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Probability Density Function of M-ary FSK Signal in the Presence of Gaussian Noise, Intersymbol Interference and Log-Normal Shadowing

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Abstract – In this paper the receiver for the demodulation of M-FSK signals in the presence of Gaussian noise, intersymbol interference and log-normal shadowing is considered. The probability density function of M-ary FSK signal in the presence of noise, interference and fading is derived. The influence of the Gaussian noise, intersymbol iterference and log-normal fading to the communication systems can seriously degrade their performance.

Keywords: M-ary Frequency Shift Keying, Probability Density Function, Gaussian noise, Intersymbol Interference, Log-Normal Shadowing,

I. INTRODUCTION

In this paper we consider a system for coherent demodulation of M-ary FSK signals in the presence of Gaussian noise, intersymbol interference and log-normal fading. These disturbances can seriously degrade the performance of communication systems [1]-[3]. We derived the probability density function of an M-ary FSK receiver output signal. In the paper [4], the performance evaluation of several types of FSK and CPFSK receivers was investigated in detail. Also, the error probability of the cross-correlator receiver for binary digital frequency modulation detection is studied using theoretical analysis and computer simulations [5]. In [6] average bitt-error probability performance for optimum diversity combining of noncoherent FSK over Rayleigh channels is determined. Performance analysis of wide-band M-ary FSK systems in Rayleigh fading channels is given in [7].

In order to view an influence of the Gaussian noise, intersymbol iterference and log-normal fading of an M-ary FSK system, we are derived the probability density function of an M-ary FSK receiver output signal. The bit error probability, the signal error probability and an outage probability can be determined by the probability density function of an output signal. Also, the moment generating function, the cumulative distribution of an output signals and the moment and variance of output signals can be derived by probability density function of output signals. Based on this, the results obtained in this paper have a great significance.

This paper is organized as follows: first section is the introduction. In the second section the model of the M-ary FSK system is defined. The expressions for the probability density function of the output signal at one time instant are obtained in the third section. Fourth section consists of the numerical results in the case M=2. The last section is the conclusion.

II. MODEL OF THE M-ARY FSK SYSTEM

The model of an M-ary FSK system, which we consider in this paper, is shown at Fig. 1. This system has M branches. Each branch consists of the bandpass filter and correlator. The correlator is consisting of multiplier and lowpass filter. That system can be used for the transmission of signals in fading indoor power line environment.

The signal at the input of the receiver is digital frequently modulated signal corrupted by additive Gaussian noise, intersymbol iterference and log-normal shadowing.

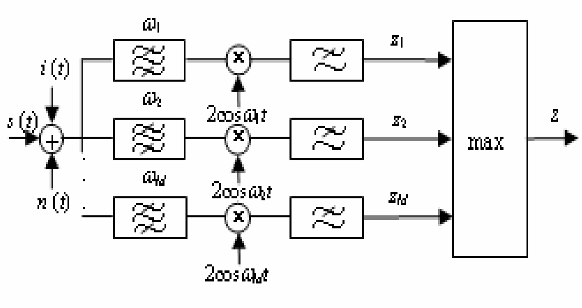

Fig.1. Block diagram of the system for coherent demodulation of M-ary FSK signal

Transmitted signal for the hypothesis H_i is:

$$s(t) = A\cos\omega_i t \qquad (1)$$

where A denotes the amplitude of the modulated signal and has log-normal distribution [8]:

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$$p_A(A) = \frac{1}{\sqrt{2\pi}\sigma_A A} e^{-\frac{(\ln A-\mu)^2}{2\sigma_A^2}}, \quad A \geq 0 \quad (2)$$

where μ and σ_A are the mean and standard deviation of this distribution.

Gaussian noise at the input of the receiver is given with:

$$n(t) = \sum_{i=1}^{M} x_i \cos\omega_i t + y_i \sin\omega_i t, \quad i=1, 2,...M \quad (3)$$

where x_i and y_i are the components of Gaussian noise, with zero means and variances σ^2.

The interference i(t) can be written as:

$$i(t) = \sum_{i=1}^{M} A_i \cos(\omega_i t + \theta_i) \quad (4)$$

where phases θ_i, have uniform probability density function.

These signals pass first through bandpass filters whose central frequencies $\omega_1, \omega_2,..., \omega_M$ correspond to hypotheses $H_1,H_2,...,H_M$. After multiplying with signal from the local oscillator, they pass through lowpass filter. The filter cuts all spectral components which frequencies are greater than the border frequency of the filter.

If $z_1,z_2, ...z_M$ are the output signals of the branch of the receiver, then the M-FSK receiver output signal is:

$$z = \max\{z_1, z_2 ... z_M\} \quad (5)$$

The probability density of output signal is

$$p(z) = \sum_{i=1}^{M} p_{z_i}(z) \cdot \prod_{\substack{j=1 \\ j\neq i}}^{M} F_{z_j}(z) \quad (6)$$

III. PROBABILITY DENSITY FUNCTION

In the case of the hypothesis H_1, transmitted signal is:

$$s(t) = A\cos\omega_1 t \quad (7)$$

while the output branch signals of the receiver are:

$$z_1 = A + x_1 + A_1\cos\theta_1 \quad (8)$$

$$z_k = x_k + A_k\cos\theta_k, \quad k=2,3,...,M \quad (9)$$

It is necessary to define the probability density functions on the output of branches and the cumulative density of these signals to obtain output probability density function of M-ary FSK receiver.

The conditional probability density functions for the signals $z_1, z_2,..., z_M$ are:

$$p_{z_1/A,\theta_1}(z_1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1-A-A_1\cos\theta_1)^2}{2\sigma^2}} \quad (10)$$

$$p_{z_k/A,\theta_k}(z_k) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k-A_k\cos\theta_k)^2}{2\sigma^2}},$$

$$k=2,3,...,M \quad (11)$$

By averaging (10) and (11) we obtain the probability density functions of the signals on the output of the branches:

$$p_{z_1}(z_1) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1-A-A_1\cos\theta_1)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_A A} e^{-\frac{(\ln A-\mu)^2}{2\sigma_A^2}} dA \frac{1}{2\pi} d\theta_1 \quad (12)$$

$$p_{z_k}(z_k) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k-A_k\cos\theta_k)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_A A} e^{-\frac{(\ln A-\mu)^2}{2\sigma_A^2}} dA \frac{1}{2\pi} d\theta_k \quad (13)$$

The cumulative distributions of the signals $z_1, z_2,..., z_M$ are:

$$F_{z_1}(z_1) = \int_{-\infty}^{z_1} \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1-A-A_1\cos\theta_1)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_A A} e^{-\frac{(\ln A-\mu)^2}{2\sigma_A^2}} dA \frac{1}{2\pi} d\theta_1 dz_1 \quad (14)$$

$$F_{z_k}(z_k) = \int_{-\infty}^{z_k} \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k-A_k\cos\theta_k)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_A A} e^{-\frac{(\ln A-\mu)^2}{2\sigma_A^2}} dA \frac{1}{2\pi} d\theta_k dz_k \quad (15)$$

The probability density function of the M-ary FSK receiver output signal in the case of the hypothesis H_1 can be obtained from:

$$p(z) = \sum_{i=1}^{M} p_{z_i}(z) \cdot \prod_{\substack{j=1 \\ j\neq i}}^{M} F_{z_j}(z) \quad (16)$$

IV. NUMERICAL RESULTATS

Now, we consider the dual branch FSK receiver because of its easy implementation and very good performances. It is employed in many practical telecommunication systems.

The probability density function, in the case of dual branch, has a form:

$$p(z)=p_{z_1}(z)\cdot F_{z_2}(z)+p_{z_2}(z)\cdot F_{z_1}(z) \qquad (17)$$

The probability density functions *p(z)*, for various values of the parameters μ, σ_A, A_i and σ, are given at Figs. 2. to 9.

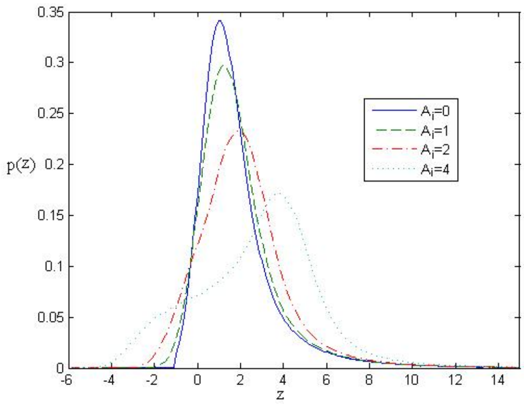

Fig. 2. The probability density functions *p(z)* for the parameters $A_i=0,1,2,4$, $\sigma=1$, $\mu=0$ and $\sigma_A=1$

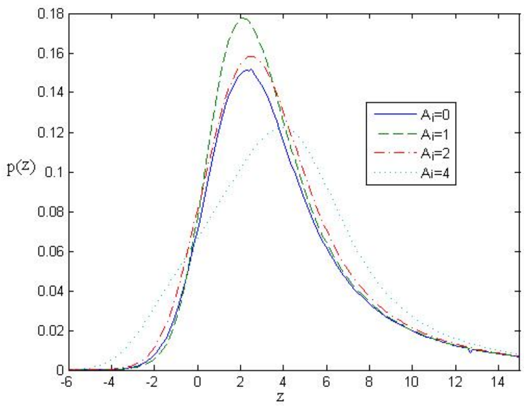

Fig. 3. The probability density functions *p(z)* for the parameters $A_i=0,1,2,4$, $\sigma=2$, $\mu=1$ and $\sigma_A=1$

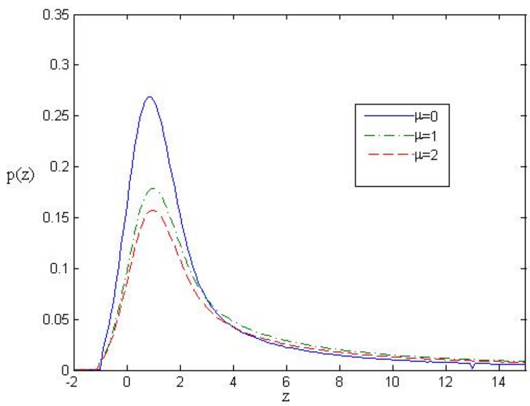

Fig. 4. The probability density functions *p(z)* for the parameters $A_i=0$, $\sigma=1$, $\mu=0,1,2$ and $\sigma_A=2$

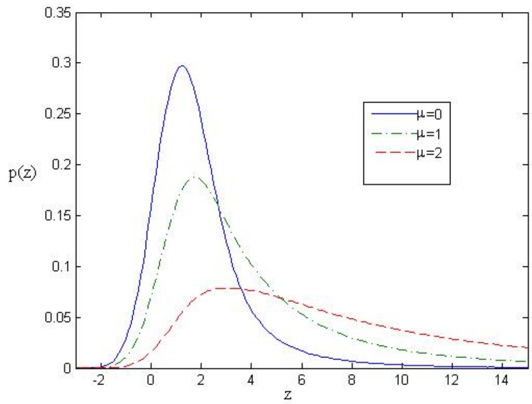

Fig. 5. The probability density functions *p(z)* for the parameters $A_i=1$, $\sigma=1$, $\mu=0,1,2$ and $\sigma_A=1$

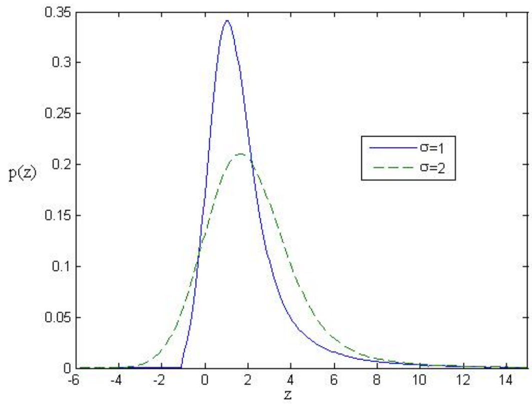

Fig. 6. The probability density functions *p(z)* for the parameters $A_i=0$, $\sigma=1,2$, $\mu=0$ and $\sigma_A=1$

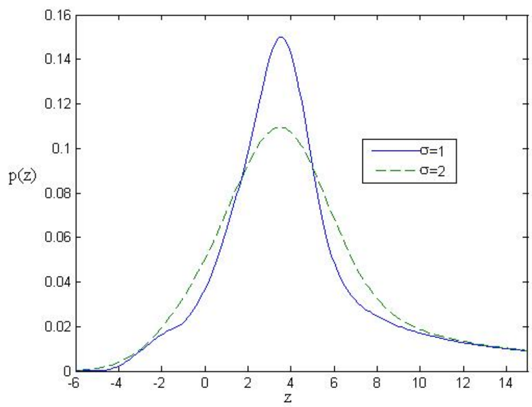

Fig. 7. The probability density functions *p(z)* for the parameters $A_i=4$, $\sigma=1,2$, $\mu=1$ and $\sigma_A=2$

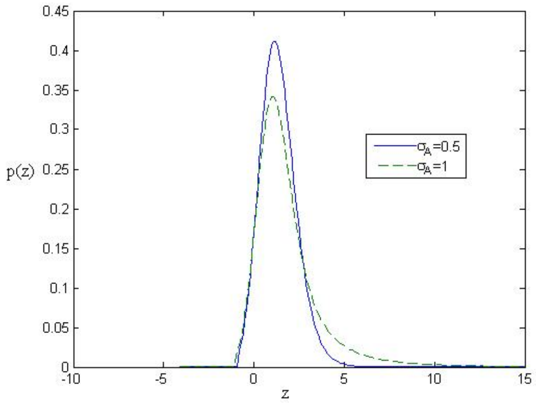

Fig. 8. The probability density functions *p(z)* for the parameters $A_i=0$, $\sigma=1$, $\mu=0$ and $\sigma_A=0.5;1$

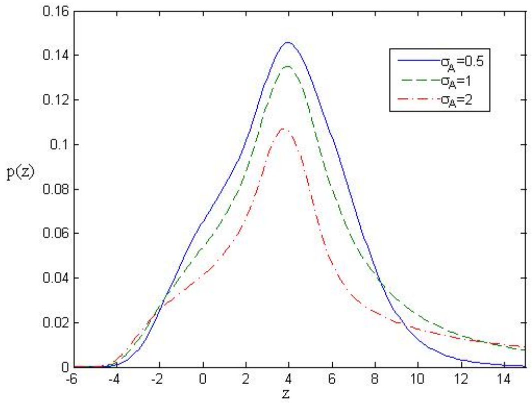

Fig. 9. The probability density functions *p(z)* for the parameters $A_i=4$, $\sigma=1$, $\mu=1$ and $\sigma_A=2$

V. CONCLUSION

In this paper, the statistical characteristics of the signal at the output of the receiver for coherent FSK demodulation are determined. The input signal of the receiver is digital frequently modulated signal corrupted by additive Gaussian noise, intersymbol iterference and log-normal shadowing. The interference appears in each receiver branch. In this paper the probability density function of an M-ary FSK receiver output signal is derived.

The bit error probability and the outage probability can be determined by the probability density function of an output signal.

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