

Modeling, Analysis and Simulation Results Regarding a Power Factor Correction Rectifier

Mircea Băbăiță¹, Viorel Popescu², Adrian Popovici³, Petru Papazian⁴

Abstract – This paper proposes a new control technique for single-phase boost power-factor-correction (PFC) rectifiers that improves the dynamic response of the converter to load steps without the need of a high crossover frequency of the voltage loop. So a low distortion of the input current is easily achieved. A 100W power-factor correction rectifier with the proposed control scheme has been designed, simulated and implemented, validating the concept.

Keywords: Power factor correction, average current control, rectifiers, power conversion

I. INTRODUCTION

The power-factor-correction rectifiers based on a boost converter are one of the most popular topologies in accordance with harmonic distortion standards, like IEC 6100-3-2, IEC 6100-3-4 and CISPR 11 [3], [4], [5]. Low harmonic distortion is achieved by using average current-mode control (ACC) [1], [2] with bandwidth of the voltage loop limited to about 20 Hz in order to properly attenuate the second line harmonic (100Hz) that appears at the output voltage of the converter [6]. As a result, the dynamic response of the output voltage to load changes is slow.

In the last years, several techniques have been proposed to overcome this problem. The basis of those methods is to eliminate the ripple at the frequency of the second line harmonic from the control signals in order to increase the voltage loop crossover frequency. The main drawback of those solutions is that they all significantly increase the complexity of the control circuit, by adding multipliers/dividers, A/D circuits, digital controllers, field-programmable gate arrays etc.

This paper proposes a new robust model-following ACC scheme (RMACC) with a high disturbance rejection and an analog implementation applied to boost PFC rectifiers. In case of a PFC rectifier, the

amplification of the output voltage ripple would be especially disturbing, because the second line harmonic present at the control signals would be amplified. The advantages of the proposed control loop applied to PFC rectifiers are:

- RMACC uses a reference model that has a low-pass nature, so that the output voltage ripple is not amplified. Therefore the contents of the second line harmonic present at the control signals is similar to that of conventional ACC, so that a low input current distortion can be achieved.
- RMACC decreases significantly the closed-loop output impedance of the PFC rectifier at low frequencies. Hence, the dynamic response of the output voltage to load steps is faster.
- RMACC does not add significant complexity to the control circuits.

The proposed control method for PFC converters is useful in those applications that request fast response of the output voltage to load steps: uninterruptible power system (UPS), power supplies systems for telecom, computers applications, etc [9].

A 100-W PFC rectifier based on a boost converter with RMACC has been designed, simulated and implemented, validating the concept.

II. DESCRIPTION OF THE RMACC

II.1. Small-Signal Model of an ACC Rectifier

The ACC scheme of a typical boost PFC rectifier with feedforward of the rectifier input voltage is shown in Fig.1. A linear small-signal model of the ACC-controlled boost PFC rectifier is shown in Fig.2, where:

R_s current sensing gains;
 β voltage sensing gains;

¹ Facultatea de Electronică și Telecomunicații, Departamentul Electronică Aplicată, Bd. V. Pârvan Nr. 2, 300223, Timișoara, e-mail mircea.babaita@etc.upt.ro;

² Facultatea de Electronică și Telecomunicații, Departamentul Electronică Aplicată, Bd. V. Pârvan Nr. 2, 300223 Timișoara, e-mail viorel.popescu@etc.upt.ro;

³ Facultatea de Electronică și Telecomunicații, Departamentul Electronică Aplicată, Bd. V. Pârvan Nr. 2, 300223 Timișoara, e-mail adrian.popovici@etc.upt.ro

⁴ Facultatea de Electronică și Telecomunicații, Departamentul Electronică Aplicată, Bd. V. Pârvan Nr. 2, 300223 Timișoara, e-mail petru.papazian@etc.upt.ro

$\hat{v}_g(s)$ rectified input voltage;
 $\hat{v}_o(s)$ output voltage;
 $\hat{i}_L(s)$ inductor current;
 V_{ref} reference voltage;

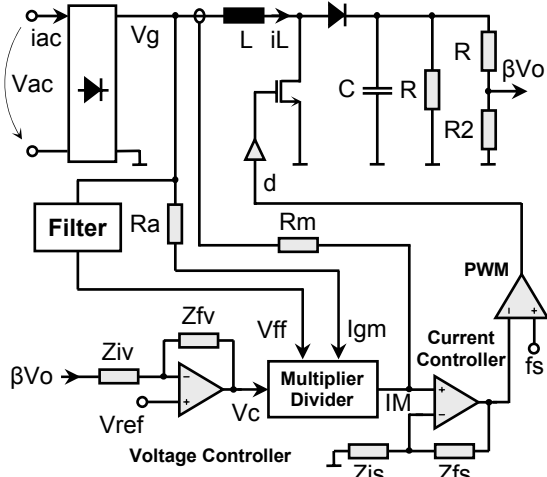


Fig.1 Typical Boost rectifier with ACC.

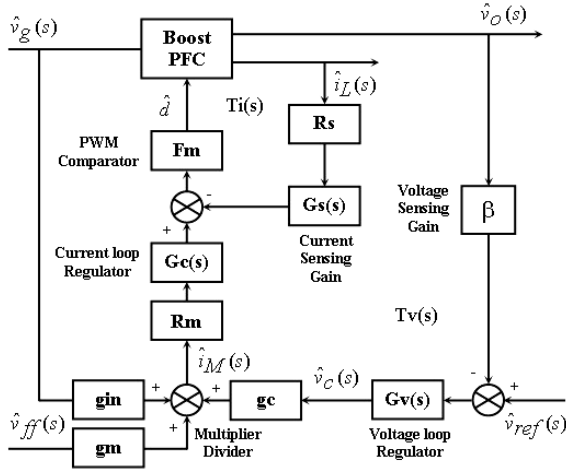


Fig.2 Small-signal model of the PFC rectifier.

F_m PWM modulator gain;
 $G_s(s)$ transfer function of the current regulator;
 $G_s(s) = \frac{Z_{fs}(s)}{Z_{is}(s)}$, $G_c(s) = 1 + G_s(s)$, (1)
 $G_v(s)$ transfer function of the voltage regulator;
 $G_v(s) = \frac{Z_{fv}(s)}{Z_{iv}(s)}$, (2)
 $T_i(s)$ loop gain of the current loop;
 $T_v(s)$ loop gain of the voltage loop;
 $\hat{i}_{gm}(s)$, $\hat{v}_{ff}(s)$, $\hat{v}_c(s)$, \hat{i}_M , small signal of the input/output multiplier-divider block;
 I_{gm} , V_{ff} , V_c , I_M , steady state input/output of the multiplier-divider block;

g_{in} , g_c , g_m equivalent gains of the multiplier-divider small signal model.

$$g_{in} = \frac{K_{ac} \cdot V_c}{V_{ff}^2} \quad (3)$$

$$g_c = \frac{K_{ac} \cdot V_{g-RMS}}{V_{ff}^2} \approx \frac{K_{ac}}{K_{ff}^2 \cdot V_{g-RMS}} \quad (4)$$

$$g_m = -2 \cdot \frac{I_M}{V_{ff}^2} \quad (5)$$

$$K_{ac} = \frac{i_{gm}(t)}{v_g(t)} = \frac{I}{R_{ac}} \quad (6)$$

$$K_{ff} = \frac{V_{ff}}{V_{g-DC}} \approx \frac{V_{ff}}{V_{g-RMS}} \quad (7)$$

Closing the current loop $T_i(s)$, the voltage regulator $G_v(s)$ must compensate an ACC power stage transfer function $VOC(s) = \hat{v}_o(s)/\hat{v}_c(s)$. This can be approximated by a first order system [1], [2], as shown in [10]. $Z_o(s) = \hat{v}_o(s)/\hat{i}_o(s)|_{\hat{v}_c=0}$ and \hat{i}_o are the ACC open-loop output impedance and the load disturbance. An approximation of $VOC(s)$ can be derived by neglecting the high-frequency dynamics:

$$VOC(s) = \frac{\hat{v}_o(s)}{\hat{v}_c(s)}|_{\hat{i}_o=0} \approx \frac{K_{ac} \cdot R_m}{K_{ff}^2} \cdot \frac{I}{2V_o} \cdot \frac{\frac{R}{RC-s}}{1 + \frac{RC}{2}s} \quad (8)$$

Do to the action of the feedforward, $VOC(s)$ doesn't depend on the input voltage V_g , K_{ac} and K_{ff} are constant.

The stability of the control system is given by the voltage loop gain, $T_v(s)$:

$$T_v(s) = G_v(s) \cdot VOC(s) \cdot \beta \quad (9)$$

The closed-loop output impedance $Z_{ocl}(s)$ is expressed by:

$$Z_{ocl-ACC}(s) = \frac{Z_o(s)}{1 + T_v(s)} = Z_o(s) \cdot S(s) \quad (10)$$

where $S(s) = 1/(1 + T_v(s))$ is the sensitivity function, being $|S(j\omega)| < 1$ up to the crossover frequency of the voltage loop, f_{cv} , and $|S(j\omega)| \approx 1$ at frequencies higher than f_{cv} . $S(s)$ expresses the disturbance rejection, being a powerful index to analyze the robust performance of a control system.

The general expression of $G_v(s)$ in conventional ACC is:

$$G_v(s) = \frac{\omega_{iv} \cdot (1 + \frac{s}{\omega_{zv}})}{\omega_{zv}} = \frac{\frac{\omega_{e-v}}{K \cdot \beta} \cdot (1 + s \frac{R_{nom}C}{2})}{s \cdot (1 + \frac{s}{\omega_{pv}})} \quad (11)$$

The zero ω_{zv} is chosen to compensate the dominant pole of the power stage, $VOC(s)$. In order to attenuate

the second line harmonic at the control signals, the pole ω_{iv} is placed around half the frequency of the output voltage ripple, i.e., around the line frequency f (50Hz). The gain ω_{iv} :

$$\omega_{iv} = \frac{\omega_{c-v}}{\frac{K_{ff}^2 2V_o R_s}{K_{ac} R_m R_{nom}} \beta} = \frac{\omega_{c-v}}{K \cdot \beta} \quad (12)$$

is chosen taking into account the desired crossover frequency, ω_{c-v} , of $T_v(s)$. R_{nom} is the load resistance at full load.

II.2. The Proposed RMACC Rectifier

The proposed RMACC scheme is shown in Fig.3.

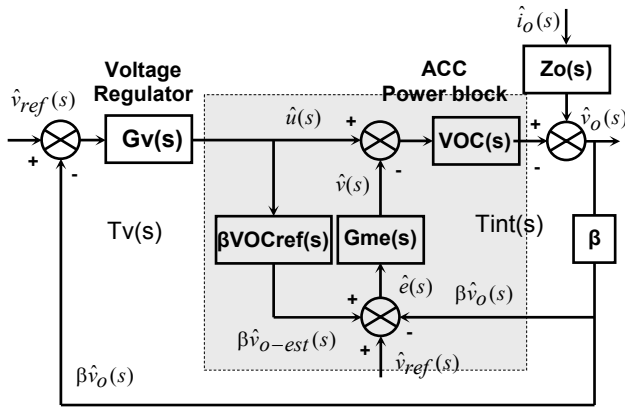


Fig.3 The proposed RMACC scheme.

After some block algebra, results the equivalent scheme presented in Fig.4, where $T_{ref}(s) = \beta G_{me}(s) VOC_{ref}(s)$. The current loop $T_i(s)$ is the same as in conventional ACC and it contains the same current regulator, $G_s(s)$, so that $T_i(s)$ is not represented in Fig.3.

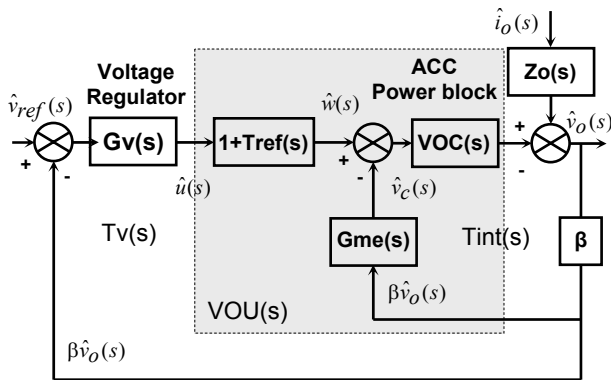


Fig.4. Equivalent scheme of the proposed RMACC.

An additional internal loop with model-following effects $T_{int}(s)$ is added before closing the outer voltage loop $T_v(s)$ with the voltage regulator $G_v(s)$. The internal loop contains two blocks: a „modeling error” PI regulator $G_{me}(s)$ and a fixed reference

model transfer function $\beta \cdot VOC_{ref}(s)$, which is low pass and first order like a conventional ACC power stage. The expression of the reference model is:

$$VOC_{ref}(s) = \frac{K_{ac} \cdot R_m}{K_{ff}^2} \cdot \frac{1}{2V_o} \cdot \frac{\frac{R_{nom}}{R_s}}{1 + \frac{R_{nom} C_s}{2}} \quad (13)$$

The output of the reference model $\beta \cdot v_{o-est}$ is an estimation of the sensed output voltage $\beta \cdot v_o$ if $VOC(s) = VOC_{ref}(s)$ and without disturbances. Thus, the signal $e(s)$ is an estimated error that represents the difference between the actual power stage and the chosen reference model. The modeling error regulator $G_{me}(s)$ is designed for the adequate loop shaping of $T_{int}(s)$. The gain of $T_{int}(s)$ at the frequency of the second line harmonic must be low enough to assure that no significant distortion appears in the line current. Therefore, the crossover frequency of $T_{int}(s)$, $f_{c-int} = \omega_{c-int} / 2\pi$, should be limited to around 10-20 Hz. The loop gain of the internal loop is:

$$T_{int}(s) = \beta \cdot G_{me}(s) \cdot VOC(s) \quad (14)$$

The intermediate transfer functions are:

$$VOW(s) = \frac{\hat{v}_o(s)}{\hat{w}_o(s)} \Big|_{\hat{i}_o=0} = \frac{VOC(s)}{1 + T_{int}(s)} \quad (15)$$

$$T_{ref}(s) = \beta G_{me}(s) VOC_{ref}(s) \quad (16)$$

This are used for the definition of the modified power stage transfer function $VOU(s)$:

$$\begin{aligned} VOU(s) &= \frac{\hat{v}_o(s)}{\hat{u}(s)} \Big|_{\hat{i}_o=0} = VOW(s) \cdot (1 + T_{ref}(s)) \\ &= VOC(s) \cdot \frac{(1 + T_{ref}(s))}{(1 + T_{int}(s))} \approx VOC_{ref}(s) \end{aligned} \quad (17)$$

$VOU(s)$ is the transfer function „seen” by the outer voltage regulator of RMACC $G_v(s)$. $T_{ref}(s)$ is a fixed transfer function and it can be defined as a „reference loop gain”, because it agrees with $T_{int}(s)$ if $VOC(s) = VOC_{ref}(s)$. The range of frequencies where $|T_{int}(j\omega)| \gg 1$ and $|T_{ref}(j\omega)| \gg 1$, the transfer function seen by the voltage regulator is a fixed one and it agrees with $VOC_{ref}(s)$, i.e., $VOC(s) \approx VOC_{ref}(s)$. Therefore, the controller of the main voltage loop $G_v(s)$ can be designed to compensate the reference model, which is a fixed transfer function. That is the basis of the model following action of the inner loop and it justifies the approximation made in the last term of (17), which is valid in the frequency range where $|T_{int}(j\omega)| \gg 1$ and $|T_{ref}(j\omega)| \gg 1$.

However, the main benefit of RMACC in this application is not the model-following effect, but the improvement of the closed-loop output impedance by means of an easy and systematic technique. The loop shaping of $T_v(s)$:

$$T_v(s) = \beta \cdot G_v(s) \cdot VOUs(s) \approx \beta \cdot G_v(s) \cdot VOUs_{ref}(s) \quad (18)$$

by means of the voltage controller must take into account that the crossover frequency is limited by the distortion of the line current. Therefore, a crossover frequency $f_{C-int} = \omega_{C-int} / 2\pi$ up to about 10-20 Hz should be chosen of $T_v(s)$.

In Fig.3 a double injection of the reference voltage in the loop can be noticed: as a reference voltage for $G_v(s)$ and as a reference of $G_{me}(s)$. The reason for this is that in steady state the integrating character of both regulators yields $V_{ref}(s) = \beta V_o$, and $\beta V_{o-est} + V_{ref} - \beta V_o = 0$, so that $\beta V_{o-est} = 0$. With this double injection of V_{ref} the output of the reference model is zero in steady state, only acting around zero in the presence of disturbances. It's an easy way to avoid the saturation of the reference model output.

II.3. Improvement of the Closed-Loop Output Impedance

With ACC and a conventional PI voltage regulator, the reduction of $Z_{ocl}(s)$ at low frequencies implies to increase the crossover frequency, f_{C-v} of $T_v(s)$, which is strongly limited by the distortion of the line current. With RMACC, $Z_{ocl}(s)$ depends not only on $G_v(s)$, but also on $T_{int}(s)$ and on $T_{ref}(s)$. Therefore, the low-frequency closed-loop output impedance can be reduced without the need of having a high f_{C-v} .

In PFC boost rectifiers with feedforward loop, the actual power stage $VOC(s)$ suffers from little variations with respect to $VOC_{ref}(s)$ around the crossover frequency of the voltage loop, i.e., $VOC(s) \approx VOC_{ref}(s)$. Therefore, if $G_v(s) = G_{me}(s)$, the loop gains will be similar i.e., $T_{int}(s) \approx T_{ref}(s) \approx T_v(s)$. In this way, a single loop shaping has to be performed for the three loop gains, simplifying the design of RMACC (Fig.4). Moreover, the closed loop output impedance can be expressed by:

$$\begin{aligned} Z_{ocl-RMACC}(s) &= \frac{\hat{v}_o(s)}{\hat{i}_o(s)} \approx \frac{Z_o(s)}{(1+T_v(s))^2} \\ &= Z_o(s) \cdot S^2(s) = Z_{ocl-ACC}(s) \cdot S(s) \quad (19) \end{aligned}$$

Both $T_{int}(s)$ and $T_v(s)$ have a low crossover frequency like the voltage loop gain in the

conventional ACC of a PFC rectifier. In spite of having low crossover frequencies, the low frequency output impedance of the PFC rectifier is lower with RMACC than with ACC, so that the dynamic response to load steps is expected to be faster.

III. DESIGN OF THE RMACC RECTIFIER

Conventional ACC and the proposed RMACC schemes have been applied to a boost PFC rectifier with: $V_{ac} = 220V$, $f = 50Hz$, $V_o = 400V$, $P_o = 100W$, $L = 1mH$, $C = 470\mu F$, $f_s = 100kHz$, $R_s = 0,2\Omega$, $\beta = 0,0125$, $K_{ac} = 1,47 \cdot 10^{-6} A/V$, $F_m = 0,19V^{-1}$, $R_m = 4,3 \cdot 10^3 \Omega$, $R_{nom} = 640\Omega$, $K_{ff} = 17,63 \cdot 10^{-3}$.

The values of L and C have been chosen so that the inductor current ripple $\Delta i_L \approx 1A$, with a holdup time $\Delta t \approx 64ms$. Δt is defined as the time at which the output voltage decreases to $V_o = 300V$ after disconnecting the line voltage.

A current regulator $G_s(s)$ designed by means of conventional loop-shaping techniques [1], [2] has been chosen. The current loop crossover frequency is about 16kHz with a phase margin of 60°. The same current regulator is used with ACC and with RMACC. The voltage loop with conventional ACC is closed with a voltage regulator. The theoretical crossover frequency with that controller is about 8 Hz. The gain of $T_v(j\omega)$ at the frequency of the second line harmonic (100 Hz) is lower than -35dB.

Due to the feedforward path, $VOC(s)$ does not depend on the input voltage around the voltage loop crossover frequency. The load variations only affect $VOC(s)$ at very low frequencies, so that the approximation $VOC(s) \approx VOC_{ref}(s)$ can be made.

$G_v(s) = G_{me}(s)$ and $T_{int}(s) \approx T_{ref}(s) \approx T_v(s)$. If the gain of $T_{int}(s)$ at 100Hz has been designed to be small, also the gain of $T_v(s)$ results as small. Following that approach, the transfer functions of the chosen regulators are:

$$G_s(s) = \frac{100000}{s} \cdot \frac{1+s/15000}{1+s/300000} \quad (20)$$

for ACC and RMACC;

$$G_v(s) = \frac{60}{s} \cdot \frac{1+s/8}{1+s/120} \quad (21)$$

for ACC and RMACC;

$$G_{me}(s) = \frac{60}{s} \cdot \frac{1+s/8}{1+s/120} \quad (22)$$

$$\beta VOC_{ref}(s) = \frac{0,85}{1+s/8} \quad (23)$$

IV. SIMULATION RESULTS

A boost PFC rectifier with the same values (Fig.5) and regulation circuits has been simulated (in CASPOC), built and tested. The control stage schematic has been built with a UC3854 PFC integrated circuit [8]. Schematic of the experimental prototype of the additional internal loop is shown in Fig.6.

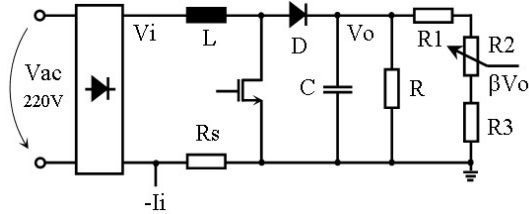


Fig.5 The used boost PFC rectifier.

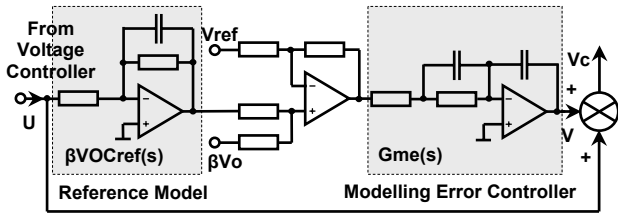


Fig.6. Schematic of the experimental prototype of the additional internal loop.

Fig.7 shows the measured gain Bode plots of the open-loop output impedance $Z_o(j\omega)$ and of the closed-loop output impedance with both ACC and RMACC $Z_{ocl-ACC}(j\omega)$ and $Z_{ocl-RMACC}(j\omega)$, respectively, with $P_o = 100W$ (full load) and $V_g = 220V$.

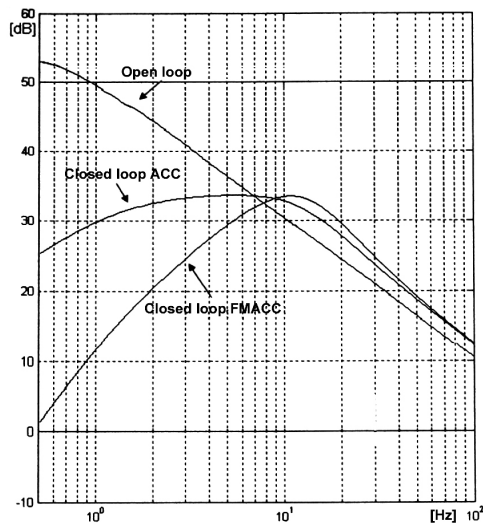


Fig.7 Module of the measured output impedance with ACC and RMACC.

An improvement of more than 20 dB at low frequencies in favor of RMACC is noticed. Note that

the output impedance of RMACC is much smaller at low frequencies than that of ACC. Therefore, the dynamic response of the output voltage to load steps is expected to be faster.

Fig.8 and Fig.9 shows the line voltage, the input current and the normalized harmonic spectrum of the line current for 220V, $P_o = 100W$ with ACC.

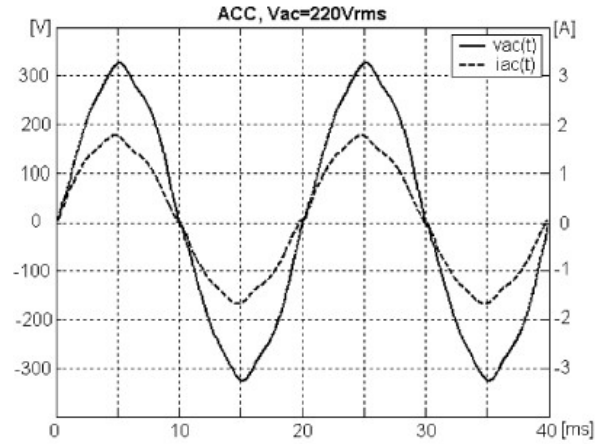


Fig.8. The line voltage and the input current with ACC.

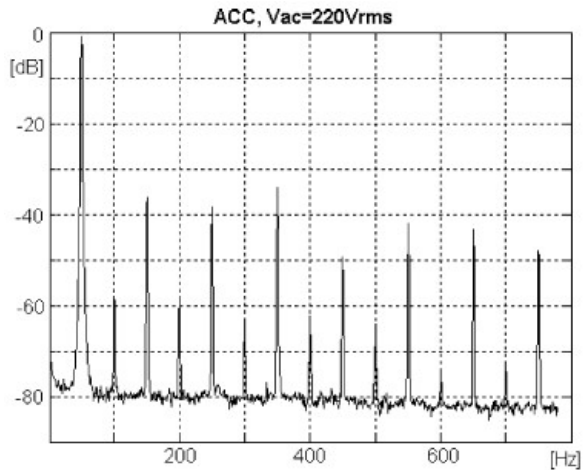


Fig.9. The input current harmonics with ACC.

Fig.10, Fig.11 and Fig.12 shows the same measurements, in the same conditions with RMACC.

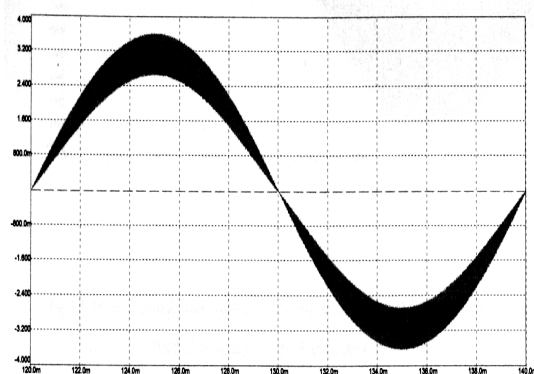


Fig.10. The input current with RMACC.

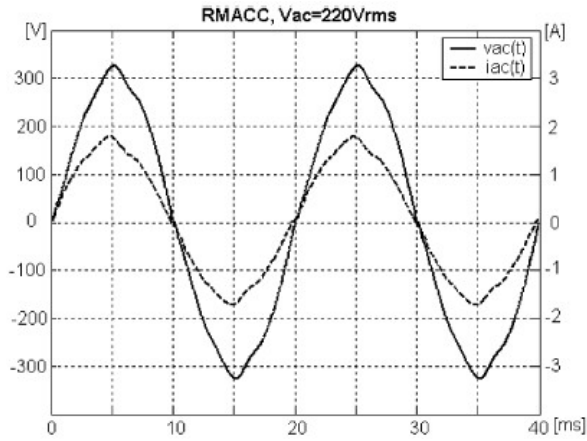


Fig.11. The line voltage and the input current with RMACC.

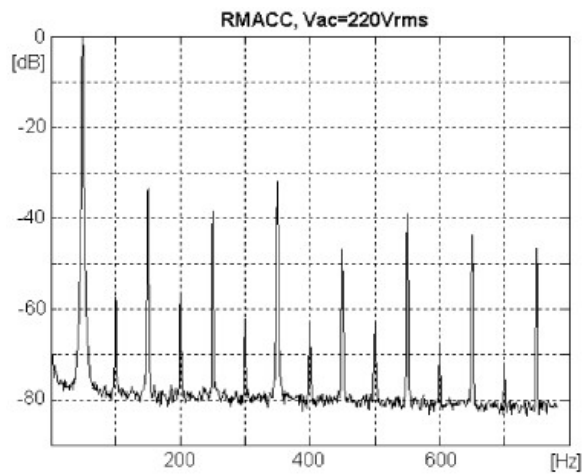


Fig.12. The input current harmonics with RMACC.

Table 1 shows the comparative experimental results of the input voltage distortions THD_v %, of the line current distortion THD_i % and of the power factor PF, with conventional ACC and with the proposed RMACC control scheme.

Table 1

Control Mode	Parameter	Line Voltage-Input Current 220V – 1,2A
ACC	THD _v	3,6%
	THD _i	6,2%
	PF	0,99
RMACC	THD _v	3,6%
	THD _i	5,8%
	PF	0,99

Note that no significant differences between ACC and RMACC are remarkable, so that their performances

from the line point of view are similar. In other words, the improvement of the closed-loop output impedance is achieved with no additional; distortion of the line current.

V. CONCLUSIONS

This paper analyzed a robust model-following ACC loop applied to a 100W boost PFC rectifier. It has been shown that the low-frequency output impedance of the converter is greatly reduced, so that the dynamic response of the output voltage to load steps is faster. The improvement of the transient response is achieved with similar values of the input current distortion and of the power factor as with conventional ACC. RMACC improves the output impedance without the need of high crossover frequencies in any of its loops, so that the control signals ripple at the frequency of the second line harmonic is easily attenuated.

The practical implementation of RMACC consists of adding an inner loop based on a low-pass first-order reference model and a conventional PI regulator, besides the outer voltage loop.

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