and comparing it to a PD controller.

In section II the system model is presented. It is a simplified model of a frictionless, 1 degree of freedom robot with no gravity acting on it. In section III a PD controller is developped for this robot using full state feedback, and the behaviour of the controlled system is studied. Section IV implements and studies a speed gradient controller for the system. Finally, some concluding remarks are given in section V. All results, in all sections, are from numerical simulation.

II. SYSTEM MODEL

A. Joint and springs

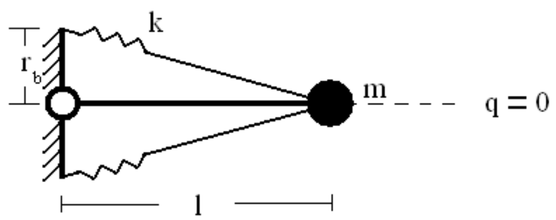

Fig. 1 Mass and fixed tendons system

We begin by considering a point mass m at the end of a rigid massless rod of length l, which can pivot around a base point. Two elastic, massless tendons have one end attached at a distance of r_b to both sides of the pivot, and the other to the point mass. The angle that the rod makes with the equilibrium position of the tendons will be called q (see fig. 1). The system is considered frictionless, and without external forces (for example, no gravity) acting on it.

Tendons can only pull, not push, on the point mass. To ease calculations, we assume that, even in the equilibrium position, both tendons are stretched by a length Δ_0, which is large enough so that at all points in the system's trajectory, both tendons pull on the mass.

First we define some length variables: the distance (d) from the point mass to an anchoring point when the point mass is at the equilibrium position (q = 0), an auxiliary variable (d_q) that is related to changes in tendon length as the angle q changes, and elongations of the two tendons due to changes of q ($\Delta_{q,k}$):

$$d=\sqrt{(l^2+r_b^2)} \tag{1}$$

$$d_q=2\,r_b\,l\sin(q) \tag{2}$$

$$\Delta_{q,1}=\sqrt{(d^2-d_q)}-d \tag{3}$$

$$\Delta_{q,2}=\sqrt{(d^2+d_q)}-d \tag{4}$$

Also denote the moment of inertia, at the pivot, of the mass connected by the rod, by J_q, and the elasticity constant of the tendons as k. Then the kinetic energy (T_q), potential energy (V_q) and therefore the formula for the acceleration are:

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$$T_q = \frac{J_q \dot{q}^2}{2} \quad (5)$$

$$V_q = \frac{k}{2}((\Delta_0 + \Delta_{q,1})^2 + (\Delta_0 + \Delta_{q,2})^2) \quad (6)$$

$$\ddot{q} = \frac{k\, r_b\, l \cos(q)}{J_q} \left(\frac{d - \Delta_0}{\sqrt{d^2 + d_q}} - \frac{d - \Delta_0}{\sqrt{d^2 - d_q}} \right) \quad (7)$$

Notice the d - Δ_0 factor. If it is 0, then the system will never accelerate; if d < Δ_0, then the mass would be pulled so that it would oscillate around q = π. Both of these situations are to be avoided, so we restrict analysis to the case d > Δ_0.

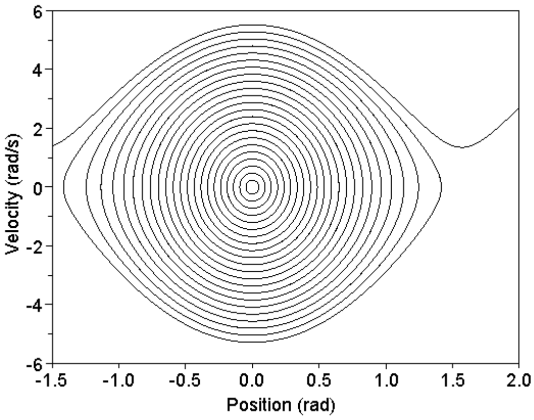

Fig. 2: Phase portrait for mass and elastic tendons

To get a feel for how the system behaves, a phase portrait was constructed (fig. 2). It shows trajectories in phase space (position, velocity), and each trajectory corresponds to a level of total energy in the system. Typically for pendulum-like oscillators, if this energy is high enough, then the point mass enters the so called "rotatory mode": it rotates around the pivot, instead of oscillating around q = 0. Of course, only trajectories where q_{max} = π/2 make sense for the system shown in fig. 1, so we will restrict analysis to this part of the phase space.

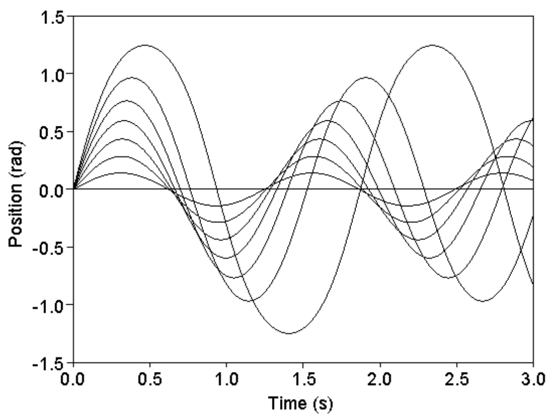

Fig. 3: Trajectories for the mass and elastic tendons system

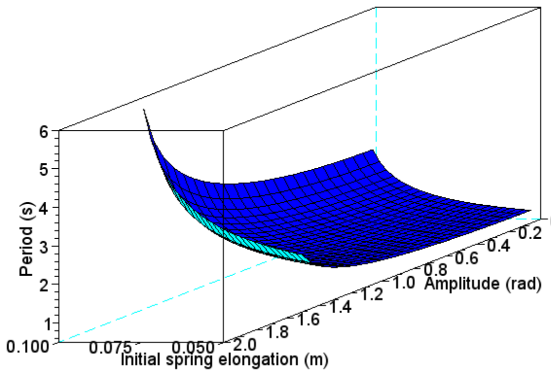

Fig. 4: Dependece of period on initial elongation and amplitude

Also visible from the phase portrait, and even more so from a plot of system trajectories (fig. 3), as amplitude increases, so does the period. Fig. 4 plots the dependency of the period on the total energy (represented by q_{max}) and Δ_0 elongation of the tendons. This will allow us to get some control over both amplitude and period of oscillations, if we can vary the Δ_0 parameter.

B. Full model

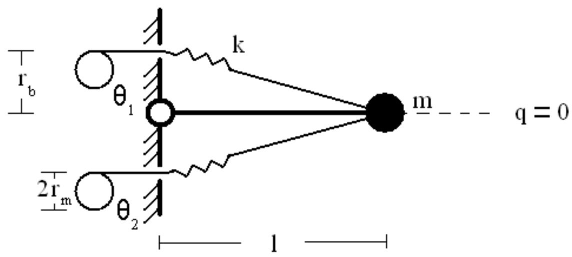

Fig. 5: System with actuated tendons

Instead of tendons attached to fixed points near the pivot, we now consider the case in which the tendons are attached to the rotors (of radius r_m and moment of inertia J_0) of two actuators (fig. 5). Call the angles that the actuators make θ_1 and θ_2. Let θ_k = 0 when the corresponding tendon is completely unrolled from the rotor. As θ_k increases or decreases away from 0, the tendon will be rolled on the rotor. We will assume that the θ_k angles will always be kept above 0, to simplify analysis.

First define some auxiliary elongation variables

$$\Delta_{\theta,1} = \Delta_0 + \theta_1 r_m \quad (8)$$

$$\Delta_{\theta,2} = \Delta_0 + \theta_2 r_m \quad (9)$$

then, by writing the kinetic and potential energy of the system we arrive to the equations of motion

$$\ddot{q}=\frac{-k}{J_q}\left(\left(\Delta_{\theta,1}+\Delta_{q,1}\right)\frac{r_b l\cos(q)}{\sqrt{d^2-d_q}}\right)+$$
$$+\frac{-k}{J_q}\left(\left(\Delta_{\theta,2}+\Delta_{q,2}\right)\frac{r_b l\cos(q)}{\sqrt{d^2+d_q}}\right) \tag{10}$$

$$\ddot{\theta}_1=\frac{1}{J_\theta}\left(\tau_1-k\,r_m\left(\Delta_{\theta,1}+\Delta_{q,1}\right)\right) \tag{11}$$

$$\ddot{\theta}_2=\frac{1}{J_\theta}\left(\tau_2-k\,r_m\left(\Delta_{\theta,2}+\Delta_{q,2}\right)\right) \tag{12}$$

III. POSITION CONTROL

Observe that the torque on the point mass depends on how the tendons are rolled by the two actuators. Therefore, we first consider the problem of controlling the tendon actuator position.

A. Control of tendon actuators

Observe that the torques exerted at each moment by the tendon on the actuators' rotors is given by:

$$\tau_{1,c}=k\,r_m\left(\Delta_{\theta,1}+\Delta_{q,1}\right) \tag{13}$$
$$\tau_{2,c}=k\,r_m\left(\Delta_{\theta,2}+\Delta_{q,2}\right) \tag{14}$$

Therefore, we consider these torques as a kind of baseline, compensation for outside influences. Once we counteract the pull from the tendons, controlling a rotor's position becomes a simple linear control problem. We use state feedback to make the rotor system be described by

$$\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix}-\begin{bmatrix}0\\ J_\theta^{-1}\end{bmatrix}\begin{bmatrix}c_1 & c_2\end{bmatrix}\begin{bmatrix}\theta-\theta_s\\ \dot{\theta}\end{bmatrix} \tag{15}$$

where c_1 and c_2 are the controller tuning parameters, and θ_s is the desired value of θ. To obtain values for them, we place the poles of the system: there will be two of them on the negative real axis, and, to get a system that is critically damped, they are equal. Then, if ρ is the value selected for the poles, we have the following expressions for c_1, c_2, and the step response of the system, s:

$$c_1=\rho^2 J_\theta \tag{16}$$
$$c_2=-2\rho J_\theta \tag{17}$$
$$s(t)=1-e^{\rho t}(1-\rho t) \tag{18}$$

To select the value of ρ, we impose the condition that the time-to-rise to 95% of the step be equal to ten times the controller period t_c (t_c = 1ms in our case):

$$0.05=e^{\rho(10t_c)}\left(1-\rho(10t_c)\right) \tag{19}$$

which brings ρ approximately equal to -474. This is so that the controller gets to sample the trajectory of the controlled system often enough, while still being reasonably aggressive.

B. Control of joint position

Consider the point mass on the rod as a linear system described by the equations:

$$\begin{bmatrix}\dot{q}\\ \ddot{q}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}q\\ \dot{q}\end{bmatrix}+\begin{bmatrix}0\\ J_q^{-1}\end{bmatrix}u \tag{20}$$

where u is a control variable (a torque) which, like in the tendon actuator case, is of the form

$$u=-c_3(q-q_s)-c_4\dot{q} \tag{21}$$

where c_3 and c_4 are controller parameters, and q_s is the desired value for q.

We select values for c_3 and c_4 in a similar fashion to the tendon actuator case, only we make this controller much less aggressive (95% rise time is 0.15s). This is so as to keep values for tendon actuator positions inside a safe window.

So we have a value of u that is known, since it was computed by the q position controller. Also, as we can see from (10) the torque u depends on the tendon actuator positions

$$u=\frac{-k}{J_q}\left(f(q)+r_m\begin{bmatrix}\frac{\partial\Delta_{q,1}}{\partial q} & \frac{\partial\Delta_{q,2}}{\partial q}\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}\right) \tag{22}$$

from which after some rearrangement of terms we obtain a condition on the actuator angles:

$$g(q,u)=\theta_1\frac{\partial\Delta_{q,1}}{\partial q}+\theta_2\frac{\partial\Delta_{q,2}}{\partial q} \tag{23}$$

where g(q, u) is a known value, and so are the derivatives of $\Delta_{q,k}$. (23) is the equation of a line, and to get values for θ_k that obey it, we find the intersection of that line with the perpendicular to it that passes through the current values of θ_k (or, we get as close to that intersection as possible, given the limits put on the actuator angles).

Fig. 6 shows the behaviour of the simulated joint with the controllers described in this section. First, we command q to follow a sine trajectory until 15s; then, we cease control of q and command the actuators to hold a certain position (note that oscillations of q continue). Finally, from 30s until the end, we command steps between +1 and -1.

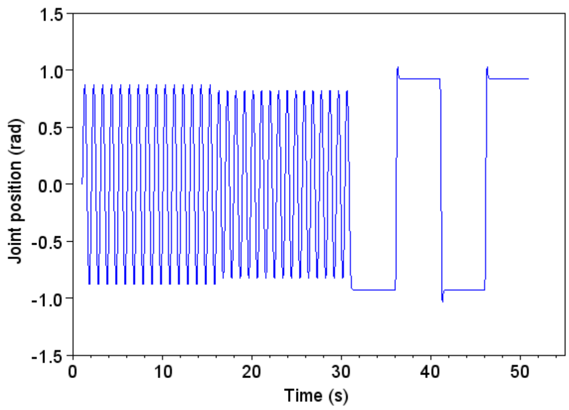

Fig. 6: Controlled joint trajectory

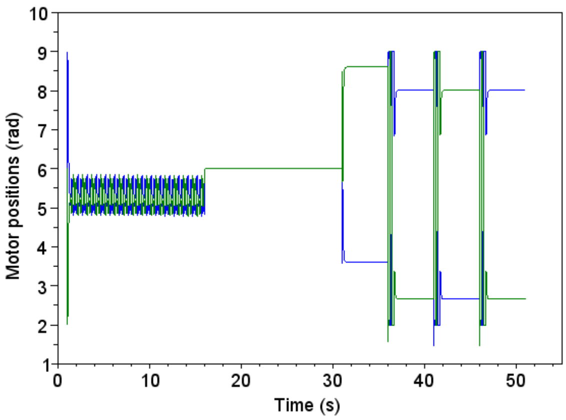

Fig. 7: Tendon actuator positions

Fig. 7 shows the values of the actuator angles in this simulation. Note the control effort for the first 15s when the goal was to follow a sine trajectory. Also note the fact that when doing steps for q, the actuator angles do hit the safety-imposed limits.

Also, observe that the system oscillates even when the actuators do not move at all. However, the amplitude of these oscillations is not controlled, and in a real system, where friction is an issue, they will eventually disappear. It is therefore necessary, if we want the system to oscillate while doing only a small control effort, to somehow pump energy into it. This is the topic of the next section.

IV. SPEED GRADIENT CONTROL OF OSCILLATION

Oscillatory systems are well suited to analysis by the Hamiltonian formulation of mechanics, and this is what we pursue here. Remember that if we introduce a new variable, the momentum (p), then Hamilton's equations are:

$$p = J_q \dot{q} \tag{24}$$

$$\dot{q} = \frac{\partial H}{\partial p} \tag{25}$$

$$\dot{p} = -\frac{\partial H}{\partial q} \tag{26}$$

where H is the total energy of the system, and p was defined in an appropriate manner for our system, if we take only q as being a degree of freedom for it.

Consider the following problem: control the point mass in such a way that it will oscillate with an amplitude of 1rad and a period of 1s. Looking at the function plotted in fig. 4, we find what elongation of the tendons at the equilibrium position is needed to make this possible. This elongation will be a combination of tendons being stretched when clamped at the actuators (Δ_0) and the tendons being stretched by being rolled by the actuators until a base position (θ_0, which will be considered a constant parameter).

We can control the joint system by changing the angle of the tendon actuators around the base position by the values $\theta_{1,u}$ and $\theta_{2,u}$: these will be what we'll consider to be the control inputs. Again, we define some auxiliary notation for elongation variables:

$$\Delta_1 = \Delta_0 + \Delta_{q,1} + \theta_0 r_m \tag{27}$$

$$\Delta_2 = \Delta_0 + \Delta_{q,2} + \theta_0 r_m \tag{28}$$

Next, find a function H will obey Hamilton's equations (that is, it can describe the equation of motion for q and p), such that it can be split into a part that doesn't depend on the control variables (H_0, energy of the "free system") and one that does (H_1):

$$H_0 = \frac{J_q \dot{q}^2}{2} + \frac{k}{2}(\Delta_1^2 + \Delta_2^2) \tag{29}$$

$$\begin{aligned} H_1 = {} & \frac{k r_m}{2}(\theta_{1,u}(r_m \theta_{1,u} + 2\Delta_1)) + \\ & + \frac{k r_m}{2}(\theta_{2,u}(r_m \theta_{2,u} + 2\Delta_2)) \end{aligned} \tag{30}$$

$$H = H_0 + H_1 \tag{31}$$

Similar to [1]. our control goal is to bring the energy of the free system to the level H_s that defines oscillations of amplitude 1:

$$Q = \frac{1}{2}(H_0 - H_s)^2 \tag{32}$$

The control goal varies with time by the formula:

$$\dot{Q} = (H_0 - H_s)\left[-\frac{\partial H_0}{\partial p}\frac{\partial H}{\partial q} + \frac{\partial H_0}{\partial q}\frac{\partial H}{\partial p}\right] \tag{33}$$

which after some substitutions and manipulations becomes

$$\dot{Q} = -(H_0 - H_s)\frac{\partial H_0}{\partial p}\frac{\partial H_1}{\partial q} \qquad (34)$$

This is the "speed" of the goal function, and we are interested in how the control variables affect this speed:

$$\nabla_u \dot{Q} = -(H_0 - H_s)\dot{q}\, k\, r_m \begin{bmatrix} \frac{\partial \Delta_{q,1}}{\partial q} \\ \frac{\partial \Delta_{q,2}}{\partial q} \end{bmatrix} \qquad (35)$$

From here, we design a control law that reduces this speed, because eventually that will make it negative and the control goal will be reached:

$$\begin{bmatrix} \theta_{1,u} \\ \theta_{2,u} \end{bmatrix} = \gamma (H_0 - H_s) sgn(\dot{q})\, k\, r_m \begin{bmatrix} \frac{\partial \Delta_{q,1}}{\partial q} \\ \frac{\partial \Delta_{q,2}}{\partial q} \end{bmatrix} \qquad (36)$$

where γ is some constant, which needs to be tuned by experiment, and sgn is a function that returns +1 or -1, depending on whether the velocity of q is positive or not. We replaced the velocity of q by it's sign because this tends to result in a faster controller, and one that doesn't need an initial "kick" to escape initial situations in which the velocity of q is 0.

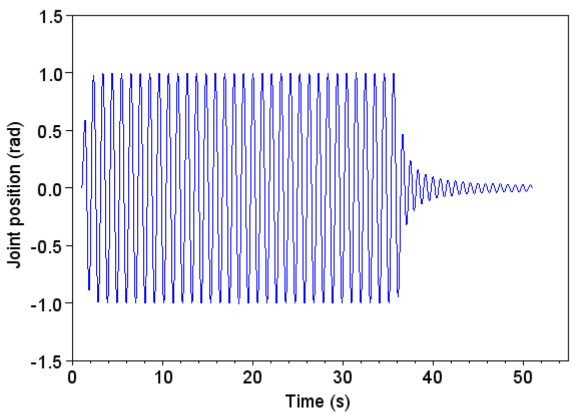

Fig. 8: Speed gradient control of joint position

Fig. 8 shows the trajectory of q under the speed gradient control. The system was commanded to oscillate with amplitude 1 until 35s, and then come back to standstill. Fig. 9 shows the positions of the tendon actuators during this time. Fig 10 shows the energy H_0 vs the desired energy H_s.

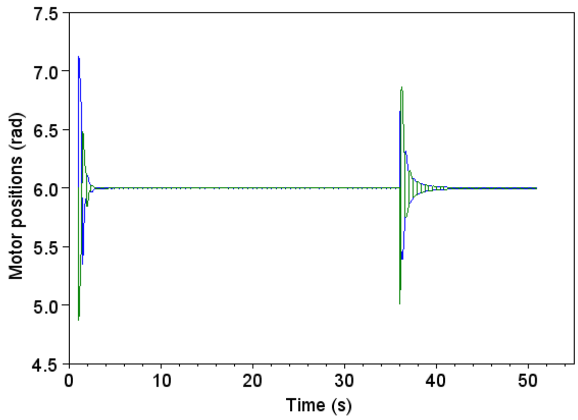

Fig. 9: Tendon actuator position

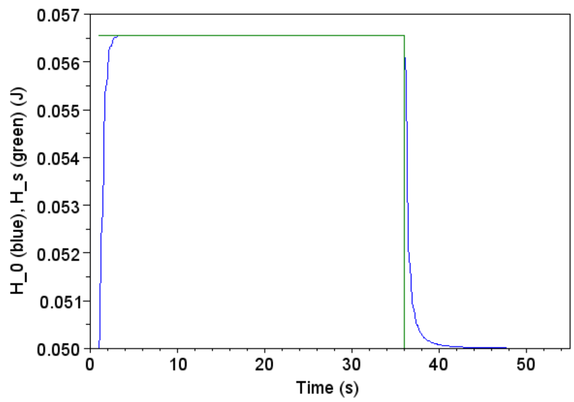

Fig. 10: Desired energy level (green) vs. actual energy level (blue)

It can be seen that the speed gradient control brings the system to the desired oscillation quickly, and then the actuators do not significantly change position anymore and thus do not need to produce much mechanical work. If maintaining an oscillatory regime is desired, the speed gradient method seems more efficient that the control of the previous section. Stopping oscillations on the other hand appears slower with this kind of control, but that may be alleviated in a real system because of friction.

V. CONCLUSIONS AND DISCUSSION

There previous sections have compared ways to control the considered system, and that speed gradient control seems especially efficient at taking advantage of natural dynamics, if oscillations are desired.

There is much work still needed before these results are applicable to a real system however. In a real system, knowledge of system parameters and states is always imperfect, and control must be augmented with some kind of estimation procedure.

However an even more basic complication arises in practical robotic systems: there will be several joints, thus several degrees of freedom of the robot, to control at once. A goal function for oscillation control in this case cannot be just the energy level, as this is not enough to constrain the oscillation of the system. It is also likely that, similarly to the case of the double pendulum, there will be energy levels for which the trajectories of a "free" chain of joints with elastic tendons will have chaotic trajectories. Controlling oscillations in this setup will be the focus of a future paper.

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