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A Combined Fusion–Diffusion Approach for Image Filtering and Enhancement

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Abstract – We propose a new method for image filtering and enhancement based on the use of partial differential equations (PDE) framework and of image fusion techniques. The restoration process is defined iteratively as a succession of diffusion and fusion steps; in each step the degraded image is first independently processed using a directional diffusion PDE tuned to different sets of parameters and then the results are combined through fusion. Fusion takes place at an intermediate results level through a weighted averaging/selection fusion rule. An experimental setup involving both synthetic and real images is used to illustrate the increased efficiency of the method for noise filtering with small scale detail, edge and junction preservation.

Keywords: diffusion, orientation, fusion, restoration.

I. INTRODUCTION

The use of partial derivatives equations (PDE) in image restoration witnessed and exponential growth in these past years. Their main advantages - increased precision due to their local or semi-local formulation, nonlinearity embedded naturally in the formulation of a PDE based filter, the strong anisotropic behavior, the coexistence of smoothing and enhancement processes acting in different directions, imposed this type of approaches as powerful alternatives to other image processing techniques for image restoration, enhancement or segmentation. The major contributions related to this research area are due to the works of Perona and Malik or Catté et al. on anisotropic diffusion filtering, Alvarez et al. on mean curvature motion like filters [1], Osher and Rudin on PDE shock filters [6], Weickert et. al. on tensor-driven diffusion processes [18], [19] or Tshumperlé and Deriche in trace based formulations of PDE driven smoothing processes [15], [16]. A brief review of the domain is presented in Section II, for a more detailed description [9], [20] and the references therein can be consulted.

The quality of the results obtained using a PDE filter is strongly influenced by the particular choice of its set of parameters and, for different choices, the output of the filter can be set to retain patterns and objects existing only at a given scale. We address this issue by proposing a method that uses fusion techniques performed at an intermediate results level for combining the results of PDE based filters in an iterative manner. The input image is processed for a pre-established number of iterations with a given PDE filter tuned to different sets of parameters - one for keeping only large scale patterns and a second one for the preservation of small scale details - then a fusion step is performed in the transform domain for injecting only pertinent information in both evolving images. By modifying iteratively the initial values of the PDEs and using an appropriate fusion rule we design a new method with an increased efficiency in edge, junction and small scale details preservation, coupled with good noise filtering capability. We are developing our method using a fusion-diffusion framework that we previously proposed in [14].

The paper is organized as follows: Section II is devoted to a brief review of the PDE based approaches for image filtering, Section III introduces our new method and Section IV includes experimental results for both synthetic, computer generated, and real images. Concluding remarks and future work directions are given in the last section.

II. DIFFUSION TECHNIQUES FOR IMAGE FILTERING AND ENHANCEMENT

Classically anisotropic diffusion techniques are modeling the image filtering process through some divergence based PDE that relays the time and spatial partial derivatives of a gray level intensity image *U(x,y,t)* [7]:

$$\frac{\partial U}{\partial t} = div[\, g(|\nabla U(x,y,t)|)\nabla U \,] \qquad (1)$$

The diffusivity function *g(.)* controls an anisotropic smoothing process induced by the PDE. A typical choice for the diffusion function is:

$$g(|\nabla U(x,y,t)|) = \frac{1}{1+\left(\frac{|\nabla U(x,y,t)|}{K}\right)^2} \qquad (2)$$

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and it relates, for each pixel of the input image, the norm of the associated gradient vector with a diffusion threshold parameter K. Equation (1) can be also expressed in terms of a directional formulation that greatly eases its interpretation:

$$\begin{cases} \frac{\partial U}{\partial t} = c_u U_{uu} + c_v U_{vv} \\ c_u = g(|\nabla U|) \\ c_v = g|\nabla U| + |\nabla U| g'(|\nabla U|) \end{cases} \quad (3)$$

In (3) $\vec{u}$ and $\vec{v}$ are denoting unitary vectors oriented along the patterns directions and, respectively orthogonal to edges.
Along with the solution time t, the diffusion threshold K acts as a scale parameter; a particular choice for K sets the edges that will be kept or even enhanced in the output of the filter. The edge enhancement process is due to the negative value of the diffusion coefficient in the $\vec{v}$ direction corrresponding to gradient vector norms greater than K [10]; on the orthogonal direction only smoothing takes place since the diffusion coefficient is always positive [7], [3].
Meanwhile the anisotropic behavior of equation (1) is governed by a scalar diffusivity function, matrix like diffusion functions can be also used for a more efficient and true separation of the filter behavior along the diffusion directions.
Matrix or tensor- driven diffusion is closely related to the work of Weickert in scale space analysis [18], [19]. The basic idea behind this class of filters is to steer the diffusion process along the eigenvectors of some diffusion matrix (a 2 x 2 square matrix for gray level images). A representative PDE filter for this type of approaches is the coherence-enhancing filter (*CED)* [18], [19]. Starting from the classical structure tensor [8]:

$$J_\rho(\nabla U_\sigma) = \begin{pmatrix} G_\rho * (\frac{\partial U_\sigma}{\partial x})^2 & G_\rho * \frac{\partial U_\sigma}{\partial x}\frac{\partial U_\sigma}{\partial y} \\ G_\rho * \frac{\partial U_\sigma}{\partial x}\frac{\partial U_\sigma}{\partial y} & G_\rho * (\frac{\partial U_\sigma}{\partial y})^2 \end{pmatrix} \quad (4)$$

obtained by a point wise Gaussian convolution of the smoothed image derivatives, a diffusion tensor is built by using an eigenvector like decomposition:

$$D = (\vec{\eta} \mid \vec{\xi}) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \vec{\eta}^T \\ \vec{\xi}^T \end{pmatrix} \quad (5)$$

The diffusion tensor possesses the same eigenvectors $\vec{\eta}, \vec{\xi}$ as the structure tensor. The two vectors are robust estimates of the mean orientation of the structures ($\vec{\xi}$) and of the orthogonal directions ($\vec{\eta}$), computed at a semi-local scale ρ. Strong anisotropic behavior (e.g. smoothing actions mainly only along edges) is achieved by modifying the eigenvalues of the structure tensor ($\lambda_{1,2}$) with:

$$\begin{cases} \mu = \lambda_1 - \lambda_2 \\ \lambda_1 \to \alpha \\ \lambda_2 \to f(\mu) = \alpha + (1-\alpha)\exp(-C/\mu^2) \end{cases} \quad (6)$$

The constant α is typically chosen equal to *0.001* whereas $f(\mu)$ is a function depending on a coherence measure μ, defined to be the difference between the eigenvalues of (4) (λ_1, λ_2). This filter is extremely efficient in processing unidirectional patterns; however it introduces strong topological modifications of the input image nearby high curvature regions (e.g. junctions and corners).
This last aspect was addressed by the authors of [15] when they proposed a series of PDE based filters more suited for handling junctions and corners. Their initial formulation of a PDE based filter corresponds to the following equation:

$$\frac{\partial U}{\partial t} = trace\left(\left(\vec{\eta}\vec{\eta}^T \frac{1}{(\lambda_1+\lambda_2+1)^{p_1}} + \vec{\xi}\vec{\xi}^T \frac{1}{(\lambda_1+\lambda_2+1)^{p_2}}\right)\begin{pmatrix} U_{xx} & U_{xy} \\ U_{xy} & U_{yy} \end{pmatrix}\right) \quad (7)$$

Similarly to (1), the behavior of (7) can be more easily understood if it is put in terms of directional derivatives:

$$\frac{\partial U}{\partial t} = \frac{1}{(\lambda_1+\lambda_2+1)^{p_1}} U_{\eta\eta} + \frac{1}{(\lambda_1+\lambda_2+1)^{p_2}} U_{\xi\xi} \quad (8)$$

(7) can be used to some extent for limiting topological modifications of the input image; for a more precise junction preservation is desired the equation was later supplemented by a curvature preserving term. Using the notation:

$$T = \vec{\eta}\vec{\eta}^T \frac{1}{(\lambda_1+\lambda_2+1)^{p_1}} + \vec{\xi}\vec{\xi}^T \frac{1}{(\lambda_1+\lambda_2+1)^{p_2}}, H = \begin{pmatrix} U_{xx} & U_{xy} \\ U_{xy} & U_{yy} \end{pmatrix} \quad (9)$$

the new formulation of this filter is [16]:

$$\frac{\partial U}{\partial t} = trace[TH] + \frac{2}{\pi}\nabla U^T \int_{\alpha=0}^{\pi} J_{\sqrt{T}a_\alpha} \sqrt{T} a_\alpha d\alpha \quad (10)$$

In (10) J stands for the Jacobian of a vector field defined by the product $\sqrt{T}a_\alpha$ with a_α denoting elementary orientations spanning the space [0, π] [16]. As argued by the authors such a formulation inhibits the smoothing process nearby corners and junctions.
Within the same framework of structure tensor steered diffusion we proposed in [10], [11] the following PDE:

$$\begin{cases} \frac{\partial U}{\partial t} = c_\xi U_{\xi\xi} + c_\eta U_{\eta\eta} \\ c_\xi = \frac{\partial}{\partial \xi}[g(U_\xi)U_\xi] \\ c_\eta = \frac{\partial}{\partial \eta}[g(U_\eta)U_\eta] \end{cases} \quad (11)$$

The PDE employs Perona Malik like functions for modulating the intensity of the diffusion processes along the directions computed by a structure tensor approach. Consequently, in contrast to (8), the filter allows both smoothing and enhancement actions to take place along the structures directions and on the orthogonal ones. Negative diffusion coefficients (i.e. enhancement processes) are obtained whenever the absolute values of the directional derivatives U_ξ and U_η are higher than the diffusion thresholds K_ξ and K_η defined over the whole image domain Ω[11]:

$$\begin{cases} K_\xi = K_\xi(t) = \inf\left\{ K : \dfrac{Card\left((x,y)\in\Omega / \left|\dfrac{\partial}{\partial\xi}U(x,y,t)\right| < K \right)}{Card(\Omega)} \geq \beta \right\} \\ K_\eta = \alpha K_\xi \end{cases} \quad (12)$$

with $\alpha, \beta \in [0,1]$.

Similar to the K parameter of the Perona Malik filter, a particular choice of the diffusion thresholds sets the junctions and edges that will be kept in the output image. Meanwhile for efficiently filtering out noise these parameters must be set to have high values, for small scale detail and junction preservation the parameters must have lower values. An asymmetric version of the filter has been introduced in [12] and a regularized version has been also defined in a manner similar to [3] for reducing the sensitivity of the filter with respect to noise:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial\xi}[g(U_{\sigma\xi})U_\xi] + \frac{\partial}{\partial\eta}[g(U_{\sigma\eta})U_\eta] \quad (13)$$

U_σ denotes in (13) a pre-smoothed version of the input image with a Gaussian kernel of standard deviation σ.

III. PROPOSED APPROACH

We develop our new approach using the fusion-diffusion framework we proposed in [13], [14]:

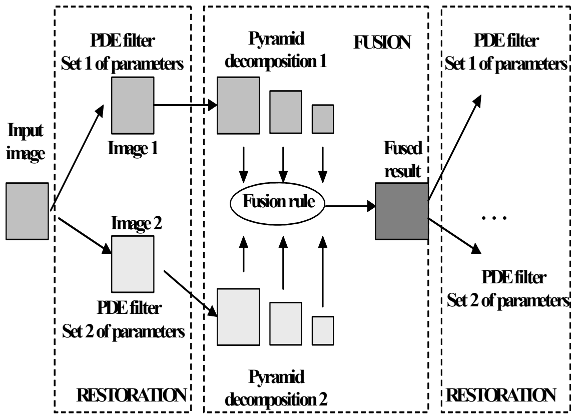


Fig. 1. Proposed approach using a diffusion-fusion framework

As the constituent diffusion filter we have choosen the PDE corresponding to equation (13), tuned to different sets of parameters. The choice is based on a previous analysis done in [10], but the method is general and can be applied to all the PDEs discussed in Section II.

Essentially, we process the same input image independently with the same filter as illustrated in Fig.2. Two different sets of parameters are used on the upper and lower branch and the intermediate results are combined through a weighted averaging /maximum selection fusion rule. By deliberately setting two different choices of parameters for the same filter we design a method that will produce intermediate outputs with different characteristics. A set of parameters corresponding to a lower diffusion threshold will lead to results in which small scale details will be kept but, for noisy images, such a choice can also lead to a less efficient noise filtering capability. On the contrary, a set of parameters corresponding to high diffusion thresholds leads to results in which noise is smoothed out efficiently but small scale details like junctions can be lost.

The image fusion framework allows us to combine these effects and we design our fusion rule to operate on a pyramid decomposition of Gaussian (G_k) and Laplacian levels (L_k) [2], [4] computed for intermediate time scales t :

$$\begin{cases} G_k = [w * G_{k-1}]_{\downarrow 2} \\ L_k = G_k - 4w * [G_{k+1}]_{\uparrow 2} \end{cases} \quad (14)$$

In (14) $\downarrow_2$ and $\uparrow_2$ are denoting respectively the down and the up sampling operators with 2 and w is a 2D smoothing kernel operating on the intermediate results.

For combining the two output images corresponding to a different set of parameters, we first express the pertinence of each result by a salience measure inspired from total variation minimization problems.

Meanwhile for the Laplacian levels of decomposition (L_k) we define an energy like measure computed over a neighborhood W of the current pixel [13]:

$$E(L_k) = \int_W |L_k| dL \quad (15)$$

for the coarsest Gaussian level G_{k+1} the salience measure is the total variation of the corresponding image computed over W:

$$TV(G_{k+1}) = \int_W |\nabla G_{k+1}| dL \quad (16)$$

When used in conjunction with edge and junction preserving filters such a measure does not penalize discontinuous solutions and is able to quantify both the edge and junction preserving properties of a PDE based smoothing/enhancement process and its noise filtering capability. Such a behavior is obtained on the Gaussian domain by explicitly setting the salience measure as in equation (16); for the Laplacian domain the gradient operator is dropped in (15) since the variation of the luminance function is quantified by the Laplacian image itself.

For analyzing the similarity between the results we employ a classical match measure defined as a normalized correlation; for the Laplacian images the measure is defined as:

$$M_{1,2}(L_{k1},L_{k2}) = \frac{2E(L_{k1})E(L_{k2})}{E^2(L_{k1}) + E^2(L_{k2})} \qquad (17)$$

with L_{k1} and L_{k2} denoting respectively the Laplacian images corresponding to the two output images at a given instant. We employ a similar match measure for the Gaussian levels by replacing the salience *(E)* with the total variation in (17).

The flowchart of the fusion rule for the Laplacian levels is shown in Fig.2 [14]:

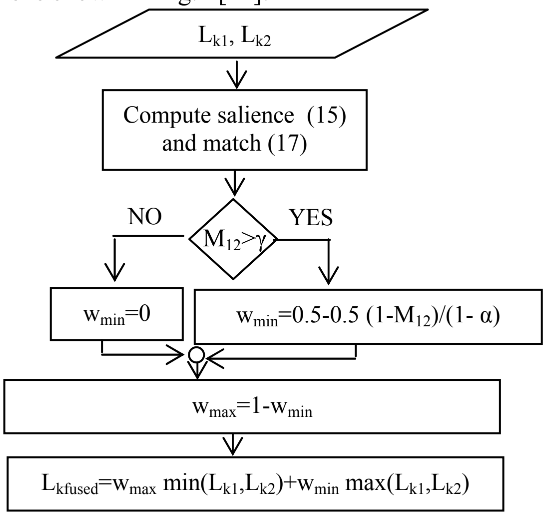


Fig. 2. Selection/weighted averaging fusion for the Laplacian levels

The fusion step introduces an extra parameter: the selection/weighted averaging threshold γ. For all the experiments presented in the next section its value was set experimentally to 0.5.

IV. EXPERIMENTAL RESULTS

A. *Synthetic images*

We first tested the efficiency of the proposed approach on computer generated synthetic images degraded by additive Gaussian noise (Fig.3 and Fig.4).

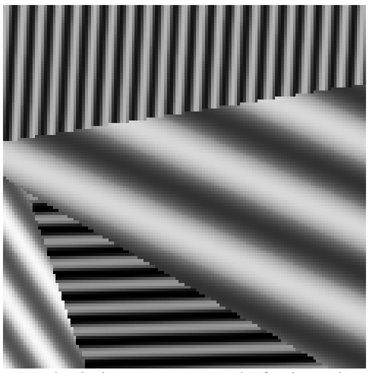


Fig. 3. Synthetic image composed of oriented patterns

The results shown in Fig.5 were computed using a unique orientation estimation step based on (4) with a 7x7 pixels Gaussian kernel support window. The numerical version of the filter corresponds to the explicit discrete scheme described in [11]; the results were computed for the same number of iterations *(n)*.

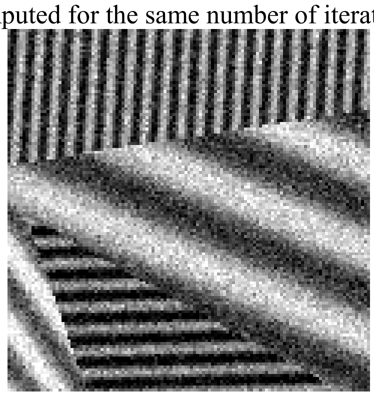
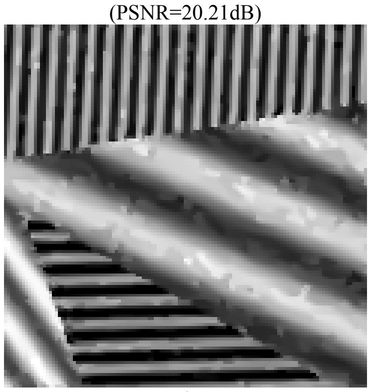
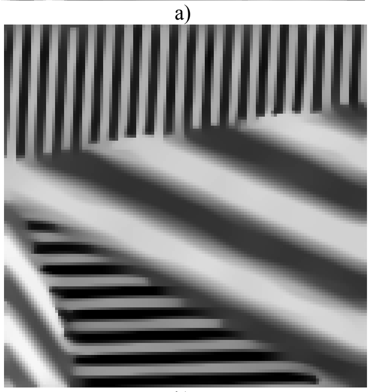


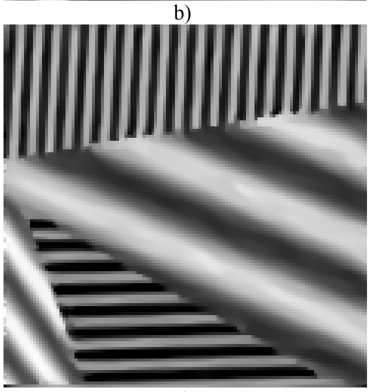
Fig. 4. Synthetic image with additive Gaussian noise (PSNR=20.21dB)



a)



b)

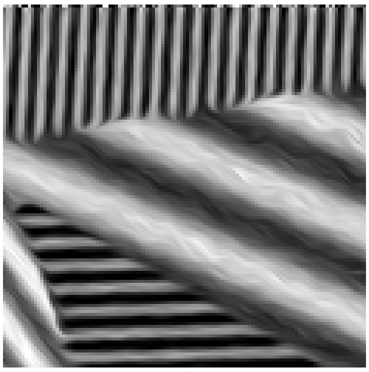


c)

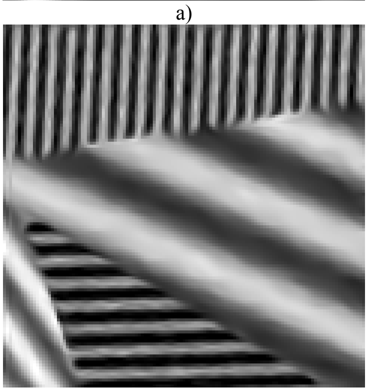
Fig. 5. Results corresponding to the noisy image in Fig.4; a) Image processed with (13) with the set of parameters: β=0.35, α=0.3, σ=0.75, n=90 iterations (PSNR=27.54 dB); b) Image processed with (13) with the set of parameters: β=0.95, α=0.3, σ=0.75, n=90 (PSNR=26.92 dB, n=90 iterations; c) Image processed with the proposed approach using the sets of parameters from a) and b) with 6 fusion steps performed each 15 iterations (PSNR=29.05 dB)

For the result shown in Fig.5.a) (PSNR=27.74dB) the diffusion threshold was deliberately chosen to have a low value; in this case a PDE filter based on (13) effectively preserves edges but has a lower efficiency in noise filtering tasks. The situation changes for the result in Fig.5.b when the diffusion threshold is set to have a relative high value (95% of the histogram corresponding to the directional derivatives along the patterns). The low frequency diagonal pattern is efficiently smoothed out but the filter introduces topological modifications by altering junction information; together with the loss of contrast, this aspect explains the lower PSNR value (26.92dB). The result from Fig. 5.c corresponds to the proposed fusion-diffusion scheme taking place by using the same filters with sets of parameters corresponding to the above two cases. Fusion was performed using 2 decomposition levels and the best result corresponded to 6 fusion steps performed at each 15 diffusion steps. The filter succeeds in combining the information from the two constituent filters and produces less contrast modifications, coupled with better junction preservation properties. The highest PSNR value is in concordance with the above remarks and with the visual aspect of the processed image.

For the same input image we also show below comparative results obtained using filters (5)-(6), (8) and (10). For filters (5)-(6) and (8) we used a full search in the space of parameters for finding the result with the highest PSNR value. The result for filter (10) was produced using the authors own implementation [17].



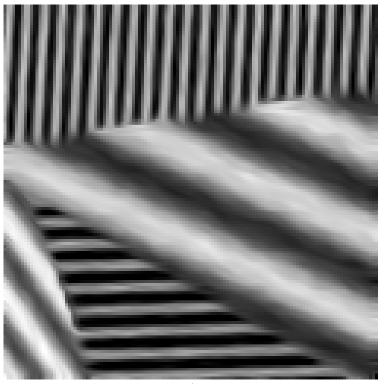
a)



b)

Fig. 6. Result corresponding to the noisy image in Fig.4; a) Image processed with (5) –(6) with the set of parameters: σ=0.5, ρ=1.5, n=17 iterations (PSNR=27.15 dB); b) Image processed with (8) with the parameters: σ=0.5, ρ=1.5, p_1=1, p_2=0.5 (PSNR=27.77 dB)

The coherence-enhancing filter (5)-(6) introduces strong topological modifications to the input image and has to be stopped quickly and the result is inferior in terms of PSNR and of visual quality.



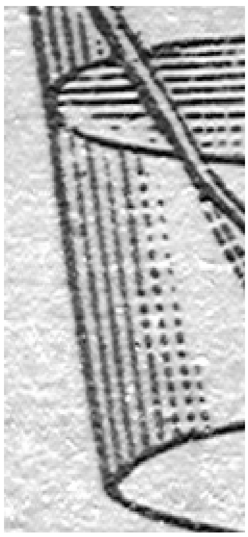
c)

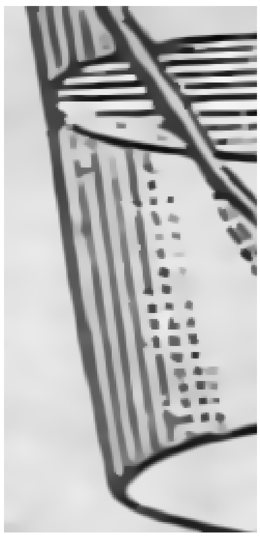
Fig. 6. Result corresponding to the noisy image in Fig.4; c) Image processed with (10) with the set of parameters: p=0.3, a=0.7,dt=10, iter=3, sigma=1.5, alfa=1.0 (see [17] for an explanation of the parameters - PSNR=27.42 dB)

Being designed to stop the smoothing process nearby high curvature regions, the filter (10) limits topological modifications of the input image when compared to the original formulation (8). However, some junctions get blurred due to the non stationarity of the input image.

B. *Real images*

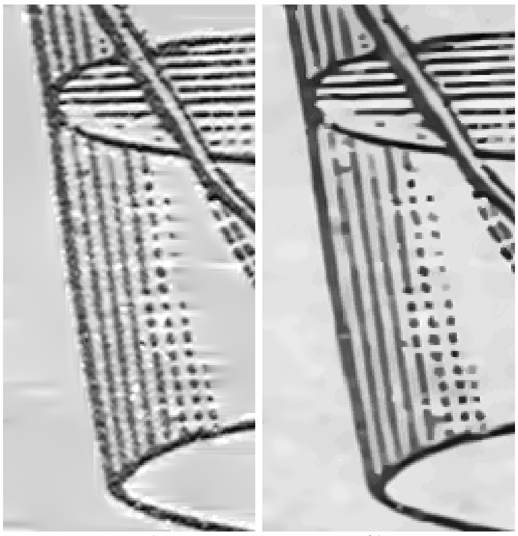
A first real image we will show comparative results on represents a fragment on a digitized old engraving and it is shown in Fig.7.a). In order to efficiently filter the background of the image, the diffusion threshold parameters have been set to relative high values (β=0.85, α=7.3). The result in Fig. 7.b) is computed with the filter (13) for a total of 60 iterations. Despite being able to smooth the background of the image, the filter also eliminates small-scale details.





a) b)

Fig. 7. Result for a real image; a) Result using (13)



a) b)

Fig. 8. Result corresponding to a real image; a) Image processed with (10) with the set of parameters: p=0.7, a=0.5,dt=10, iter=50, sigma=2.0, alfa=1; b) Result using the proposed method

The result from Fig.8.a) was obtained using the discrete filter implementation of equation (10) provided by the authors in [17]. The parameters were tuned in order to preserve most of the relevant objects in the image; the less efficient filtering nearby edges is due to the fact that (10) uses local contrast modulated smoothing and the smoothing process is strongly diminished in intensity for high contrast regions.

Finally we present for the same image a result obtained using the proposed method. The first set of parameters is identical to the one used for producing the image in Fig.7.b. For the second set of parameters we simply modified the diffusion threshold in the $\vec{\xi}$ directions to 50% of the value used for the first set (i.e β=0.42). The number of fusion steps was 4 and each restoration step corresponded to 15 iterations. For the same observation scale (n=60 iterations) as the result in Fig. 7.b, the proposed method produces better visual results. This is due to the fact that at each fusion step the pertinent results corresponding either to the set of parameters with lower values either to the second one are injected in the initial values for the next diffusion step.

V. REMARKS

We proposed a new image restoration method based on the use of directional diffusion and fusion techniques. Through application samples we showed that by using image fusion and appropriate fusion rules, the results of the same filter corresponding to different set of parameters can be coherently combined for producing better final results. The approach compares favorably with other approaches developed using the PDE framework with tensor structure based orientation estimation methods.
The method can be generalized to any other PDE filter for increasing its robustness with respect to the choice of parameters or for combining different PDEs as in [13].
Future work will be devoted for analyzing the influence of using more evolved wavelet based fusion techniques.

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