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Real Single Tone Frequency Estimation by PHD and Filtering

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Abstract – In this paper we propose a single real tone frequency estimator for the case of low signal-to-noise ratios. The starting point is the Pisarenko Harmonic Decomposition method due to its moderate complexity, which is combined with filtering of the data sequence in order to increase the signal-to-noise ratio. We provide results of computer experiments that support the proposed algorithm.
Keywords: **frequency estimation, Pisarenko Harmonic Decomposition, filtering**

I. INTRODUCTION

The Pisarenko Harmonic Decomposition (PHD) is a well known frequency estimation method [1..4], a signal processing problem relevant in many fields of modern electrical engineering (see [9] in this issue for further comments).

For signals with only one sinusoid embedded in noise, the highest peak in the DFT-based periodogram provides the maximum likelihood (ML) parameter estimates (lowest estimate variance). However, the cumbersome calculations involved rules out the ML from practical estimators.

Many authors studied the statistical properties of the Pisarenko estimator for multiple- or single-frequency signals, in order to show its asymptotical unbiasedness and to calculate its variance [5..7]. One of its important features is the moderate computational complexity.

Motivated by this and by another work where we have studied the improvement of a known single frequency estimation by bandpass filtering the data sequence [8], in this paper we tested the performances of a combination of PHD and filtering to signals embedded in white noise at low signal-to-noise ratios (*SNR*'s), when simple estimators like the PHD do not give useful results. Namely, we applied the PHD to the initial data sequence in order to find an initial estimate, and used this estimate as central frequency for a bandpass, noise-rejection filtering of the data sequence. Then, we used again the PHD on the filtered data in order to find a better estimate. We applied several times the filtering - PHD combination until no significant improvement occurred.

In the next section we briefly review the theory, and in Section III we present computer experiments results. Conclusions are drawn in the last section.

II. THEORY

We consider a sinusoid embedded in white noise:

$$x(n) = \alpha\cos(\omega_0 n + \varphi) + q(n)\,, \quad n = 1, 2, ..., N \qquad (1)$$

where the amplitude $\alpha>0$, (angular) frequency $\omega_0 \in (0, \pi)$ and phase $\varphi \in [0, 2\pi)$ are deterministic but unknown constants. The noise $q(n)$ is white and Gaussian, uncorrelated with the signal, with zero-mean and unknown variance σ^2.

The signal-to-noise ratio is $SNR = \alpha^2/(2\sigma^2)$.

The PHD estimate for a single real tone is given by [6]

$$\hat{\omega} = \cos^{-1}\left(\frac{r_2 + \sqrt{r_2^2 + 8r_1^2}}{4r_1}\right) \qquad (2)$$

where r_k is the sample covariance, and it is calculated with

$$r_k = \frac{1}{N-k}\sum_{n=1}^{N-k} x(n)x(n+k)\,, \quad k=1, 2. \qquad (3)$$

We propose the following algorithm for estimating the frequency. First, we use the PHD applied to the input signal $x(n)$ for obtaining an initial estimate $\hat{\omega}_0$ of the frequency ω_0. The frequency estimate is used to filter the signal $x(n)$ with a selective filter centered at $\hat{\omega}_0$. We have considered the second-order noise rejection filter with the following transfer function

$$H(z) = \frac{z^2}{z^2 - 2\rho\cos(\omega_r)z + \rho^2}\,, \qquad (4)$$

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where ρ is a parameter smaller but close to unity whose value is chosen experimentally and ω_r is the central frequency. The purpose of filtering the input signal $x(n)$ is to increase the signal-to-noise ratio [8,9]. Although after filtering the noise in no more white and consequently does not fit into the PHD theory, the application of this estimation procedure to the filtered data sequence leads to a better estimate of the frequency (in the sense of a smaller mean square error), as computers experiments have shown. Then, we use this new estimate as ω_r in (4) and repeat the two steps until the difference between the mean square errors of two successive estimates becomes insignificant.

III. EXPERIMENTAL RESULTS AND COMPLEXITY ANALYSIS

Several experiments have been performed on the procedure described in the last section using (1) with $\alpha = \sqrt{2}$ and $\varphi = 0$. In order to prove its effectiveness, we present two results at different signal-to-noise ratios. Both results are averages of 1000 independent runs.

In Fig. 1 the results are for *SNR*=3 dB, *N*=100 and ρ=0.9, while in Fig. 2 they are for *SNR*=0 dB, *N*=200 and ρ=0.9. The larger value for *N* in the second case is motivated by the lower value of the *SNR*.

The Cramer-Rao lower bound is also represented for reference purposes [4]. The highest mean square error corresponds to the PHD. An important improvement is obtained after the application of a first combination filtering – PHD and a further improvement by repeating the procedure *p* times.

Experiments show that a larger value of *p* than 5 does not improve the results significantly. A favorable counterpart of the small value of *p* is the limitation of the complexity of the algorithm.

The estimator variance depends on frequency, a property inherited from the PHD [6].

In order to evaluate the complexity of the proposed algorithm, we note that one PHD requires 2*N*-3 real additions, 2*N*+1 real multiplications and 5 special operations (one square root, one division, one inverse of the cosine function, one multiplication by 8 and

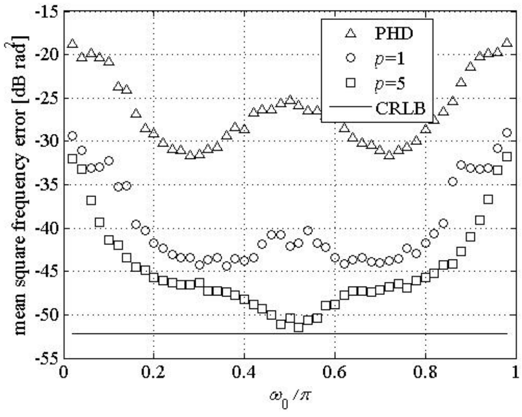

Fig.1 Mean square frequency errors versus frequency for *SNR*=3 dB, *N*=100, ρ=0.9.

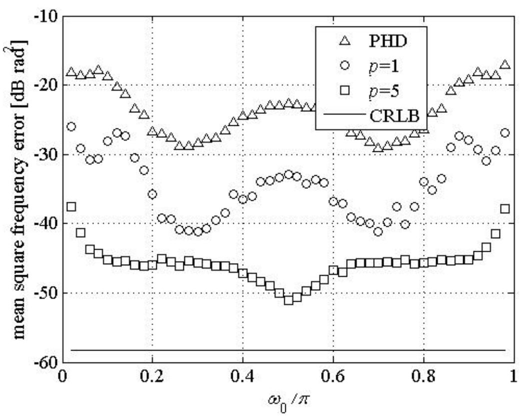

Fig.2 Mean square frequency errors versus frequency for *SNR*=0 dB, *N*=200, ρ=0.9.

one multiplication by 4), while filtering involves 2*N*-3 real additions and 2*N*-2 real multiplications (ρ^2 and 2ρ can be calculated off-line and stored, while $\cos(\hat{\omega}_0)$ is provided by the PHD). If filtering is applied *k* times, then the PHD is applied *k*+1 times. We get a total of (2*N*-3)(2*k*+1) real additions, 2*N*(2*k*+1)-2*k*+1 real multiplications and 5(*k*+1) special operations. As we have shown that, for efficiency reasons, *k* has to be taken much smaller than *N*, the overall complexity is of the order of *N* (i.e. *0*(*N*)), like the PHD itself.

IV. CONCLUSIONS

We have proposed an algorithm for the estimation of the frequency of a real sinusoid embedded in white noise. We started from a well known, moderate complexity algorithm, the Pisarenko Harmonic Decomposition and performed repeated bandpass filtering followed by PHD frequency estimation. We have shown experimentally that our algorithm behaves well at low signal-to-noise ratios, when the PHD itself does not give good results. Furthermore, the complexity of the proposed algorithm is of the order of the input data length.

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