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Direction-of-Arrival Estimation in case of Uniform Sensor Array using the MUSIC Algorithm

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Abstract – The resolution of a signal direction of arrival (DoA) estimation can be enhanced by an array antenna system with innovative signal processing. Super resolution algorithms take advantage of array structures to better process the incoming signals. This paper explores the eigen-analysis category of super resolution algorithm. The MUSIC (MUltiple SIgnal Classification) algorithm can be applied to estimation of the direction of arrival with a receiver formed by Uniform Linear Array (ULA) or Uniform Circular Array (UCA) antenna. In this paper, I analyze the performances obtained in both situations through computer simulation.
Keywords: array antenna, direction of arrival estimation, MUSIC algorithm.

I. INTRODUCTION

The need for Direction-of-Arrival estimation arises in many engineering applications including wireless communications, radar, radio astronomy, sonar, navigation, tracking of various objects, rescue and other emergency assistance devices. In its modern version, DoA estimation is usually studied as part of the more general field of array processing. Much of the work in this field, especially in earlier days, focused on radio direction finding – that is, estimating the direction of electromagnetic waves impinging on one or more antennas [1].
There are many different super resolution algorithms including spectral estimation, model based, eigen-analysis. The various DoA estimation algorithms are Bartlett, Capon, Min-norm, MUSIC and ESPRIT. The MUSIC algorithm is one of the most popular and widely used subspace-based techniques for estimating the DoAs of multiple signal sources. The conventional MUSIC algorithm involves a computationally demanding spectral search over the angle and, therefore, its implementation can be prohibitively expensive in real-world applications. The uniform circular array (UCA) is able to provide 360º of coverage in the azimuth plane and has uniform performance regardless of angle of arrival. Thus, sometimes, UCA is more suitable than uniform linear array (ULA) for applications such as radar, sonar, and wireless communications.[2]

The direction of arrival (DOA) estimation of multiple narrowband signals is a classic problem in array signal processing. An array antenna system with innovative signal processing can enhance the resolution of a DoA estimation. An array sensor system has multiple sensors distributed in space. This array configuration provides spatial samplings of the received waveform. A sensor array has better performance than the single sensor in signal reception and parameter estimation [3].
Sensor array processing techniques have attracted considerable interest in the signal processing society. These techniques have focused mainly on high-resolution direction-of-arrival estimation. Generally, the choice of DOA estimator is made adequately in accordance with the array geometries used.
In this paper, a computer simulation programs using Matlab were developed to evaluate the direction-of-arrival performance of MUSIC and UCA-MUSIC algorithms. We consider 1-D (azimuth) angular estimation of the noncoherent narrowband source located at the same elevation angle with element array.

II. ULA – MUSIC ALGORITHM

In wireless transmission, the receiving antennas can collect more signals that can be emitted by several sources, as shown in Fig. 1. An important fact is the direction of arrival estimation of signals received from different sources.

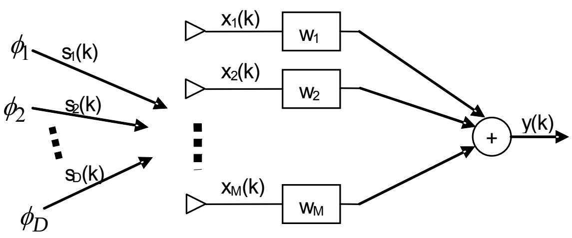

Fig. 1. Receiver with uniform linear *M*-element array

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It can be observed that the D signals arrive from D directions. They are received by an array of M elements with M potential weights.

Many of the DoA algorithms rely on the array correlation matrix. In order to simplify the notation let us define the M x M array correlation matrix R_{xx} as [4]:

$$\begin{aligned} R_{xx} &= E\left[\bar{x}\cdot\bar{x}^H\right] = E\left[\left(\bar{A}\bar{s}+\bar{n}\right)\left(\bar{s}^H\bar{A}^H+\bar{n}^H\right)\right] \\ &= \bar{A}E\left[\bar{s}\cdot\bar{s}^H\right]\bar{A}^H + E\left[\bar{n}\cdot\bar{n}^H\right] \\ &= \bar{A}R_{ss}\bar{A}^H + R_{nn}. \end{aligned} \quad (1)$$

where R_{SS} represents the source correlation matrix (D x D elements), $R_{nn} = \sigma_n^2 I$ represents the noise correlation matrix (M x M elements), and I represents the identity matrix (N x N elements).

Given M-array elements with D-narrowband signal sources and uncorrelated noise, we can make some assumptions about the properties of the correlation matrix: is an MxM Hermitian matrix. The array correlation matrix has M eigenvalues ($\lambda_1, \lambda_2, \ldots, \lambda_M$) along with associated eigenvectors $\bar{E} = \left[\bar{e}_1 \bar{e}_2 \cdots \bar{e}_M\right]$ If the eigenvalues are sorted from smallest to largest, we can divide the matrix $\bar{E}$ into two subspaces: $\bar{E} = \left[\bar{E}_N \bar{E}_S\right]$. The first subspace $\bar{E}_N$ is called the noise subspace and is composed of $M - D$ eigenvectors associated with the noise, and the second subspace $\bar{E}_S$ is called the signal subspace and is composed of D eigenvectors associated with the arriving signals. The noise subspace is an M x $(M - D)$ matrix, and the signal subspace is an $(M$ x $D)$ matrix.

The MUSIC algorithm is based on the assumption that the noise subspace eigenvectors are orthogonal to the array steering vectors, $\bar{a}(\phi)$, at the angles of arrival $\phi_1, \phi_2, \ldots, \phi_D$. Because of this orthogonality condition, one can show that the Euclidian distance $d^2 = \bar{a}^H(\phi)\bar{E}_N\bar{E}_N^H\bar{a}(\phi) = 0$ for each and every arrival angle $\phi_1, \phi_2, \ldots, \phi_D$. Placing this distance expression in the denominator creates sharp peaks at the angles of arrival. The MUSIC pseudospectrum is:

$$P_{MUSIC}(\theta) = \frac{1}{\bar{a}(\phi)^H \bar{E}_N \bar{E}_N^H \bar{a}(\phi)}. \quad (2)$$

III. UCA – MUSIC ALGORITHM

Consider a UCA consisting of M identical elements uniformly distributed over the circumference of a circle of radius r. Assume that D narrowband sources, centered on wavelength λ, impinge on the array from directions ϕ_i (i=1, … ,D), respectively, where $\phi_i \in [0, 2\pi)$ is the azimuth angle measured from the x-axis counter-clockwise. Figure 2 depicts a receiver formed by an uniform circular array with incident planewaves from various directions.

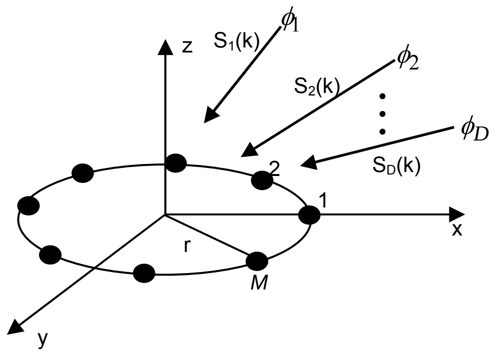


Fig. 2. Receiver with uniform circular M-element array

The $Mx1$ vector received by the array is expressed by[5]:

$$y(k) = \bar{C}\,\bar{A}(\phi)s(k) + n(k). \quad (3)$$

where $\bar{A}(\phi) = [\bar{a}(\phi_1)\ldots\bar{a}(\phi_D)]$ is the MxD matrix of the steering vectors, $s(k) = [s_1(k), \ldots, s_D(k)]^T$ is the $Dx1$ signal vector, $n(k) = [n_1(k), \ldots, n_M(k)]^T$ is the $Mx1$ noise vector. The signal vector $s(k)$ and the vector $n(k)$ of the additive and spatially white noise are assumed to be statistically independent and zero-mean. The MxM matrix $\bar{C}$ is the mutual coupling matrix. Due to the circular symmetry, a model for the mutual coupling matrix of UCAs [6] can be a complex symmetric circulant matrix. The steering vector with mutual coupling can be modeled as:

$$\tilde{a}(\phi) = \bar{C}\,\bar{a}(\phi). \quad (4)$$

The covariance matrix $\bar{R}$ of the received data is constructed and an eigendecomposition of $\bar{R}$ results in a signal and noise subspace:

$$\begin{aligned} \bar{R} &= E\{x(k)x^H(k)\} = \\ &= \bar{E}_S\Lambda_S\bar{E}_S^H + \bar{E}_n\Lambda_n\bar{E}_n^H \end{aligned}. \quad (5)$$

where $\bar{E}_S$ and $\bar{E}_n$ denote the signal and noise subspace eigenvectors and the diagonal matrices Λ_S and Λ_n contain the signal subspace and noise subspace eigenvalues. The MUSIC algorithm estimates the DOAs from the D deepest nulls of the UCA - MUSIC function:

$$f_{UCA-MUSIC}(\phi) = \tilde{a}^H(\phi)\bar{E}_n\bar{E}_n^H\tilde{a}(\phi). \quad (6)$$

IV. SIMULATION RESULTS

In this section, computer simulations are provided to substantiate the performance analysis. In all cases, the impinging angles of the sources are relative to the broadside of a uniform array. It is considered that the source is placed on the direction $\phi = +20^\circ$

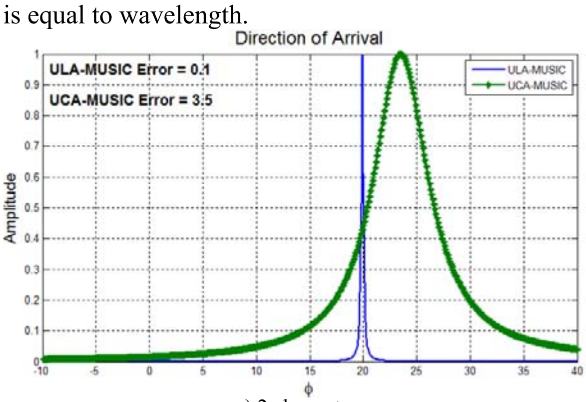
The additive background noise is assumed to be spatially and temporally white complex Gaussian with zero – mean, having the 0.1 variance value.
The space between two adjacent array elements is one half of a wavelength for uniform linear array, and, respectively, for uniform circular array can be determined by:

$$d = 2r\sin(\pi / M) . \qquad (7)$$

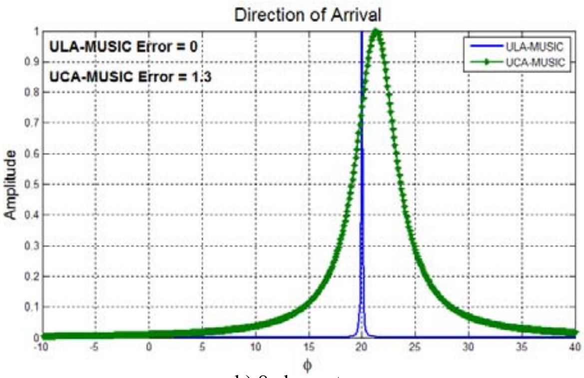
where r is the UCA radius. The angle between two consecutive sensors can be found:

$$\phi' = 2\pi / M . \qquad (8)$$

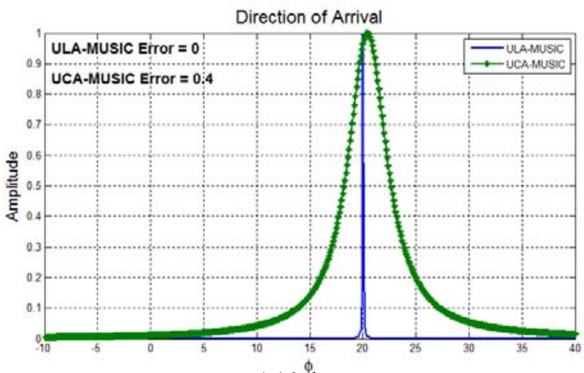
First, we consider 1000 samples of data were taken from the array and SNR = 1 dB. The direction of arrival estimation in case of ULA and UCA antenna respectively, is presented in Fig. 3, for the number of array elements equal to 2, 8 and 16. The UCA radius is equal to wavelength.



a) 2 element array



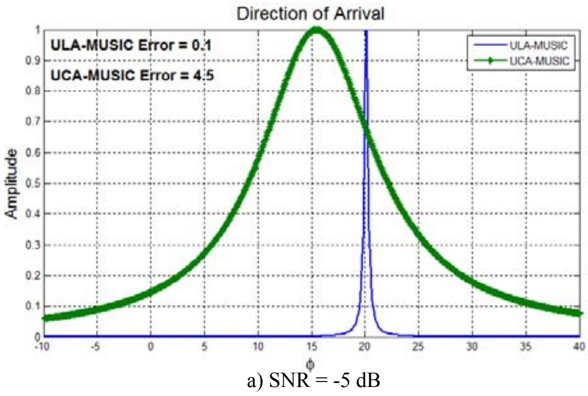
b) 8 element array



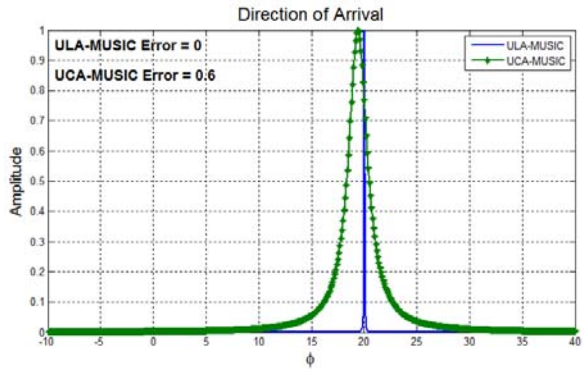
c) 16 element array

Fig. 3. DoA in case of ULA and UCA antenna, using MUSIC algorithm

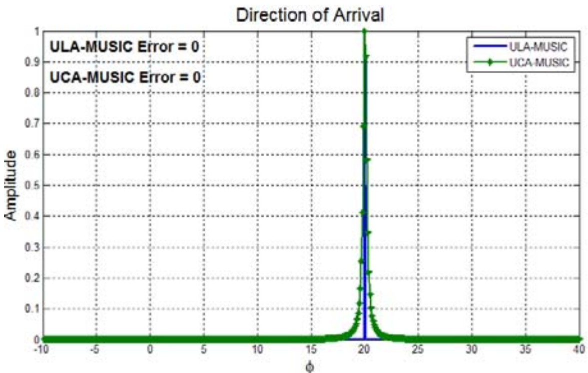
As the number of array elements increases, can be seen that in the UCA antennas case best performances are obtained.
Next, the performance of the array antennas was discussed at various SNRs and the results are plotted in Fig. 4. The number of array element is equal to 8.



a) SNR = -5 dB



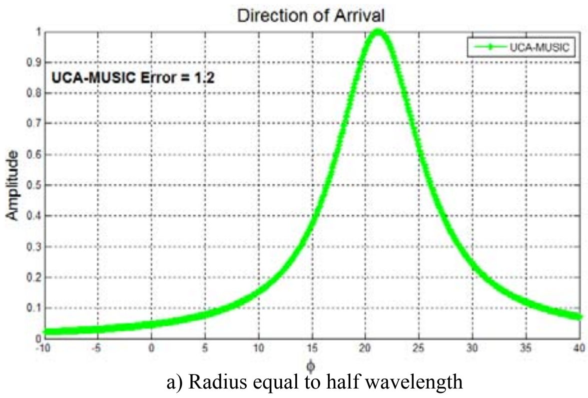
b) SNR = 5 dB



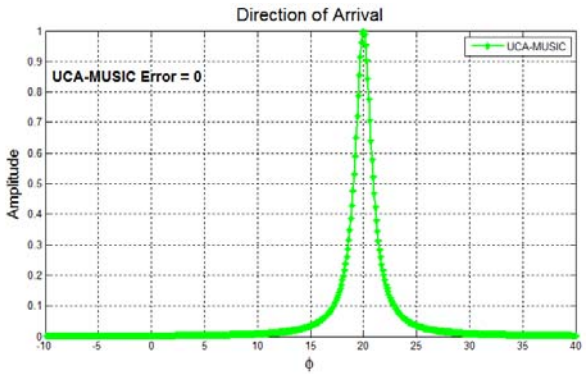
c) SNR = 10 dB

Fig. 4. DoA in case of ULA and UCA antenna, using MUSIC algorithm, for different SNRs

It is observed that the UCA antennas can be used when SNR is greater than 10 dB, for the number of array elements equal to 8. If the number of elements increases, the SNR can be small than 10 dB.
Further it is study the influence of the circle radius size that the elements are located on the performances of DoA. For this case, we consider UCA antennas with 8 elements and the SNR value is equal to 1 dB. The results obtained for different values of the radius are plotted in Fig. 5.



a) Radius equal to half wavelength



b) Radius equal to 2λ

Fig. 5. DoA in case of UCA antenna,
using MUSIC algorithm, for different values of radius

Can be seen that, for a greater radius, are obtained good performances.

V. CONCLUSIONS

In this paper, I give extensive computer simulation results to demonstrate the performances of the UCA antennas obtained in case of DoA estimation. The estimation of direction of arrival for one source of signal is implemented by using the MUSIC algorithm. The performances of DoA estimation obtained in case of ULA antenna are better than in case of UCA antenna. It is observed that the UCA antenna is not so powerful, but in some cases the results obtained with this system are acceptable.

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