

OTA-C BIQUAD CELLS EMULATION OF LC
LADDER FILTERSAndrei Câmpeanu and János Gal¹

Abstract – The paper presents a method to design continuous-time filter based on the node-voltage simulation of LC ladder structures. The node voltages are used as variables to generate transfer functions. The OTA-C biquad cells are realized in distributed-feedback configuration and use node current injection to permit multiple input action. Both all-pole and finite-transmission-zeros LC low-pass and high-pass filters are implemented. Applying standard low pass to bandpass frequency transformation to OTA-C circuits capacitors, bandpass and bandstop filters are easily obtained from their low-pass or high-pass active filters prototypes.

Keywords: filter synthesis, OTA-C biquads

I. INTRODUCTION

Passive double loaded ladder networks are commonly used as prototypes for active filters because their low sensitivity is preserved by active simulations. Nevertheless, the well known „leap-frog” active filter synthesis technique, based on the simulation of current-voltage relationships existing in a LC-ladder prototype [1], [2] do not solves satisfactorily the simulation of finite-zeros filters [3], the resulting circuits in these cases having usually floating capacitors which are responsible for parasitic bottom-plate capacitances and undesirable non-observable poles.

An alternative to classical simulation method proposed in [4], uses the mesh currents description of the passive circuit. This approach uses coupled-biquad filter cells to implement mesh equations of the passive network. In order to implement finite-zeros passive filters simulation by the mesh currents method in circuits consisting only from operational amplifiers with grounded inputs, capacitors and resistors, we used succesfully in [5], a universal multiple-inputs biquad cell based on the well-known Tow-Thomas structure. In this case, each biquad cell implements the currents equations that take place in one of circuit meshes.

The aim of the present paper is to extend the area of coupled-biquad method to OTA-C circuits. We investigate also the use of the node-voltage description of the passive ladder network as an equivalent alternative to the mesh current simulation

method. With that end in view, an important effort was done to establish an optimal OTA-C biquad cell that suits best to coupled-biquad applications. Therefore, the OTA-C biquads must permit multiple inputs, contain only grounded capacitors and realize all types of filtering functions. These requirements can be fulfilled only by connecting three or four additional OTAs to the three OTA basic biquad circuit [3], [6]. This does not represent a major overhead, since in integrated circuits, OTAs can be fabricated very efficiently and economically.

II. SYNTHESIS PROCEDURE

Consider a general RLC ladder network with n nodes, as shown in Fig. 1. The work of the passive network can be described using the voltages in the nodes of the network: V_1, V_2, \dots, V_n . The most general form of the equation that can be written for any node of the network is

$$V_k = \frac{I_k}{Y_k^n} + \frac{Y_{2k-2}}{Y_k^n} V_{k-1} + \frac{Y_{2k}}{Y_k^n} V_{k+1}, \quad k = 1, 2, \dots, n \quad (1)$$

where Y_k^n is the total admittance connected to node k of the network ($Y_k^n = Y_{2k-2} + Y_{2k-1} + Y_{2k}$). I_k represents the value of current sources connected to the node. Only for $k=1$, $I_1 \neq 0$ and $V_0 = 0$. In the last node of the network: $V_{k+1} = 0$, $k = n$.

According to eq. (1), the ladder filter network can be implemented using RC active circuits cells with multiple inputs. Moreover, as the denominators of all the terms on the right side of (1) are identical, Y_k^n , a unique active circuit structure with multiple inputs can be used to implement the desired functions. Depending on the position of the node in the RLC

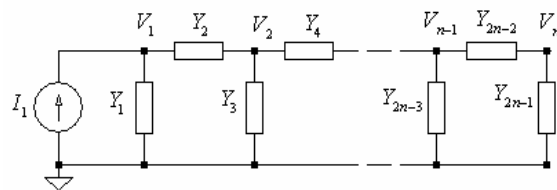


Fig. 1 Using node voltages to describe the work of a RLC ladder network

¹ Facultatea de Electronică și Telecomunicații, Departamentul Comunicații Bd. V. Pârvan Nr. 2, 300223 Timișoara, e-mail andrei.campeanu@etc.upt.ro, janos.gal@etc.upt.ro

network, these functions are first or second order RC active circuits transfer functions. Referring to node k , we will denote the corresponding transfer functions in eq. (1) as

$$T_{ik}(s) = \frac{1}{Y_k^n}; T_{k-1,k}(s) = \frac{Y_{2k-2}}{Y_k^n}; T_{k+1,k}(s) = \frac{Y_{2k}}{Y_k^n} \quad (2)$$

Consequently, an active filter cell will realize the equation

$$V_k = T_{ik}I_k + T_{k-1,k}(s)V_{k-1} + T_{k+1,k}V_{k+1}, \quad k=1, 2, \dots, n \quad (3)$$

There are two forms of RLC ladder filter structures: T-shaped and π -shaped circuits. Both of them can be implemented by this method, but it is normal to choose for simulation the most efficient one, vewing that the emulation of first-order transfer functions uses almost the same number of OTAs as second-order function cells. From this point of view, π -shaped circuits are the choice, because, with the exception of the last node in an even-order filter, all transfer functions in (3) are second-order rational functions.

III. BIQUAD OTA-C FILTER BUILDING BLOCKS

First and second-order filter cells are needed to implement eq. (3). Since the solutions for first-order sections are obvious, we will focus on second-order solutions used in our approach. A convenient OTA-C biquad filter cell is a two-inputs device able to perform second-order functions with identical poles but different zeros regarding the inputs. Multiple inputs is obtained by the technique of node current injection [3], which apply the input voltages through extra single-ended input OTAs to some circuit nodes. The OTA-C biquad itself has not the simplest structure, vewing the necessity of an inpedent tuning of parameters ω_o and Q . We choosed a distributed-feedback (DF) two-integrator loop structure with four OTAs and two grounded capacitors [3].

A. Finite-Q OTA-C Biquad Cell

These cells are used to emulate network voltage equations for nodes which total admittance Y_k^n

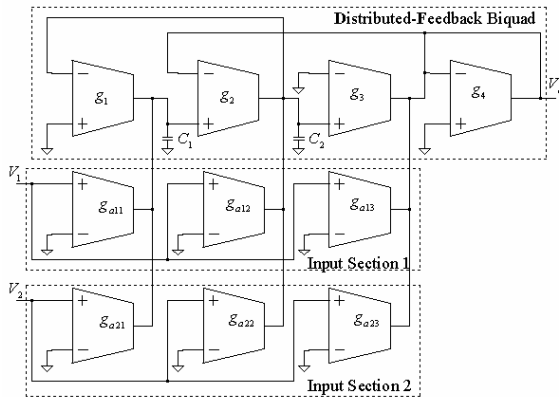


Fig. 2 Double-input finite-Q OTA-C biquad cell used in coupled-biquad filter implementation

includes a real part. It's the case in a double terminated RLC ladder filter for the first and the last node of the network. The complete circuit is shown in Fig. 2. Choosing for convenience $C_1 = C_2 = C$ and $g_1 = g_2 = g$, the transfer function of the circuit is given by

$$V_o = \sum_i^2 \frac{\pm \frac{g_{ai3}C^2}{g^2g_4}s^2 \pm \frac{g_{ai2}g_3C}{g^2g_4}s \pm \frac{g_{ai1}g_3 \pm g_{ai3}}{gg_4}}{\frac{C^2}{g^2}s^2 + \frac{g_3C}{g_4g}s + 1} V_i \quad (4)$$

Depending on the node voltage equation that is emulated, the circuit can realize any kind of second order relationship established at the terminal nodes of a low-pass or high-pass double-terminated passive ladder filter. The sign \pm simply means both non-inverting and inverting inputs of OTAs in the *Input Sections* can be used, depending on application. Not all OTAs in the *Input Sections* are used simultaneously. The shape of the numerator of the transfer function in (4) dictates which OTA remains, $g_{aij} \neq 0$, and which OTA is removed, $g_{aij} = 0$. It is seen that the circuit can realize a low-pass filter if $g_{ai2} = g_{ai3} = 0$ and a bandpass filter for $g_{ai1} = g_{ai3} = 0$. The single shortcoming of this structure is the coefficient difference matching needed to realize a high-pass filter: $g_{ai1} = -(g_4/g_3)g_{ai3}$, $g_{ai2} = 0$. In defiance of this, any filter characteristics can be realized, making the architecture the best suited for our application.

B. Infinite-Q OTA-C Biquad Cell

For network nodes to which are connected only reactances, the corresponding OTA-C biquad cells exhibit infinite-Q second order transfer functions. It is a situation met in all internal nodes of an RLC ladder filter. Though, it were possible to use the circuit in Fig. 2 by removing the feedback between g_4 and g_2 , we preferred to eliminate the drawback of this structure which we talk about before, by using the circuit shown in Fig. 3.

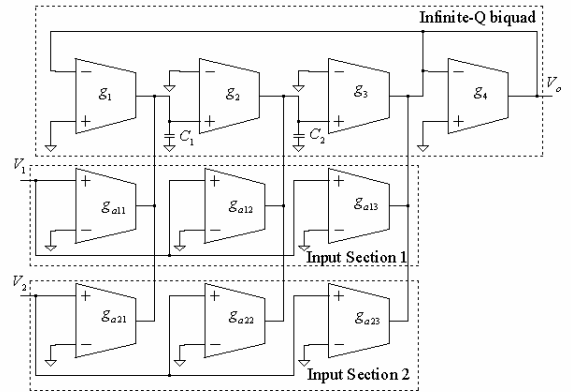


Fig. 3 Double-input infinite-Q OTA-C biquad cell used in coupled-biquad filter implementation

To write the transfer function, the same simplifications like in (4) will be made: $C_1 = C_2 = C$, $C_1 = C_2 = C$. Now, the transfer function of the infinite-Q OTA-C biquad cell is given by

$$V_o = \sum_i \frac{\pm \frac{g_{ai3} C^2}{g^2 g_3} s^2 \pm \frac{g_{ai2} C}{g^2} s \pm \frac{g_{ai1}}{g}}{\frac{g_3 C^2}{g^2 g_4} s^2 + 1} V_i \quad (5)$$

There are no more coefficient difference matching involved, thus the architecture is insensitive to variations in transconductances values. As in previous case, not all OTAs in the *Input Sections* are used simultaneously, and the change of sign in the terms from the numerator simply means to pass from a non-inverting input to an inverting input of OTAs in the *Input Sections*. More simplifications are obtained if simple low-pass and bandpass characteristics must be implemented. It's the case of low-pass all-pole filters, where the four OTAs biquad structure, g_1, g_2, g_3, g_4 is reduced to g_1, g_2 .

IV. LOW-PASS AND HIGH-PASS FILTERS DESIGN EXAMPLES

The emulation of RLC ladder filter networks by the node-voltage simulation method consists in an adequate coupling of biquad OTA-C cells introduced in the previous Section. Viewing the large number of OTAs involved by the use of cells in Fig. 2 and Fig. 3, it is obvious that OTA-C coupled-biquad implementation represents an alternative only in those areas where the „traditional” leap-frog method do not perform well: finite-zeros passive ladder filters emulation. The complexity of a filter synthesis depends on the choice of the prototype network and, as we pointed before, π -shaped filter networks are best suited to our purposes.

The design procedure of an OTA-C filter based on the node-voltage emulation method starts by the selection of an appropriate RLC ladder prototype. Next, a description of passive network work is made by its set of node-voltage equations. Then, each equation is replaced by one of OTA-C biquad cells from the previous Section, or by a first-order OTA-C filter. Drawing a parallel between the passive network equations and the transfer functions of OTA-C filter

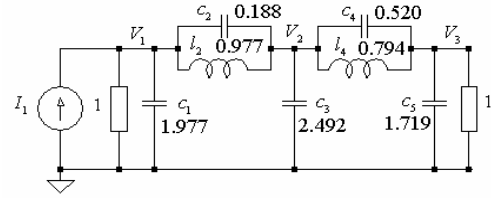


Fig. 4 Fifth-order elliptic low-pass passive ladder filter used as prototype in node-voltage emulation method.

cells, the values of OTAs transconductances and capacitances are computed. Finally, the neighbouring filter cells are connected in accordance with passive filter equations.

To illustrate the node-voltage approach, a fifth-order low-pass elliptic passive filter prototype will be used to obtain equivalent OTA-C coupled-biquad filters. The LC ladder prototype shown in Fig. 4 has 1dB ripple in the passband and minimum 55db attenuation in the stopband. Node-voltage analysis leads to the following system of equations:

$$\begin{aligned} V_1 &= T_{01}I_1 + T_{21}V_2 = \frac{sl_2I_1 + (s^2c_2l_2 + 1)V_2}{s^2(c_1 + c_2)l_2 + sl_2 + 1} \\ V_2 &= T_{12}V_1 + T_{32}V_3 \\ &= \frac{\left(s^2c_2 \frac{l_2l_4}{l_2 + l_4} + \frac{l_4}{l_2 + l_4}\right)V_1 + \left(s^2c_2 \frac{l_2l_4}{l_2 + l_4} + \frac{l_2}{l_2 + l_4}\right)V_3}{s^2(c_2 + c_3 + c_4) \frac{l_2l_4}{l_2 + l_4} + 1} \quad (6) \\ V_3 &= T_{23}V_2 = \frac{(s^2c_4l_4 + 1)V_2}{s^2(c_4 + c_5)l_4 + sl_4 + 1} \end{aligned}$$

Each pair of these transfer functions (f.i. T_{01} and T_{21}) have the same poles but different zeros. This enables the realization of each equation in (6) by one OTA-C

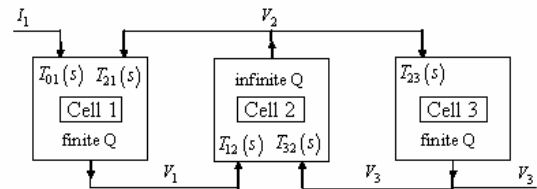


Fig. 5 Block diagram of the node-voltage coupled-biquad active filter emulation of ladder filter in Fig. 4. The various transfer functions are given in the text.

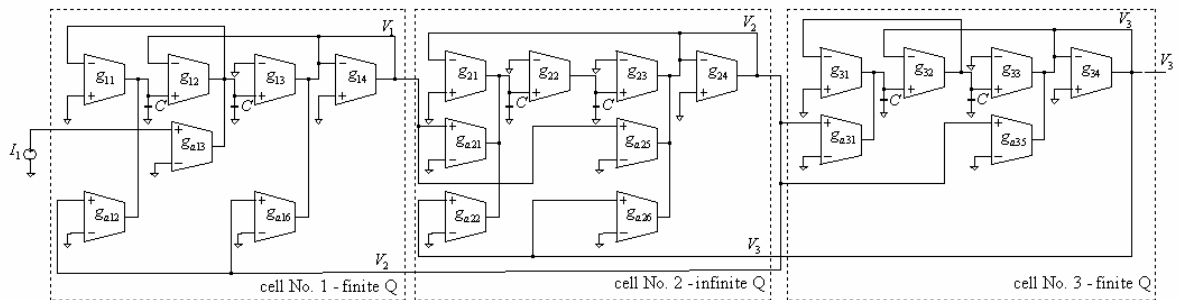


Fig. 6 The OTA-C realization of fifth-order elliptic low-pass ladder prototype filter in Fig. 4.

biquad circuit cell.. The result is that for the fifth-order filter under discussion, three OTA-C coupled biquad cells are used for a total of 21 OTAs. The block diagram of the circuit is given in Fig. 5 and a complete schematic of OTA-C coupled biquad filter is shown in Fig. 6. Denoting the cut-off frequency of the active low-pass filter by ω_o and choosing C as the value of the capacitors, Table 1 gives the design relationships for the first two cells of the filter.

As a matter of fact, the design of a high-pass filter equivalent to the prototype in Fig. 4 is very simple, and leads after the standard frequency transformation and the use of the design procedure developed in this Section to a circuit quite identical with the low-pass structure shown in Fig.6. Table 1 presents the design equations of first two cells of a fifth-order elliptic OTA-C coupled-biquad high-pass filter. The negative sign used in the table for the transconductance of g_{a2} OTA means that signals V_2 and V_3 are applied on the inverting inputs of g_{a12} respectively g_{a31} OTAs instead of their non-inverting inputs. It is the single difference between the low-pass implementation in Fig. 6 and the high-pass realization.

To verify the method described above, SPICE simulations were performed. The fifth-order low-pass elliptic passive filter prototype was used to design the circuit in Fig. 6 for low-pass and high-pass filters. Here, in Fig. 7 we only give the simulated result of a high-pass OTA-C circuit having the cut-off frequency of 10kHz.

V. CONCLUSIONS

The node-voltage emulation method reveals as a powerful method of simulation of pasive ladder filters by active filters, being equivalent from the point of view of performances with the mesh current approach.

Table 1 Design relationships used for cells 1 and 2 in low-pass (LP) respectively high-pass (HP) OTA-C filter realizations of the coupled-biquad schematic of Fig. 6.

OTA' s g_m	Cell No. 1 – finite Q		Cell No. 2 – infinite Q	
	LP Filter	HP Filter	LP Filter	HP Filter
g_1	$g_{11L} = \omega_o C / \sqrt{(c_1 + c_2)} l_2$	$g_{11H} = \omega_o C \sqrt{(c_1 + c_2)} l_2$	$g_{21L} = \omega_o C / \sqrt{(c_2 + c_3 + c_4)} \frac{l_2 l_4}{l_2 + l_4}$	$g_{21H} = \omega_o C \sqrt{(c_2 + c_3 + c_4)} \frac{l_2 l_4}{l_2 + l_4}$
g_2	g_{11L}	g_{11H}	g_{21L}	g_{21H}
g_3	$\sqrt{l_2 / (c_1 + c_2)} g_{11L}$	$\sqrt{l_2 / (c_1 + c_2)} g_{11H}$	g_{21L}	g_{21H}
g_4	g_{11L}	g_{11H}	g_{21L}	g_{21H}
g_{a1}	-	-	$l_4 / (l_2 + l_4) g_{21L}$	$c_2 / (c_2 + c_3 + c_4) g_{21H}$
g_{a2}	$c_1 / \sqrt{(c_1 + c_2)} l_2 g_{11L}$	$-c_1 / \sqrt{(c_1 + c_2)} l_2 g_{11H}$	$l_2 / (l_2 + l_4) g_{21L}$	$c_4 / (c_2 + c_3 + c_4) g_{21H}$
g_{a3}	g_{11L}	g_{11H}	-	-
g_{a4}	-	-	-	-
g_{a5}	-	-	$c_2 / (c_2 + c_3 + c_4) g_{21L}$	$l_4 / (l_2 + l_4) g_{21H}$
g_{a6}	$c_2 / (c_1 + c_2) g_{11L}$	g_{11H}	$c_4 / (c_2 + c_3 + c_4) g_{21L}$	$l_2 / (l_2 + l_4) g_{21H}$

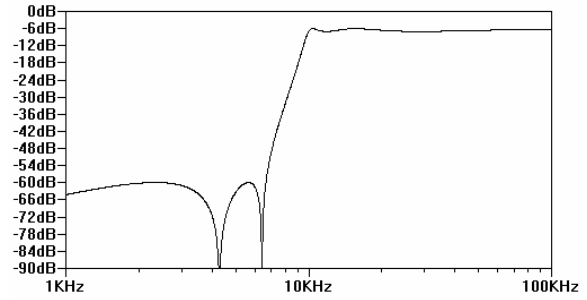


Fig. 7 Simulated magnitude response of high-pass OTA-C coupled-biquad implementation of low-pass prototype of Fig. 4

The method implements the passive network by the interconnection of multiple inputs biquad cells. The paper proposes two general OTA-C biquad cells that suits well in all cases of low-pass and high-pass network filter implementations. Fifth order low-pass and high-pass coupled biquad filters were designed and simulated.

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