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A New Method for Fast and Accurate Evaluation of PCB Parasitics

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Abstract – Printed Circuit Boards (PCBs) are more than just interconnection structures, but rather passive circuits that influence the quality of signals they carry. Especially for high speed designs, the parasitic capacitances and inductances introduced by the PCB traces cannot be ignored, as they have a significant impact on both the signal integrity and electromagnetic compatibility of the designed circuit. This paper presents a new method for fast evaluation of those parasitics, which aims to offer the accuracy of field solving simulations at the speed of analytical estimations.

Keywords: PCB, parasitic, microstrip, stripline, statistically-enhanced analytical.

I. INTRODUCTION

As electronic applications become more and more complex and operate at higher frequencies, it becomes increasingly important to take into consideration the signal integrity (SI) and electromagnetic compatibility (EMC) concerns early in the design stage. The most efficient way to fulfill this task is to use software tools that are able to evaluate SI & EMC prior, during and after the PCB design in completed. For pre-layout and post-layout analysis there are well established analysis techniques based on numerical solving of the electromagnetic field equations, which are able to provide accurate solutions. The main drawback of those analysis techniques is that they are computational intensive, as they require both computer resources and analysis time in order to provide valid solutions. Another difficulty regarding field-solving simulators is that they are "specialist" software, with a slow learning curve and high price tag, so not always convenient to use. Yet another deficiency of field-solvers is that they are exclusively off-line tools, meaning that due to high computational costs they cannot provide solutions fast enough to assist the engineer during the actual design process, when it is mostly needed, so they are useful only for preliminary analysis and post-layout validation.

Another approach to SI & EMC problem implies the analysis of the equivalent circuit obtained by evaluating the parasitics introduces by the PCB. It is a simplified analysis which cannot compete with field solvers in terms of accuracy, but they can provide useful guidelines for the design process when they are mostly needed, which is during the design stage. Since the solution is given by an analytical formula with no iterations required, they work considerably faster than field solvers giving almost instantaneous solutions.

There is a vast literature presenting analytical estimations for various parasitic extraction problems, with relative errors usually ranging from ±1% to ±10% [2, 4, 5, 6, 9]. This paper presents a method to greatly improve the accuracy of such analytical estimations, further called "Statistically Enhanced Analytical" (SEA). It uses an analytical approximation as an initial estimation and a low order polynomial interpolation based on a reduced number of reference points obtained using numerical analysis.

II. INITIAL ESTIMATION

The initial solution is given by an analytical approximation of the electrical parameter that needs to be calculated, which might be:

- The characteristic impedance (Z0) or delay (TD) of a microstrip or stripline transmission line
- The characteristic capacitance (C0) or inductance (L0) of a microstrip or stripline transmission line
- The characteristic impedance, delay, capacitance or inductance of coupled transmission lines.

Various solutions for those problems are presented in literature, with a more insight on this being presented in [7].

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As a typical example, we might consider the characteristic impedance of microstrip transmission lines, which according to [5] is given by:

$$Z0_A(W,T,\varepsilon_r,H) = \frac{60}{\sqrt{0.475\cdot\varepsilon_r + 0.67}} \cdot \ln\left(\frac{5.98\cdot H}{0.8\cdot W + T}\right) \quad (1)$$

, where W is the trace width, T the trace thickness, ε_r the dielectric constant of the board substrate and H its height. This is not a very good estimate, as figure 1 shows that the relative error of this solution compared to field solver results is of several percents.

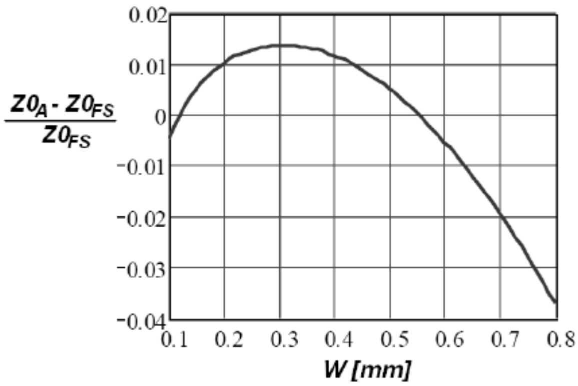


Fig. 1 The relative error of an analytical approximation for microstrip characteristic impedance ($Z0_A$), as compared with field solver solutions ($Z0_{FS}$)

Our method will use this analytical approximation only as an initial estimation, which will be corrected by statistical means.

III. THE "SEA" METHOD

If one tries to use interpolation to accurately estimate the parasitic components introduced by the interconnection structure, one will find himself surmounting the same problem as numerically solving the field equations: the accuracy comes with high computational costs, this time not in processing time but in memory space and access time, so interpolating Z0(W,T, ε_r,H) – which is a function of four variables – is not a very practical idea. Instead our method will interpolate the relative error of Z0, the function presented in figure 1, which gives several advantages:

- There is an initial guess about the solution
- A low degree polynomial interpolation is enough to significantly improve the accuracy of the analytical estimation
- Only a small number of reference point must be calculated and stored
- Once the reference points are stored in the computer memory, the computational overhead of correcting the analytical estimation is insignificant.

We will further denote the relative error of the analytical solution as:

$$E_{Z0} = \frac{Z0_A - Z0_{FS}}{Z0_{FS}} \quad (2)$$

By estimating the value of E_{Z0} for any given W, T, ε_r and H, one my calculate the corrected value of Z0 as:

$$Z0_{AC} = \frac{Z0_A}{1 + E_{Z0}} \quad (3)$$

It must be stated at this point that, although Z0 is a function of four variables, any practical implementation may take advantage of the fact that, for any given interconnection layer of a PCB, T, ε_r and H are in fact constants, so the characteristic impedance is reduced to a function of just one variable: Z0(W).

Our solution for the estimation of $E_{Z0(W)}$ – which just like Z0(W) becomes a function of only the trace width – is to use a low degree Lagrange interpolation, defined by the equation:

$$\hat{E}_{Z0}(W) = \sum_{i=0}^{n}\prod_{\substack{j=0\\ j\neq i}}^{n} E_{Z0}(W_i)\cdot\frac{W - W_j}{W_i - W_j} \quad (4)$$

, where $\hat{E}_{Z0}(W)$ represents the estimated value of $E_{Z0}(W)$, which will be used in equation 3 to calculate the characteristic impedance, and $E(W_0) \ldots E(W_n)$ are the reference points calculated with high accuracy, using an electromagnetic field solver.

The Lagrange interpolation on equally spaced points $W_0 \ldots W_n$ tends to induce oscillation of the estimated $\hat{E}_{Z0}(W)$, one possible solution to this problem being to interpolate in the Chebyshev nodes [1], the roots of the first type Chebyshev polynomials, defined recursively by:

$$\begin{cases} T_0(x) = 1 \\ T_1(x) = x \\ \ldots \\ T_{K+1}(x) = 2x\cdot T_K(x) - T_{K-1}(x) \end{cases} \quad (5)$$

The Chebyshev nodes of order n are given by the general solution:

$$x_K = \cos\left(\pi\cdot\frac{2K-1}{2n}\right) \quad (6)$$

The Chebyshev nodes are always contained in the closed interval [-1, 1], and since we need them to determine the reference widths in the interval [W_{MIN}, W_{MAX}], a linear transform must be involved, defined by:

$$LT(x) = \frac{W_{MAX} + W_{MIN}}{2} + x\cdot\frac{W_{MAX} - W_{MIN}}{2} \quad (7)$$

In our implementation we used a 3rd degree Lagrange interpolation, which according to eq. (6) and (7) gives the Chebyshev nodes:

$$\begin{cases} W_0 = 0.962 \cdot W_{MIN} + 0.038 \cdot W_{MAX} \\ W_1 = 0.691 \cdot W_{MIN} + 0.309 \cdot W_{MAX} \\ W_2 = 0.309 \cdot W_{MIN} + 0.691 \cdot W_{MAX} \\ W_3 = 0.038 \cdot W_{MIN} + 0.962 \cdot W_{MAX} \end{cases} \tag{8}$$

Using the 3rd degree Lagrange interpolation defined by eq. (4) in the Chebyshev nodes defined by eq. (8) for the analytical estimation given by eq. (1), the corrected value of the characteristic impedance of a microstrip line, calculated according to eq. (3), is given by:

$$Z0_A(W,T,\varepsilon_r,H) = \frac{60}{\sqrt{0.475 \cdot \varepsilon_r + 0.67}} \cdot \ln\left(\frac{5.98 \cdot H}{0.8 \cdot W + T}\right) \cdot \frac{1}{0.142 \cdot W^3 - 0.427 \cdot W^2 + 0.232 \cdot W + 0.979} \tag{9}$$

If we compare this result against the field-solver solution we obtain the relative error presented in figure 2, which is more than an order of magnitude lower than that presented in figure 1.

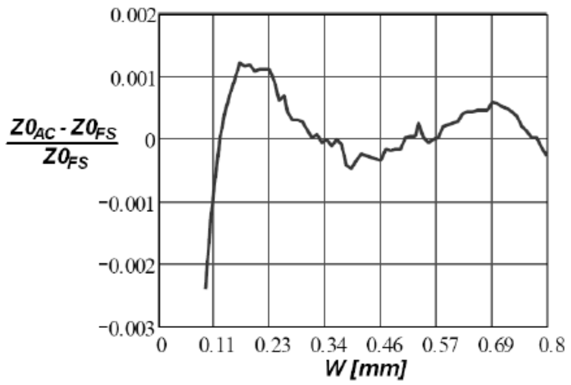


Fig. 2 The relative error of the SEA solution for microstrip characteristic impedance ($Z0_{AC}$), as compared with field solver solution ($Z0_{FS}$)

IV. MULTIVARIATE EXTENSION

The one-dimensional Lagrange interpolation is useful for calculating the parasitic introduces by a single transmission line, but often coupling parameters must be determined, such as the odd and even characteristic impedance, coupling capacitance or coupling inductance. In this situation, even after restricting the calculations to a single layer, the function that needs statistical correction is a function of two variables: W (with of the coupled traces, assumed the same for both) and S (spacing between traces). A more general formulation of the problem would be: given a function of two variables f(x,y), find the Lagrange interpolation function of order n.

For bi-dimensional interpolation, a matrix of (n+1) x (n+1) reference points must be known, and the solution works as follows:

1. First interpolate one-dimensional n+1 times, keeping x=X constant and giving the reference values f(X,y0), … f(X,yn).
2. Then calculate f(X,Y) by one-dimensional interpolation keeping y=Y constant, between the reference values previously determined.

A graphical illustration of this process is given in figure 3.

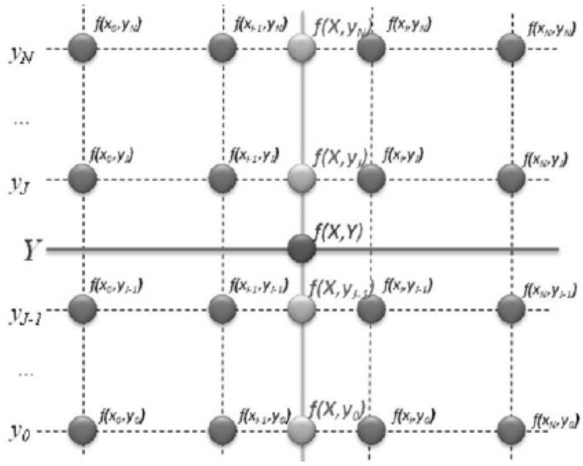


Fig. 3 Bi-dimensional interpolation

A bi-dimensional interpolation of order n is equivalent to n+2 one-dimensional interpolations, so for the 3rd degree Lagrange interpolation we need 16 reference points and 5 polynomial evaluations, which is not a significant computational overhead. If we apply this statistical correction to the odd characteristic impedance solution for microstrip lines given in [2], we obtain the relative error compared with field solver results illustrated in figure 4.

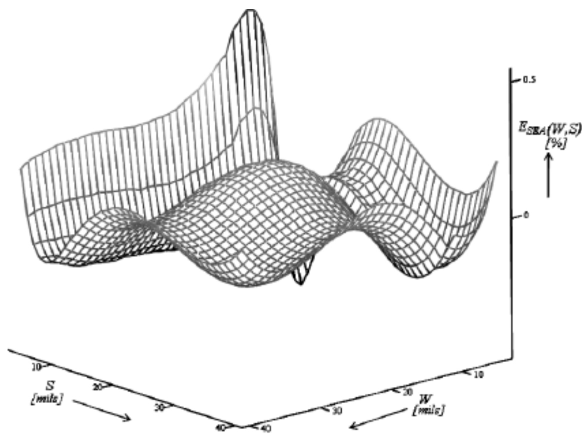


Fig. 4 Relative error of the SEA results for the odd characteristic impedance of microstrip transmission lines, using bi-dimensional Lagrange interpolation of 3rd degree

The initial analytic estimation given in [2] gives relative errors in the domain (-2%, 4%), while after SEA correction this reduces to about ±0.5%, which is a significant improvement considering that it only required 5 polynomial calculations.

V. IMPLEMENTATION

In order to determine the parasitics of a single transmission line, one only needs no know the characteristic impedance (Z0) and propagation time (TD), the rest being calculated as follows:

- Characteristic capacitance:

$$C0 = \frac{TD}{Z0} \qquad (10)$$

- Characteristic inductance:

$$L0 = TD \cdot Z0 \qquad (11)$$

Similarly, for coupled line parasitics it is enough to determine the odd and even characteristic impedance and propagation time, the rest being calculated as follows:

- Characteristic capacitance:

$$C11 = \frac{1}{2} \cdot \left(\frac{TD_{even}}{Z_{even}} + \frac{TD_{odd}}{Z_{odd}} \right) \qquad (12)$$

- Mutual capacitance:

$$C12 = \frac{1}{2} \cdot \left(\frac{TD_{even}}{Z_{even}} - \frac{TD_{odd}}{Z_{odd}} \right) \qquad (13)$$

- Characteristic inductance:

$$L11 = \frac{1}{2} \cdot \left(TD_{even} \cdot Z_{even} + TD_{odd} \cdot Z_{odd} \right) \qquad (14)$$

- Mutual inductance:

$$L12 = \frac{1}{2} \cdot \left(TD_{even} \cdot Z_{even} - TD_{odd} \cdot Z_{odd} \right) \qquad (15)$$

Our implementation uses the above formulas to determine the parasitic of any single or coupled transmission line, based on analytical estimations described in [2, 4, 6, 9] and 3rd order Lagrange interpolation in the Chebyshev nodes. In is a two-stages implementation:

A. Initialization:

The initialization stage is executed at the startup of the program and calls a numerical field solver to calculate the reference point, which then stores in the computer memory as polynomial coefficients. The initialization stage differs for one-dimensional functions (figure 5.a) and for two-dimensional functions (figure 5.b)

For each parameter that the program must calculate, a different initialization stage is involved and a different set of coefficients is stored.

B. On-line calculations:

The on-line calculations are done each time one of the electrical parameters of a single or coupled transmission lines must be determined, and also differs for one-dimensional functions (figure 6.a) and two-dimensional functions (figure 6.b).

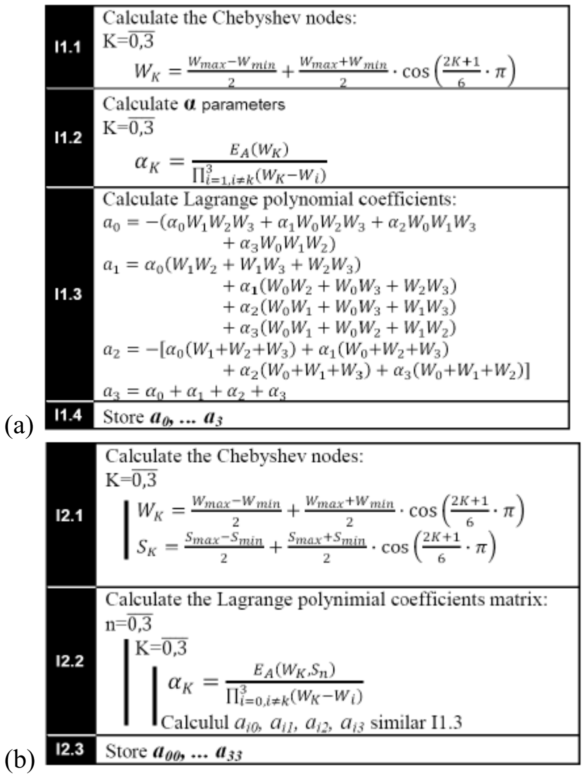


Fig. 5 SEA initialization stage: (a) for one-dimensional functions (b) for two-dimensional functions

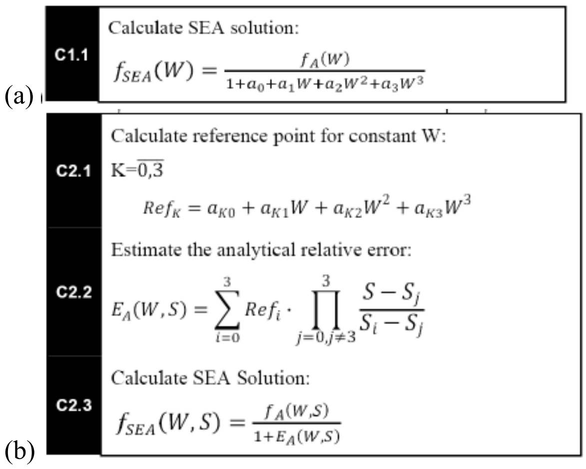


Fig. 6 SEA calculations stage: (a) for one-dimensional functions (b) for two-dimensional functions

The SEA solutions are calculated only for the primary parameters of the transmission lines: Z0 and TD for single lines, Z0odd, TDodd, Z0even and TDeven for coupled transmission lines. For all the other parameters, the equations (10)-(15) are used.

The implementation was done in Visual Basic and resulted in a dynamic link library (DLL file) which contains all the data structure and calculation functions required, so that the SEA method may be implemented in any CAD program. For numerical calculations of the reference points the "SI8000" tool from Polar Instruments was accessed programmatically, which is a 2D field solver for transmission lines calculations.

VI RESULTS AND CONCLUSIONS

Since the on-line calculations involved by the SEA method involves only low-order polynomial evaluations, the computational cost of its implementation is not an issue worth discussing. The accuracy of the solutions is the most important qualitative indicator, and in order to evaluate it we conducted a statistical analysis. We compared SEA results with field-solver results for 500 random generated values for W,T, ε_r, H and S, and used as indicator the confidence level curve, as illustrated in figure 7 for the characteristic impedance of microstrip lines.

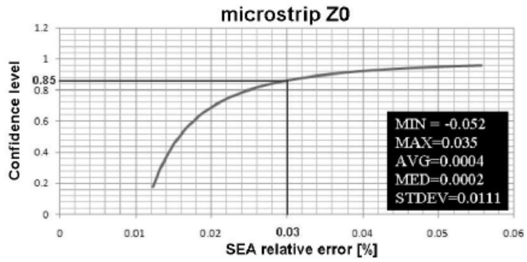


Fig. 7 Confidence level for microstrip Z0

A confidence level of 0.85 for the relative error of 0.03% means that a relative error smaller than 0.03% will be obtained with a probability of 0.85 (or alternatively 85%). Figure 7 also presents the minimum, maximum and average relative errors introduced by SEA, and also the median and standard deviation. Figure 8 presents the confidence level for the other primary parameters of single transmission lines.

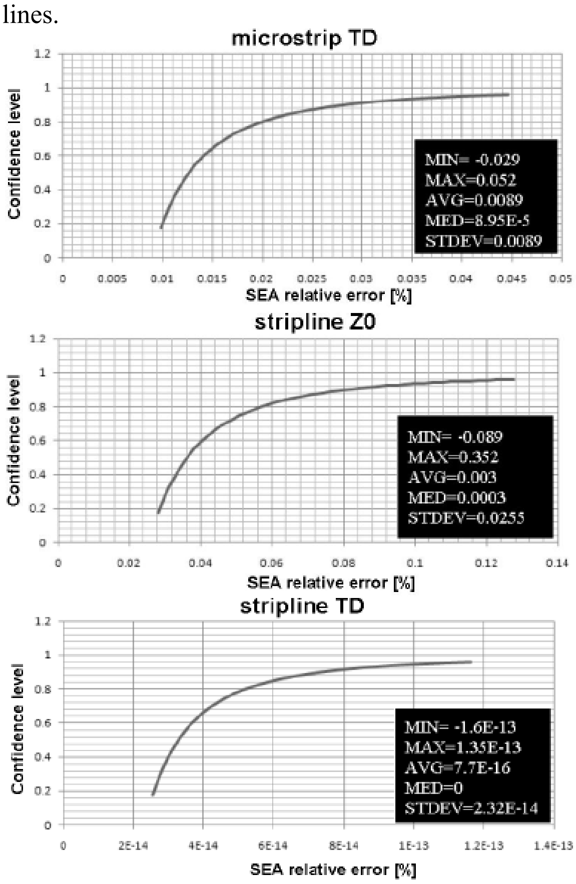
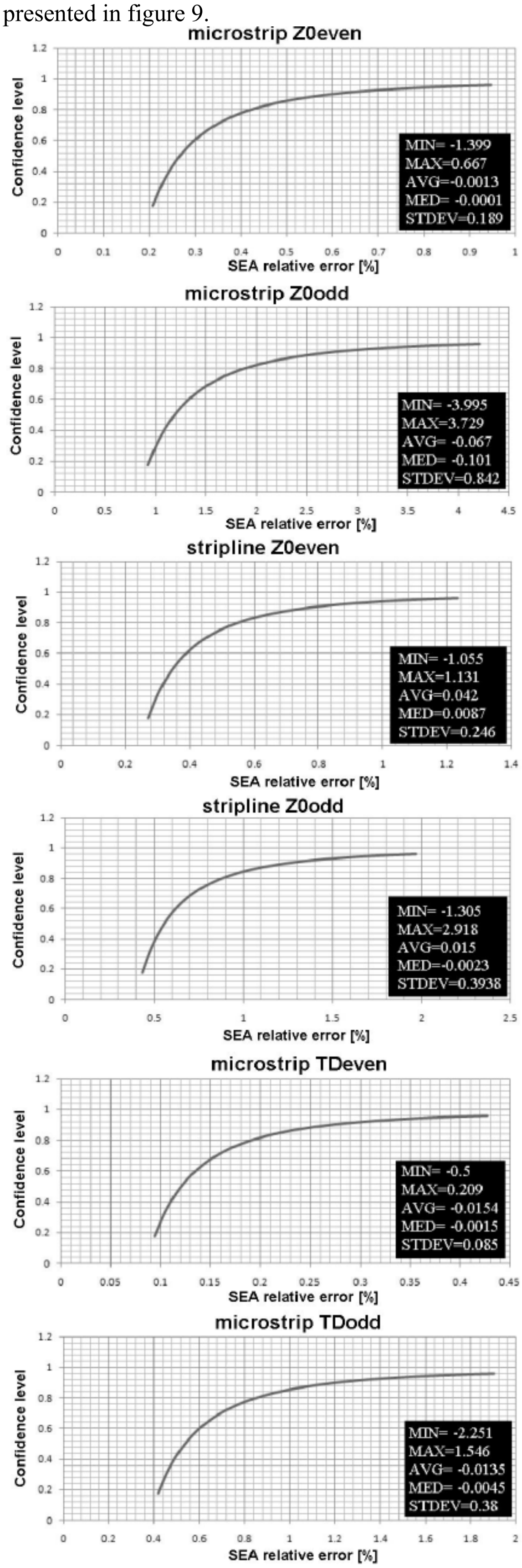


Fig. 8 Confidence level for single transmission line parameters:

For single-line electrical parameters only one dimensional interpolation is required, so the confidence levels are very high, with relative errors below 0.1% having a probability higher than 97%. The results for coupled transmission lines are presented in figure 9.



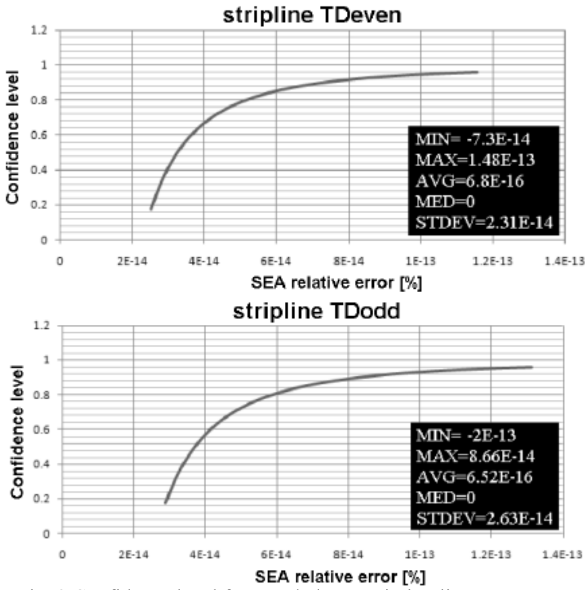


Fig. 8 Confidence level for coupled transmission line parameters

For coupled transmission lines the confidence level of the SEA method is not as high as for single lines, partly because the initial analytical solution is less accurate and partly because of the two-dimensional Lagrange interpolation. As a general conclusion, it can be stated that the SEA method can provide much more accurate solutions for on-line electrical characterization of printed circuit board, as compared with analytical approximations, with a very low computational overhead. The main goal of the proposed method is to enable high-accuracy signal integrity analysis during the PCB design process, where it is mostly needed.

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