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Pipeline Identification in a TDOA Experiment

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Abstract - The time difference of arrival (TDOA) related to single input/two output systems has many practical applications. Using a kind of system identification applied to a water pipeline, this paper proves that the supposed linear relation between TDOA and the phase angle of the cross-spectral power density of the output signals is valid only in a limited frequency range. This conclusion shows the importance of low frequency components in the measured leak signals for TDOA estimation and leak localization.
The model proposed for system identification can be utilized with the main advantage of taking the correlation between the extraneous noise signals into account.
Keywords: Water pipelines, Leak signals, Identification, TDOA, Cross-spectral power density

I. INTRODUCTION

The time difference of arrival (TDOA) estimation is generally formulated as a single input/multiple output problem [1]. Particularly, if only two signals are measured, one talk about a single input/two output system. The representation of such system can be seen in Fig.1.

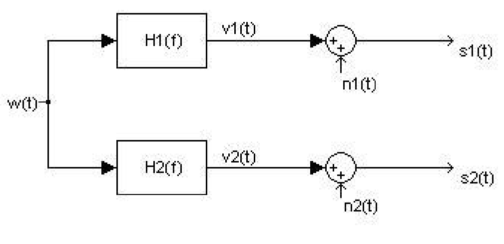

Fig.1 Single input/two output system

The leak localization in water pipelines is often based on the fundamental procedure of TDOA determination [7]. For $H_1(f) = 1$ and $H_2(f) = \alpha \cdot \exp(-j2\pi f \tau_1)$, the measured signals $s_1(t)$ and $s_2(t)$ are given by

$$\left.\begin{aligned} s_1(t) &= w(t) + n_1(t) \\ s_2(t) &= \alpha \cdot w(t-\tau_1) + n_2(t) \end{aligned}\right\} . \qquad (1)$$

The constants α and τ_1 represent the attenuation factor and the time difference to be determined, respectively. The extraneous noise terms $n_1(t)$ and $n_2(t)$ are assumed to be uncorrelated with each other and with the leak noise $w(t)$. Under these assumptions, one can show that the cross-correlation and the cross-spectral power density of the measured signals can be expressed using the autocorrelation and the spectral power density of the leak noise $w(t)$:

$$R_{s1s2}(\tau) = \alpha \cdot R_{ww}(\tau - \tau_1); \qquad (2)$$

$$G_{s1s2}(f) = \alpha \cdot \exp(-j2\pi f \tau_1) \cdot G_{ww}(f). \qquad (3)$$

According to these relations, the time delay τ_1, i.e. the time difference of arrival (TDOA) of $s_1(t)$ and $s_2(t)$, can be estimated using either the cross-correlation function or the cross-spectral density function where TDOA appears in the linear phase angle [2], [3], [4] :

$$\theta_{s1s2}(f) = 2\pi \cdot f \cdot \tau_1 . \qquad (4)$$

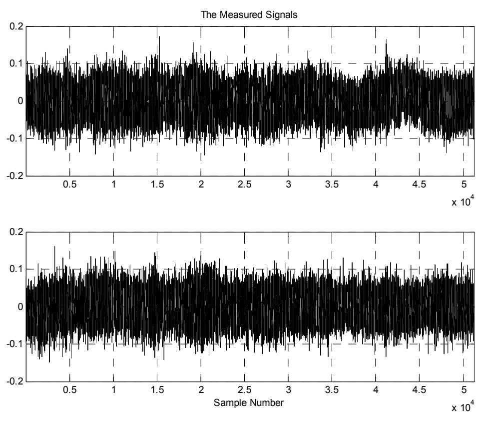

Fig.2 Experimental leak signals

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For example, a pair of leak generated signals, $s_1(t)$ and $s_2(t)$, each containing 51200 samples, measured at a water pipeline, are presented in Fig.2. The position of the maximal value of their cross-correlation function, pictured in Fig.3, is an estimation of the TDOA of these signals.

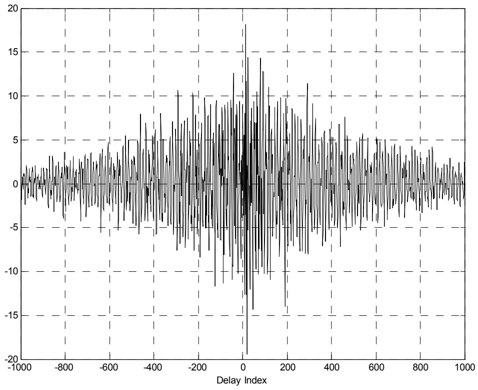

Fig.3 $R_{s1s2}(\tau)$ cross-correlation function

However, the linear relation (4) between phase angle $\theta_{s1s2}(f)$ and TDOA τ_1 was obtaining under simplifying assumptions related to model (1). If these conditions are not fulfilled, the precision of time delay estimation is affected, not only in the spectral power representation but also in the equivalent method based on the cross-correlation.

Using a kind of system identification, this paper investigates the relation between phase angle $\theta_{s1s2}(f)$ and TDOA of the signals $s_1(t)$ and $s_2(t)$, in a particular case of a water pipeline experimental setup.

II. PIPELINE IDENTIFICATION

One considers the pipeline model shown in Fig.4, which differs from that represented in Fig.1.

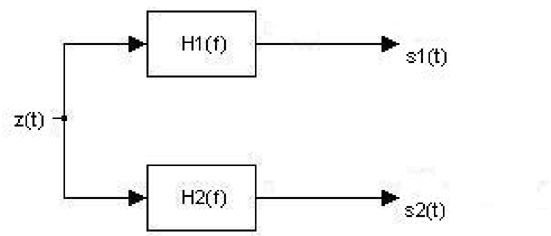

Fig.4 Single input/two output system with input signal including the extraneous noise

Thus, the input signal

$$z(t) = w(t) + n(t) \qquad (5)$$

is the sum of the leak noise, $w(t)$ and the extraneous input noise, $n(t)$. The pipeline sections between the leak and the sensors measuring the noise corrupted signals $s_1(t)$ and $s_2(t)$ are modeled by the constant parameter linear systems with frequency response functions $H_1(f)$ and $H_2(f)$ or the corresponding weighting functions $h_1(t)$ and $h_2(t)$, respectively.

One cannot do a proper identification of the transfer functions $H_1(f)$ and $H_2(f)$ because the input signal $z(t)$ is unknown [5]. However, the measured signals $s_1(t)$ and $s_2(t)$, are obtained by convolution operations between $z(t)$ and the weighting functions $h_1(t)$ and $h_2(t)$:

$$s_i(t) = \int_0^{\infty} h_i(\tau) \cdot z(t-\tau) dt, \quad i = 1,2. \qquad (6)$$

We can try using several realizations of a possible input noise $z(t)$ in order to reverse equation (6) and find $h_i(t)$ by deconvolution. The problem is that unlike convolution (abbreviation “conv”), deconvolution (abbreviation “deconv”) has not a unique result. The result h depends on a certain remainder, R, according to the relation

$$[h_k, R_k] = deconv(s, z_k) \qquad (7)$$

so that

$$s = conv(h_k, z_k) + R_k. \qquad (8)$$

Equations (6) and (8) are identical if the reminder $R_k = 0$. Practically, the deconvolution relation (7) was implemented using several hundreds of input noise sequences z_k, until a white noise realization with mean zero and variance one was found, assuring very small reminders for both output signals $s_1(t)$ and $s_2(t)$. This sequence is represented in Fig.5. The weighting sequences h_1 and h_2, shown in Fig.6, were obtained using in (7) the particular sequence z_k which assures $R_k \cong 0$. Only the first 50 samples from 512 values determined for the weighting functions are represented in Fig.6.

Now we can find an empirical formula for h_1 and h_2. So, by inspection, the weighting functions appear to be a decaying cosine wave:

$$h(t) = -C \cdot e^{-a \cdot t} \cdot \cos(\omega \cdot t + \varphi). \qquad (9)$$

Using a MATLAB computer program, the best-fit parameters C, a, ω and φ were found. Thus, for $h_1(t)$ the estimated values are: $C_1 = 6{,}8 \cdot 10^{12}$;

$a_1 = 1100$ Hz; $\omega_1 = 451220$ rad/s; $\varphi_1 = -0{,}38$ rad while for $h_2(t)$ one obtained: $C_2 = 18{,}9 \cdot 10^{12}$; $a_2 = 1050$ Hz; $\omega_2 = 451210$ rad/s; $\varphi_2 = -0{,}1$ rad.

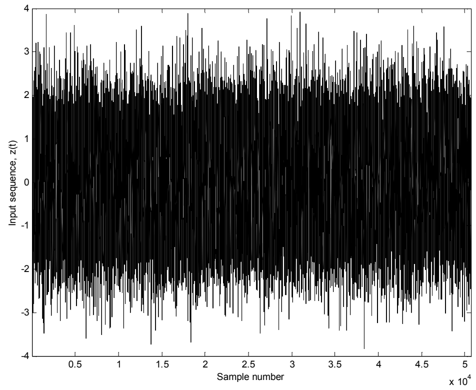

Fig.5 Input sequence
assuring very low deconvolution reminders

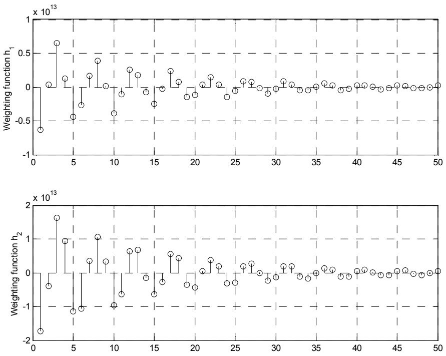

Fig.6 The first 50 samples
of the weighting sequences h_1 and h_2

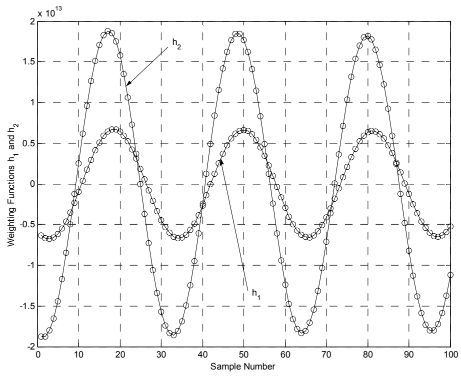

Fig.7 The first 100 values of the weighting functions sampled with $F_S = 2{,}25\text{MHz}$

In Fig.6 the weighting functions are sampled with $F_S = 15\text{kHz}$, corresponding to a sampling period $T_E \cong 6{,}66666 \cdot 10^{-5}\,\text{s}$. Using the empirical formula (9) one can represent the weighting functions sampled with an arbitrary frequency. Thus, Fig.7 shows the weighting functions sampled with $F_S = 2{,}25\text{MHz}$ i.e. $T_E \cong 0{,}044444 \cdot 10^{-5}\,\text{s}$. Both functions h_1 and h_2 exhibit a pronounced oscillatory character.

III. FREQUENCY DOMAIN INTERPRETATIONS

In order to facilitate some interpretations, it is useful to derive the Laplace transfer functions $H_1(s)$ and $H_2(s)$, corresponding to the weighting functions $h_1(t)$ and $h_2(t)$. Thus, one can observe that the time function (9) can be written as

$$h(t) = -C \cdot \cos\varphi \cdot \left[e^{-a \cdot t} \cdot \cos(\omega \cdot t)\right] + \\ + C \cdot \sin\varphi \cdot \left[e^{-a \cdot t} \cdot \sin(\omega \cdot t)\right]. \qquad (10)$$

Thereafter, using the Laplace transforms pairs [5]:

$$e^{-a \cdot t} \cdot \sin(\omega \cdot t) \Leftrightarrow \frac{\omega}{(s+a)^2 + \omega^2};$$

$$e^{-a \cdot t} \cdot \cos(\omega \cdot t) \Leftrightarrow \frac{s+a}{(s+a)^2 + \omega^2},$$

we obtain the Laplace transfer function

$$H(s) = \frac{(-C \cdot \cos\varphi) \cdot s}{s^2 + 2a \cdot s + a^2 + \omega^2} + \\ + \frac{-a \cdot C \cdot \cos\varphi + \omega \cdot C \cdot \sin\varphi}{s^2 + 2a \cdot s + a^2 + \omega^2}. \qquad (11)$$

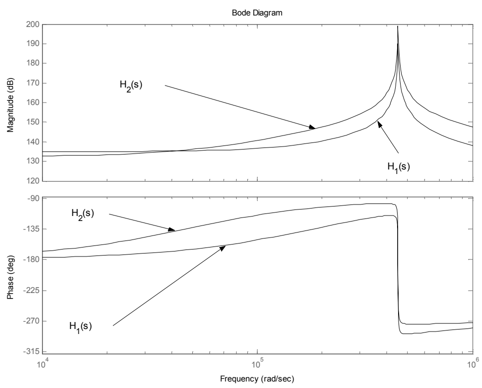

Fig.8 Bode plots for $H_1(s)$ and $H_2(s)$

With particular values for the constants C, a, ω and φ, the general expression (11) gives the transfer functions $H_1(s)$ and $H_2(s)$. The Bode plots for $H_1(s)$ and $H_2(s)$ are presented in Fig.8. The most remarkable feature revealed by the magnitude and phase plots in Fig.8 is the resonance at a high frequency (about 72 kHz). However, the high frequency ranges ($10^4 \div 10^6$ rad/s), represented in Fig.8, remain unexplored under normal working conditions of the pipeline. Therefore we are rather interested in the Bode plots at lower frequencies. These are shown in Fig.9.

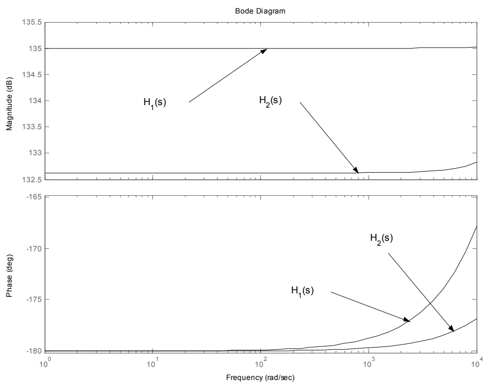

Fig.9 Bode plots for $H_1(s)$ and $H_2(s)$, at low frequencies

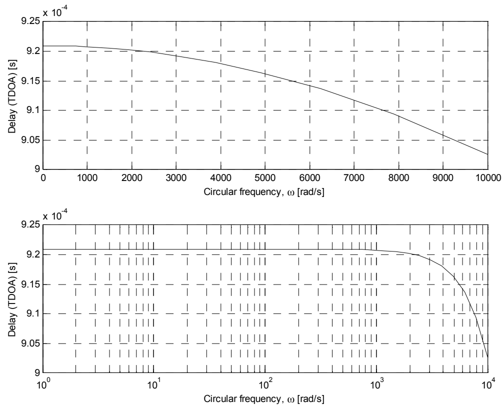

Fig.10 The ratio $\theta_{s1s2}(f)/(2\pi \cdot f)$, in linear (top) and logarithmic (bottom) frequency scale

The magnitude plots in Fig.9, show quasi-constant amplification at low frequencies ($10^0 \div 10^3$ rad/s) and an increasing tendency of $|H_2(s)|$ for the decade ($10^3 \div 10^4$ rad/s). On the other side, the phase difference between the two transfer functions increases with frequency. According to (4), the phase difference $\theta_{s1s2}(f)$, i.e. the phase difference in Fig.9, should be proportional to $2\pi \cdot f = \omega$. However, the ratio $\theta_{s1s2}(f)/(2\pi \cdot f)$ represented in Fig.10 in linear as well in logarithmic scale, shows a decreasing tendency. But, in (4) this ratio is a constant, τ_1, or the TDOA to be measured. Fig.10 shows that (4) must be generalized in the form

$$\theta_{s1s2}(\omega) = F(\omega) \cdot \Delta\tau . \qquad (12)$$

In (12), $\Delta\tau = \tau_2 - \tau_1$, stands for the TDOA between $s_2(t)$ and $s_1(t)$, while $F(\omega)$ is a nonlinear frequency function. Only at low frequencies, the approximation $F(\omega) \cong \omega$ is justified.

This analysis puts into evidence the importance of low frequency components of the measured signals in TDOA measuring experiments. Especially, in the signal pre-processing step, when rejection of low frequency components is used in order to assure that the measured signals are stationary, the rejection operation must restricted to the necessary minimum. This recommendation is important if the TDOA is to be determined from the position of the maximum of the cross-correlation function. Alternatively, one can think of determining the nonlinear function $F(\omega)$ in (12), using the identification procedure described in this paper, in order to improve the TDOA determination from the cross-spectral power density of the measured leak signals.

The identification procedure used in this paper refers to the block diagram shown in Fig.4. This suggests a new model for the one input/two output model of TDOA determination:

$$\left.\begin{aligned} s_1(t) &= \alpha_1(f) \cdot [w(t-\tau_1) + n(t-\tau_1)] \\ s_2(t) &= \alpha_2(f) \cdot [w(t-\tau_2) + n(t-\tau_2)] \end{aligned}\right\} \qquad (13)$$

Unlike (1), the proposed model (13) takes the dependence of attenuation on frequency into account. The new model also assures the extraneous noise signals acting on the physical pipeline arrangement are correlated. This is certainly a more realistic assumption than the classical lack of correlation related to model (1), especially when the geometrical dimensions of the experimental arrangement are small. However, the utilization of the proposed model remains a task for future research.

4 Conclusion

The classical single input/two output model and the associated simplifying assumptions lead to a linear relation between TDOA and the phase angle of the cross-spectral power density of the output signals. Using a kind of system identification applied to a water pipeline experimental setup, this paper proves that the supposed linear relation is valid only in a limited frequency range. Thus, the importance of low frequency components in the leak signals in put into evidence.

The single input/two output model proposed for system identification can be utilized as an alternative to the classical one, with the advantage of describing the correlation between the extraneous noise signals. This assumption is more realistic, especially if the geometrical dimensions of the experimental setup are small.

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