Buletinul Ştiinţific al Universităţii "Politehnica" din Timişoara

Seria ELECTRONICĂ şi TELECOMUNICAŢII
TRANSACTIONS on ELECTRONICS and COMMUNICATIONS

Tom 51(65), Fascicola 2, 2006

A New Quasi Shift Invariant Non-Redundant Complex Wavelet Transform

Ioana Adam[1], Marius Oltean[2], Mircea Bora[1]

Abstract
The property of shift-invariance associated with the property of good directional selectivity are important for the application of a wavelet transform in many fields of image processing. Unfortunately, the classical discrete wavelet transform is shift-variant. All modified algorithms proposed in the literature for the computation of a shift invariant transform are less or more redundant and difficult to implement, and consequently thorny to use in signal processing applications. In this paper, we propose a new, quasi shift-invariant wavelet transform, without redundancy and easy to implement.

1. INTRODUCTION

A wavelet transform (WT), is shift-sensitive if an input signal shift causes an unpredictable change of the transform coefficients.

Shift-sensitivity is an undesirable property because it implies the impossibility to distinguish between wavelet transform coefficients corresponding to input signal shifts.

The shift-sensitivity of the Discrete Wavelet Transform (DWT) is generated by the down-samplers used for its computation.

The property of shift-invariance associated with the property of good directional selectivity are important for the application of a wavelet transform in many fields of image processing including denoising, de-blurring, super-resolution, watermarking, segmentation and classification.

In the next section, several quasi shift-invariant WTs, proposed in the literature are presented. Our transform is introduced and explained in section 3. Simulation results are presented in section 4, in order to illustrate the degree of shift invariance of the proposed transform. In the final section, a few conclusions are exposed and future possible research directions on the subject are indicated.

2. TYPES OF WAVELET TRANSFORMS

There are in the literature some wavelet transforms which are shift-invariant or quasi shift-invariant. In the following, some of them are presented.

A. UDWT

Since down-samplers in the DWT implementation create shift-sensitivity, Mallat [1], Beylkin [2], Coifman and Donoho [3] and Guo [4], devised the un-decimated DWT (UDWT), which is a wavelet transform without down-samplers. Although the UDWT is shift-insensitive, it has high redundancy, caused by the absence of down-samplers. Unfortunately, the high redundancy incurs a massive storage requirement that makes the UDWT inappropriate for most signal processing applications. Another disadvantage of the UDWT comes from the fact that it requires the implementation of a large number of different filters.

B. Shift Invariant Discrete Wavelet Transform

Lang, Guo, Odegard, Burrus and Welles [4] have proposed a new shift-invariant but very redundant wavelet transform, named Shift Invariant Discrete Wavelet Transform, SIDWT. Their proposition is based on a translation invariant algorithm proposed by Coifman and Donoho [3]. The computation of this transform implies the consideration of all circular shifts of the input signal. After the computation of the DWT of every shifted version of the signal, this method requires the shifting back (or unshifting) and averaging over all results obtained.

C. Cycle Spinning

The method introduced by Coifman and Donoho in [3] and called Cycle Spinning (CS) was conceived to suppress the artefacts in the neighbourhood of

[1] Ph. D Students, [2] Teaching Assistant, Faculty of Electronics and Telecommunications, Communications Departement, Bd. V. Pârvan Nr. 2, 300223 Timişoara, e-mail ioana.adam@etc.upt.ro

discontinuities introduced by the classical DWT, and it implies the rejection of the translation dependence. For a range of shifts, data (time samples of a signal) is shifted (right or left as the case may be), the DWT of shifted data is computed, and than the result is un-shifted. Doing this for a range of shifts, and averaging the several results so obtained, a quasi shift-invariant discrete wavelet transform is obtained. The degree of redundancy of this transform is proportional to the number of shifts of the input signal produced. Cycle spinning over the range of all circular shifts of the input signal is equivalent to SIDWT.

D. Dual Tree Complex Wavelet Transform

Abry [5], first demonstrated that approximate shiftability is possible for the DWT with a small, fixed amount of transform redundancy. He designed a pair of real wavelets such that one is approximately the Hilbert transform of the other. This wavelet pair defines a complex wavelet transform (CWT). For explaining that such a transform is complex, consider the pair of DWT trees associated with the wavelet pair already mentioned. A complex wavelet coefficient is obtained by interpreting the wavelet coefficient from one DWT tree as being its real part, whereas the corresponding coefficient from the other tree is interpreted as its imaginary part. This transform is represented in figure 1.

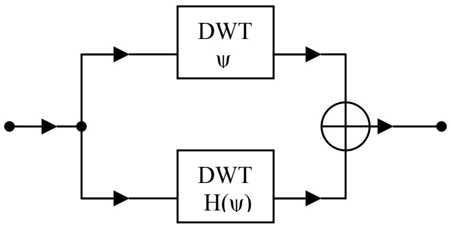

Figure 1. Abry's CWT.

Kingsbury [6] developed the dual tree complex wavelet transform (DTCWT), which is a quadrature pair of DWT trees, similar to Abry's wavelet transform (see figure 1). The DTCWT coefficients may be interpreted as arising from the DWT associated with a quasi-analytic wavelet. Both DTCWT and Abry's transform are invertible and quasi shift-invariant; however the design of these quadrature wavelet pairs is quite complicated and it can be done only through approximations.

E. Mapping-based Complex Wavelet Transform

Fernandes, van Spaendonck and Burrus have introduced, in [7], a two-stage mapping-based complex wavelet transform (MBCWT) that consists of a mapping onto a complex function space followed by a DWT of the complex mapping computation. The authors of this article have observed that the DTCWT coefficients admit also another interpretation: they may be interpreted as the coefficients of a DWT applied to a complex signal associated with the input signal. The complex signal is defined as the Hardy-space image of the input signal. As the Hardy-space mapping of a signal is impossible to compute, they have defined a new function space called the Softy-space, which is an approximation to Hardy-space.

The advantages of this method are:

- controllable redundancy of the mapping stage that offers a balance between the degree of shift sensitivity and the transform redundancy;
- the possibility to use any mother wavelet for the computation of the DWT in the transform implementation, which provides flexibility to this transform.

3. ANALYTIC DISCRETE WAVELET TRANSFORM

In this paper, we propose a new complex wavelet transform, similar to the DTCWT but easier to implement. It involves computing a single DWT but, instead of applying it to the original signal we apply it to the analytical signal associated with our input signal. The analytical signal associated with the signal x is defined as $x_a=x+iH\{x\}$, where $H\{x\}$ denotes the Hilbert transform of the input signal.

In the following, this transform will be called analytic discrete wavelet transform, ADWT. The equivalence between the DTCWT and the ADWT is illustrated in figure 2.

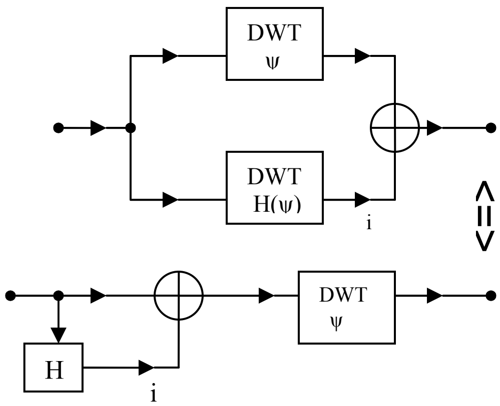

Figure 2. The equivalence between the DTCWT (top) and the ADWT (bottom).

In [8], Simoncelli has defined a new measure of the shift-invariance, called "shiftability". According to their definition, a transform is shiftable if and only if any subband energy of the transform is invariant under input-signal shifts. Although weaker than shift invariance, shiftability is important for applications because it is equivalent to interpolability, which is a property ensuring the preservation of transform-subband energy under input-signal shifts.

4. SIMULATIONS

In order to evaluate the shift-invariance performance of our transform, we introduced a new criterion: the degree of shift invariance. In order to calculate this measure, we calculate the energies of every set of detail coefficients (at different decomposition levels) and of the approximation coefficients, corresponding to a certain delay (shift) of the input signal samples. This way, we obtain a sequence of energies at each decomposition level, each sample of this sequence corresponding to a different shift. Then the mean m and the standard deviation d of every energy sequence are computed. Our degree of invariance is defined as:

$$Grad = 1 - d/m \tag{1}$$

We perform the normalization with respect to the mean of the energy sequence because we want the values of the degree of invariance to be within the interval [0, 1], for better interpreting it.

If the transform is shift-invariant, then the value of its degree of invariance is 1 because the standard deviation of the energy sequence is zero in this case. The reciprocity is not guaranteed. There are quasi shift invariant wavelet transforms with the degree of shift-invariance equal to 1 that are not perfectly shift-invariant. However, generally, when the transform is not shift-invariant the value of this degree of invariance is smaller than 1. This observation is also sustained by experimental work.

We consider that the degree of shift invariance is an objective way of analysing the shift invariance of a transform.

In the simulations purpose, we used as input signal a unitary step, like in [6]. In fact, 16 different unitary steps were used. They were generated one from another by delaying with a sample. Each unitary step is composed of 1024 samples. The number of iterations used for the computation of the DWT was 3. We repeated the simulations for several mother wavelets commonly used in the literature (Daubechies, Symmlet and Coiflet).

In the first set of simulations we have compared the degree of shift invariance of our transform with the degree of shift invariance of the DWT.

In the second set of simulations we have compared the degree of shift invariance of our transform with the degree of shift invariance of the CS with a various number of cycle spins and for a variety of spinning steps (a spinning step is the number of samples the signal is shifted once).

In table 1 we present a comparison between our transform and the DWT. This comparison is based on the values of the degree of shift invariance calculated for the approximation coefficients obtained after the 3rd iteration of the DWT computation algorithm (Scaling fn., level 3), for the detail coefficients obtained after the 3rd iteration (Wavelets level 3), for the detail coefficients obtained after the 2nd iteration

	The degree of shift invariance	
Decomposition level	ADWT	Classical DWT
Scaling fn. level 3	0.8594	0.7552
Wavelets level 3	0.9981	0.7878
Wavelets level 2	0.9982	0.8265
Wavelets level 1	0.9992	0.9236

Table 1. A comparison between the proposed WT and the DWT with respect to the degree of shift-invariance

(Wavelets level 2) and for the detail coefficients obtained after the 1st iteration (Wavelets level 1). By recomposing all these signals, the initial step signal should be obtained. Mother wavelet used was Daubechies-10 (with five vanishing moments). In order to isolate the coefficients corresponding to each level, after the computation of the DWT, we put all the complex coefficients corresponding to the other levels to zero, by applying a "mask" on the sequence obtained after DWT computation. For a better understanding of this procedure, we illustrate in figure 3 the system used for the analysis of the shift-invariance at the 3rd decomposition level of the ADWT.

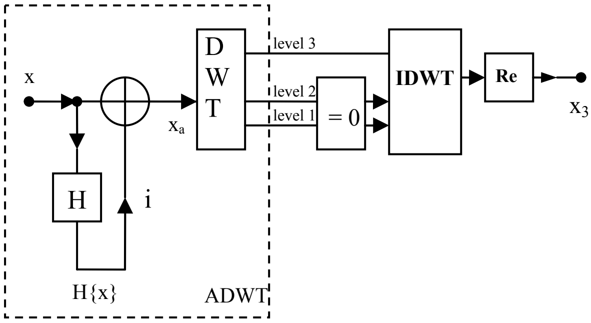

Figure 3. The system used for the shift-invariance analysis of the third level of the wavelet decomposition.

The first experiment already described is illustrated in figure 4. The results obtained using the proposed WT are presented in figure 4 a) and the results obtained using the classical decimated DWT in figure 4 b). It can be observed that the DWT is not shift-invariant. The ADWT is quasi shift-invariant. It can be observed that the ADWT is quasi shift-invariant. That is, for shifted version of the same signal applied to the transform's input, we obtained shifted-like versions of the signal reconstructed following the steps indicated in figure 3.

In fig. 5 we show the dependency of the degree of shift invariance of the proposed WT with respect to the regularity of the mother wavelet used for its computation. We investigated the Daubechies family, each element being indexed by its number of vanishing moments. As the curve illustrated in figure

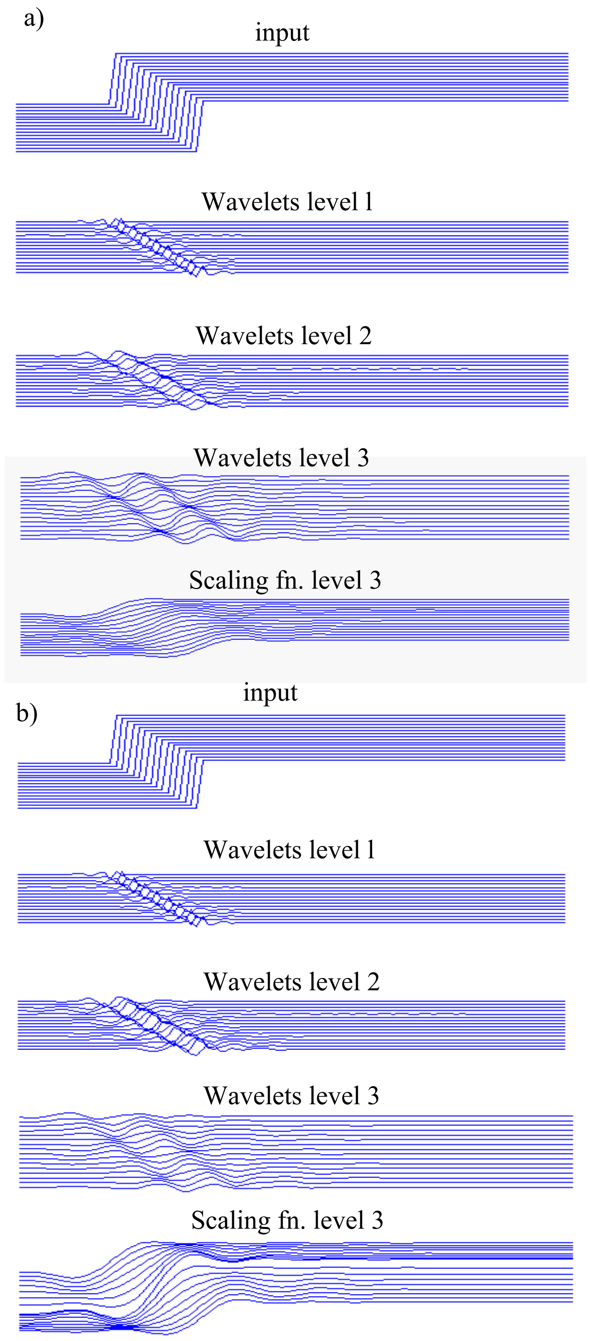

Figure 4. A comparison between the ADWT (a) and the DWT (b).

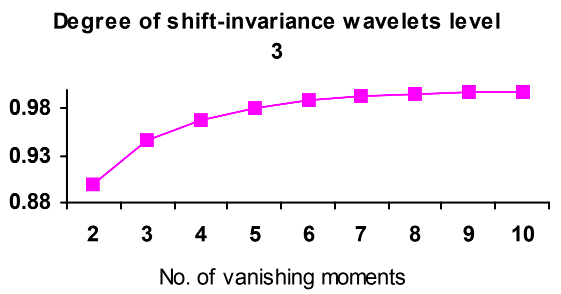

Figure 5. The dependency of the degree of shift-invariance of ADWT on the regularity of the mother wavelet used. for its computation.

Symmlet, 10	ADWT	CS step=1 64 delays	CS step=1 512 delays
Redundancy	Non redundant	64	512
Scaling fn. level 3	0,8594		0,7551
Wavelets level 3	0,9962	0,9962	0,9995
Wavelets level 2	0,9963	0,9965	0,9996
Wavelets level 1	0,9992	0,9985	0,9998
Daubechies, 10	ADWT	CS step=1 64 delays	CS step=1 512 delays
Scaling fn. level 3	0,8594	0,7551	0,7551
Wavelets level 3	0,9981	0,9965	0,9996
Wavelets level 2	0,9982	0,9968	0,9996
Wavelets level 1	0,9992	0,9985	0,9998

Table 2. A comparison between two quasi shift-invariant WTs, the ADWT and the CS.

5 indicates, the degree shift-invariance increases with the regularity of the mother wavelets used. In table 2 we present a comparison between our transform and the CS. It can be observed, analyzing this table, that the ADWT is equivalent to the CS with redundancy 64, from the degree of shift-invariance point of view. This in an excellent result, given that our transform is non-redundant, since for L samples to the input of ADWT, we still get L complex samples in the wavelet domain.

5. CONCLUSION

In this paper we propose a new complex non-redundant quasi shift-invariant WT. A new measure of the degree of shift-invariance of a WT is introduced. The degree of shift-invariance of the proposed transform is studied using this new measure. We show, on an illustrative example chosen, that the ADWT is equivalent from the degree of shift-invariance point of view with the CS with redundancy 64, when both WTs are applied to a signal having a duration of 1024 samples. This research will be continued on the following directions:

- a comparison of the degree of shift-invariance obtained applying the proposed WT with the degree of shift-invariance obtained applying other WTs like the DTCWT or the MBCWT.
- The generalization of ADWT in 2D.
- The construction and the study of a new 2D ADWT with improved directional selectivity, 2D ADWTIDS.
- The implementation and the study of a new 2D ADWTIDS with enhanced diversity, 2D ADWTIDSED.
- The construction and the study of a wavelet packets transform inspired by the 2D ADWTIDSED.

- The utilization of the 2D AWFTIDSED for the de-blurring and denoising of SONAR images.

ACKNOWLEDGEMENT

The authors want to thank Professor Jean-Marc Boucher and Associated Professor Sorin Moga from ENST-Bretagne, for the fruitful discussions on the topic of this paper, discussions developed during the conferences sustained in our university. The results reported here were obtained in the framework of two Romanian research programs granted by CNCSIS and directed by Associated Professor Dorina Isar and Professor Alexandru Isar. The authors would also like to express a special word of gratitude for the later, who carefully guided us during our research work.

REFERENCES

[1] S.Mallat, “Zero-crossings of a wavelet transform”, *IEEE Trans. Information Theory*, vol. 37, pp. 1019 - 1033, July 1991.
[2] G. Beylkin, “On the representation of operators in bases of compactly supported wavelets”, *SIAM J. Numer. Anal.*, vol. 29, no. 6, pp. 1716 - 1740, 1992.
[3] R. Coifman and D. Donoho, “Translation-invariant de-noising”, *Wavelets and Statistics*, A. Antoniadis and G. Oppenheim Eds, Springer-Verlag, pp. 125-150, New York, 1995.
[4] M. Lang, H. Guo, J. E. Odegard, C. S. Burrus and R. O. Wells Jr., “Noise reduction using an undecimated discrete wavelet transform”, *IEEE Signal Processing Lett.*, vol.3, no.1, pp. 10-12, Jan. 1996.
[5] P. Abry, "Transformées en ondelettes-Analyzes multirésolution et signaux de pression en turbulence", Ph.D.dissertation, Université Claude Bernard, Lyon, France, 1994.
[6] Nick Kingsbury, “Complex Wavelets for Shift Invariant Analysis and Filtering of Signals”, *Applied and Computational Harmonic Analysis*, 10, 234-253, 2001.
[7] Felix C. A. Fernandes, Rutter L.C. van Spaendonck and C. Sindey Burrus, “A New Framework for Complex Wavelet Transforms”, *IEEE Transactions on Signal Processing*, vol. 51, no. 7, pp.1825–1837, July, 2000.
[8] E.P.Simoncelli, W.T. Freeman, E.H.Adelson and D.J.Heeger, “Shiftable multi-scale transforms”, *IEEE Trans. on Inform. Theory*, vol. 38, pp. 587 – 607, March 1992.