In-Depth Analysis of Wavelet Modulation Performance in Flat Fading Channels: Choosing the Wavelets Mother

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Abstract – This paper presents an in-depth investigation of DWT- based OFDM, by analyzing the influence of the wavelets mother choice on the BER performance for the transmission in a flat fading channel. Simulations made show the importance of this parameter, especially if the channel is rapidly changing in time. The best results under the considered scenario are provided by the Haar wavelet, while for other families (Daubechies, Symmlet, Coiflet), the number of vanishing moments of the wavelets mother may be relevant to a certain extent.
Keywords: WOFDM, flat fading, wavelets mother

I. INTRODUCTION

Wavelets represent a successful story of the last decade in signal processing applications. Thus, these signals, with some highly desirable properties, are currently widely used in applications as compression, denoising, segmentation, in-painting or classification. By the other hand, in data communications, the same successful story can be assigned to multi-carrier modulation techniques. The principle of the Orthogonal Frequency Division Multiplexing (OFDM) is employed in a large number of transmission standards, over wired and wireless channels: WiFi, WiMAX, DAVB or ADSL.

The wavelet based OFDM (WOFDM), sometimes referred to as wavelet modulation is the point where the above concepts meet with each-other. Although they are widely used in signal processing, few wavelets applications are known in data transmission. The idea that gathers the two concepts is to use wavelet signals as carriers in a multi-carrier data transmission.

Multi-carrier transmissions transmit N data symbols simultaneously, using a large number of subcarriers. This way, a longer symbol (or "block") is composed, which carries the information contained by all the N symbols. By demodulating a single multi-carrier block, a decision is made on all the N symbols. Thus, the average data rate is kept constant, while the fact that the multi-carrier symbol duration is long provides an increased resilience against the inter-symbol interference (ISI) [1].

The key point that allows the demodulation of the subcarriers is their orthogonality. In the classical OFDM, the subcarriers are complex exponentials (sine and cosine waves), having the form:

$$subc(k) = e^{jk2\pi f_0 t} \qquad (1)$$

In practice, the family $subc(k)$ is generated using digital signal processing algorithms, namely the Inverse Fast Fourier Transform (IFFT) [2]. Despite its excellent behavior in unfriendly channel environments, OFDM raises some practical problems which are difficult to overcome. Thus, the bandwidth is increased by the use of a cyclic prefix and the transmission is highly sensitive to the Doppler spread introduced by the time variant channels. Next, the OFDM systems are very sensitive to the narrow band interferences. Furthermore, the synchronization in time and frequency is critical for the system performance and the peak-to-average ratio of the signal is very large due to the non-constant nature of the envelope. Finally, the out-of-band rejection of such a signal is not satisfactory, since the OFDM spectrum is made of sinc functions, whom sidelobes contain an important amount of energy.

Recent research has shown that some of these drawbacks may be counteracted if wavelet signal were used instead of the complex exponential carriers [3-5]. Furthermore, focusing on the BER performance, the author of this paper highlighted significant BER reduction in WOFDM compared to OFDM, under some specific scenarios. This proves that is worthwhile to conduct an extensive research on this technique and to perform a detailed analysis of some of its parameters. Thus, the practical implementation of wavelet modulation relies on the Discrete Wavelet Transform (DWT). This transform is implemented using digital filter banks, according to Mallat's well known algorithm [6] and it has two important parameters: the wavelet mother employed and the number of iterations (decomposition levels) used in computation. The first of these parameters is extensively analyzed in this paper. Simulations are made in flat fading scenarios for a large set of wavelets, commonly used in DWT implementation:

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Haar, Daubechies, Symmlet and Coiflet. These investigations revealed that the best performance is achieved by the Haar- based transform and that, in general, shorter-time wavelets provide better results, especially in fast fading conditions.

An overview of WOFDM is made on Section II. Next, the simulation model is extensively presented. In section IV, simulation results are shown and commented. Some conclusions and new research directions are drawn in the last section.

II. WOFDM OVERVIEW

The WOFDM principle is the same as in the classical OFDM: the signal is composed of a large number of orthogonal subcarriers. Only that, unlike in (1), the subcarriers are wavelet mothers and scaling functions. Their ortogonality, very important for the demodulation, is expressed in (2):

$$\langle \Psi_{j,k}, \Psi_{m,n} \rangle = \begin{cases} 1, & \text{if } j = m \text{ and } k = n \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where the family $\Psi_{j,k}(t)$ is described by equation (3):

$$\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}} \psi(2^{-j}t - k) \quad (3)$$

Equation above corresponds to a dyadic form for the wavelet family, where the wavelets from one level are scaled versions of wavelets from the previous level, the scaling function being a power of 2. An analysis of this equation shows us that higher the value of j, poorer the time resolution and better the frequency localization of the wavelets. Similar to the OFDM case, the practical implementation of the modulator in the wavelet-based technique may rely on the IDWT. This implementation is done using the Mallat's filter bank algorithm [6]. If, theoretically, j and k in (3) can be any integer number, the practical implementation limits these parameters to finite values. Thus, considering that we have a N samples discrete time input signal, then we define the maximum value of j as being $J=log_2N$. In this case, $j = 1$ corresponds to the best time rezolution which can be achieved, whereas $j=J$ corresponds to the poorest time and to the best frequency resolution.

In practice, the number of IDWT iterations is oftentimes lower than thge maximual value J. If we consider only L iterations, then the poorest time resolution will correspond to the wavelet $\psi(2^{-L}t)$. At this scale, we also retrieve the scaling function $\varphi(2^{-L}t)$ involved in the IDWT computation. The task of this scaling function is to provide (together with the wavelet functions) a complete representation

of the signals from L^2. Thus, the continuos-time signal $s(t)$ may be represented as a sum of approximation and detail wavelet coefficients (4):

$$s(t) = \sum_{j=1}^{L} \sum_{k=1}^{2^{J-j}} w_{j,k} \Psi_{j,k}(t) + \sum_{k=1}^{2^{J-L}} a_{L,k} \varphi_{L,k}(t) \quad (4)$$

Equation 4 is close to the computation of the Inverse Discrete Wavelet Transform (IDWT).

The parameter L in (4) is a measure of the number of IDWT iterations. Higher this value is, longer duration are the wavelets that compose the signal in relation (4), and fewer the wavelet coefficients transmitted at each scale.

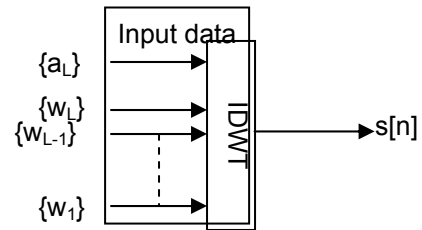The implementation of equation 4 is shown in fig.1.

Fig. 1: WOFDM implementation using IDWT.

The input data vector (symbols to be transmitted) may be imagined as:

$$data = [\, a_L, w_L, w_{L-1}, ..., w_1 \,] \quad (5)$$

where a_L and w_L are vectors of length 2^{J-L} and w_1 is a vector of length 2^{J-1}. Every transmission scale j has its own subcarrier set $\{\Psi_{j,k}(t)\}$. At a fixed scale, the used subcarriers occupy the same frequency band, but they are separated in time. By the other hand, at different scales, the spectral content of the subcarriers is different. In our example, Ψ_L and φ_L have a low frequency and narrow band content, whereas half of the bandwidth is carried by the high-pass $\psi_1(t)$ carriers. The spectra of these subcarriers are displayed in figure 2, for the Haar wavelet. Three iterations were considered in the modulator. Fig. 2 confirms that the largest bandwidth corresponds to the wavelets used in the first iteration of IDWT algorithm, where we have the shortest carriers and where most of the symbols are transmitted. The wavelet mother and the scaling function involved in (4) do not have, in general, a known analytical expression. As already mentioned, DWT is implemented using quadrature mirror filter banks, using Mallat's famous algorithm [6].

Exactly as for OFDM, the demodulator is implemented by the intermediate of the direct algorithm, DWT. If we take a closer look to what the
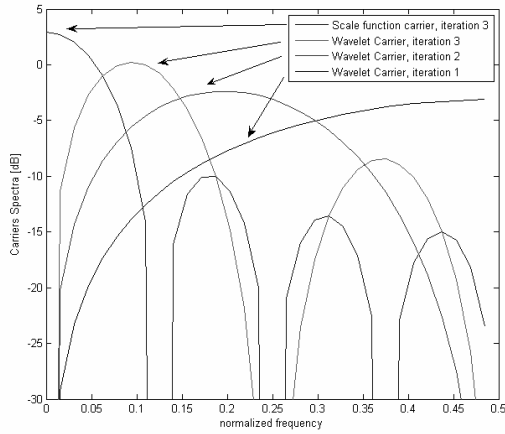
Fig.2: Haar carriers spectra.

direct wavelet transform does, we can figure out the demodulator as a bank of multipliers, followed by integrators. Thus, one may notice that the signal samples are multiplied by wavelets at different scales *j* and locations *k* and by scale functions at the poorest time resolution, then the signal is integrated over a symbol period. For example, assuming ideal channel, the wavelet coefficients transmitted at scale *j* and position *k* can be computed as in (6):

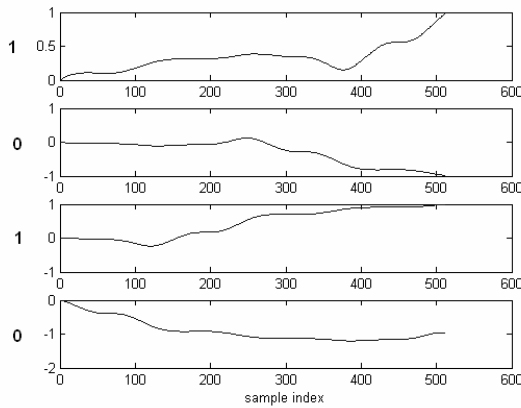$$w_{j,k} = \int s(t) \cdot \psi^*_{j,k}(t) \cdot dt \qquad (6)$$

Fig.3: DWT demodulator output signal.

The signal at the output of the integrator (based on which the data symbols are estimated) is shown in a graphical example (fig. 3). A sequence of four data bits (1,0,1,0) is identified based on the demodulator output at the end of the symbol duration. Note that the description above is only a conceptual view of the demodulator. In practice, the wavelet and detail coefficients are computed by successively filtering the input signal using high-pass and low pass-filters [6].

III. SIMULATION MODEL

A. *Transmitter model*

The model used for simulation purposes is shown in fig. 4. The data to be transmitted is a sequence of equally probable bipolar data symbols (+1 and -1), corresponding to a BPSK modulation. Every block of *N=1024* symbols is brought to the input of the IDWT modulator. For the IDWT computation, two parameters were taken into account: the wavelets
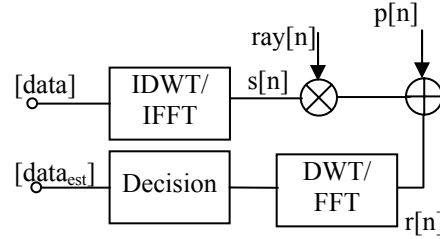

Fig.4: Baseband implementation of a WOFDM system.

mother used, and the number of iterations of this transform. This paper focuses on the first of these parameters. Thus, as wavelet carriers, several wavelet families were tested: Haar, Daubechies, Symmlet and Coiflet. These wavelets are widely used for the DWT computation. They have the advantage of a simple implementation, using a free toolbox of Matlab functions, called Wavelab [7]. For all these wavelets, there is another parameter which must be taken into account: the number of vanishing moments [6]. Related to our research, it is interesting to remark that, generally speaking, higher this number is, longer the time support and more compacted is the wavelet carrier in frequency. The extreme case is the Haar wavelet, with only one vanishing moment. This wavelet has the shortest time support among all other wavelets, detail which will become meaningful when the simulation results will be discussed.

B. *Channel model*

The channel simulation gathers two different effects: AWGN noise and flat Rayleigh fading. The first type of noise affects the signal additively and the second one multiplicatively. The Rayleigh fading models the effect of the multipath propagation in a radio channel [8]. There are two types of analysis that can be applied to this phenomenon: the spectral analysis and the statistical analysis. The term "flat", used before to describe the simulation scenario, refers to the spectral properties of the fading. The flat fading scenario is based on the assumption that all the multipath replicas of the transmitted signal arrive at receiver during the transmission interval of one single symbol. The flatness corresponds to the channel spectra under this assumption.

By the other hand, the radio channel changes its response during the time. The variance in time of the radio channel's behavior can be expressed by the Doppler shift parameter (f_d), which depends on the relative motion between transmitter and receiver (assuming mobile communications) and on the carrier frequency used for transmission. Usually, a normalized version of this parameter it's used:

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$$f_m = f_d \cdot T_S \qquad (7)$$

where T_S is the duration of a serial symbol. Two values were considered for this parameter: $f_m=0.005$ and $f_m=0.05$. In slow fading scenarios, T_S must be much smaller than the coherence time of the channel expressed as:

$$T_C = \frac{0.423}{f_d} \qquad (8)$$

Taking into account equations 7 and 8, our worst case scenario ($f_m=0.05$) leads to a coherence time T_C which is approximately 8 times higher than T_S. In the best case (the lowest Doppler shift), the coherence time is 80 times longer than the symbol duration.

These values apparently meet the slow fading assumption, where the channel stays unchanged for the duration of a symbol. Though, when evaluating the channel behavior, one should take into account that in multi-carrier communications the transmitted block (WOFDM symbol) is much longer that the serial symbols at the modulator's input. Since the whole data vector is required at demodulator to identify the transmitted symbols, one may consider that the WOFDM block duration is N times longer than the serial symbols brought at system's input. Note that in these conditions, the channel response changes several times during the transmission of one block.

In order to simulate the time variance of the radio channel, the fading samples must approximate as accurately as possible the theoretical auto-correlation function:

$$R[n] = J_0(2\pi f_m \cdot |n|) \qquad (9)$$

, where J_0 is the zero order Bessel function [9].

Regarding the statistical properties of the fading exhibited by the radio channel, a largely used model is the Rayleigh probability density function (pdf). If a constant envelope signal will be transmitted through a time variant radio channel (for example a pure sine wave), then the envelope of the received signal can be described using the Rayleigh pdf:

$$pdf(x|\sigma) = \frac{x \cdot e^{-x^2/2\sigma^2}}{\sigma^2} \qquad (10)$$

Under this model, the signal arrived at receiver can be expressed as:

$$r(n) = s(n) \cdot ray(n) + p(n) \qquad (11)$$

, where $p(n)$ represents the AWGN noise, $s(n)$ is the multi-carrier signal computed in equation (4) and $ray(n)$ is the Rayleigh distributed sequence from (10), with $\sigma^2 = 1/2$. This choice allows a straightforward computation of the signal to noise ratio (SNR). By definition, SNR measure is used in purely AWGN channels, but the previous choice allows extending this term for the more complex case of the Rayleigh fading channels. Thus, the signal affected by fading $s(n)ray(n)$ has the same energy as the useful signal.

C. *The receiver*

As explained in the previous section, the receiver demodulates the signal using the DWT algorithm. The detector is based on a simple threshold comparison, taking into account the bipolar nature of the useful signal.

D. *The end-to-end scenario*

The data is transmitted in blocks of N=1024 symbols. Every simulation is carried out for 10000 blocks. After detection, the BER is computed at different SNR values. The SNR is estimated using the formula:

$$SNR = \frac{\sum_n |s[n]|^2}{\sum_n |p[n]|^2} \qquad (12)$$

The BER curves are represented as a function of E_b/N_0, where E_b is the bit energy, whereas N_0 represents the uni-dimensional power spectral density of the white noise. Expressed in dB, the two measures may be related by:

$$SNR = \frac{E_b}{N_0} + 3 \quad [dB] \qquad (13)$$

The above presented scenario is repeated for every tested wavelet mother.

IV. RESULTS AND DISCUSSIONS

The fist goal of our simulations is to compare the performance achieved by using different wavelet mothers in flat fading. The results are slightly different for the values of the Doppler taken into account. In the first scenario, we consider $f_m=0.005$. This value meets the slow fading condition, previously explained. One wavelet was chosen from each family and one iteration for the IDWT computation. The results are displayed in figure 5. The best results are by far obtained using the Haar wavelet (7dB of gain at a BER of 10^{-2}), whereas there is no significant difference between the other tested wavelets. These results are confirmed for $f_m=0.05$ (figure 6). This time, the Daubechies wavelet has the
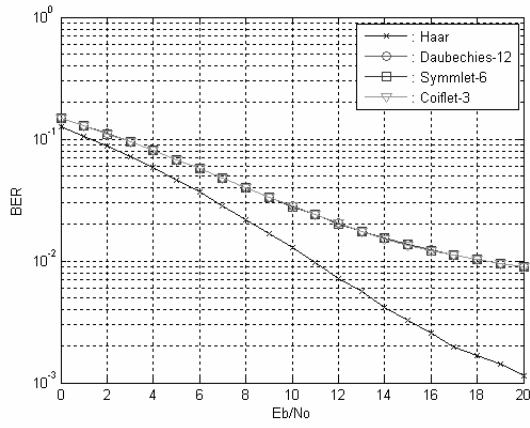
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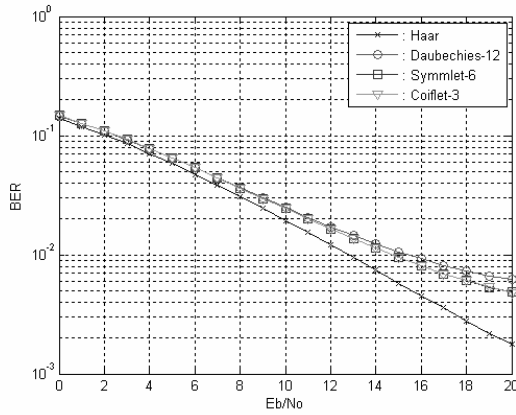Fig. 5: WOFDM performance for different wavelet mothers, f_m=0.005.

Fig. 6: WOFDM performance for different wavelet mothers, f_m=0.05.

worst result. The explanation of the superior performance of the Haar wavelet resides in the waveform of this carrier. The above simulations were made for one iteration of the IDWT. This means that only the wavelets from the best time resolution were taken into account. At this scale, the Matlab representation of the Haar wavelet is composed of only two samples. Comparing this value with other tested wavelets, we notice that the Haar carrier duration is at least six times shorter than all the other wavelets. Considering $L=1$ and $a_{1,k}$ and $w_{1,k}$ being two samples of the input data vector (*data[n]*), then every sample of the received signal *r(n)* only depends on the two transmitted symbols (one approximation and one wavelet coefficient). Consequently, the demodulator may decide what symbols were transmitted based only on two consecutive received samples. This minimizes the influence of the fading sequence *ray(n)* and leads to the excellent results of the Haar wavelet compared to the other wavelets. For conformity, we must however remark that the flatness of the fading favorizes the wavelets which have a shorter duration. Indeed, their spectrum is very large (see figure 2), and a frequency selective channel would affect more these kind of carriers than longer duration wavelets.

The next objective of our simulations was to test, for the same type of wavelets mother, the influence of the number of vanishing moments. Simuations made

allow us to draw two conclusions. First, no significant effect of this parameter can be highlighted for the slow fading scenario. Next, at f_m=0.05, the Daubechies family was the only one where some differences were observed (4 dB of gain for the wavelet with the shortest time support compared to the longest one). Some results are displayed in the figures 7 and 8 (for the Daubechies wavelets) and a compendium of the results for all the tested wavelets and f_m=0.05 is illustrated in Table 1.
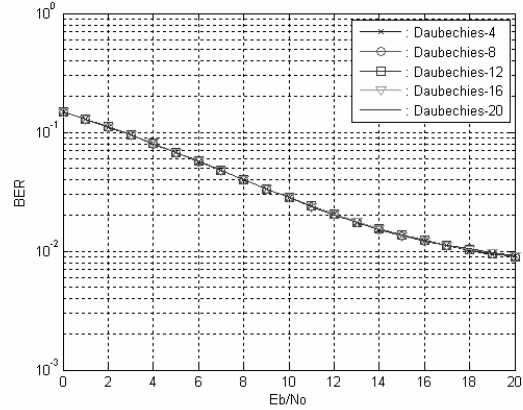

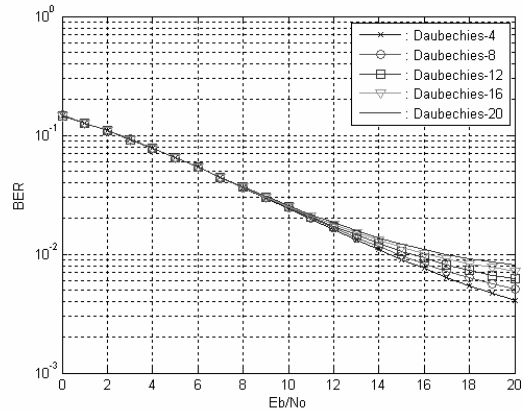Fig. 7: The influence of the number of vanishing moments (Daubechies family), f_m=0.005.

Fig. 8: The influence of the number of vanishing moments (Daubechies family), f_m=0.05.

Table 1 shows that, excepting the Haar wavelets, significant differences introduced by the number of vanishing moments can be highlighted only at high SNRs. Excepting the first row of table 1 (which refers to the Haar carrier), the other "best results" are highlighted. The displayed results enforce the idea that for the flat, time variant channel, the compactness in time of the wavelet carriers leads to improved performance. Thus, for *Eb/No=12* and above, it is the Daubechies-4 wavelet which achieves the second performance (after Haar). We may assume that at this SNR, the negative impact of the Doppler spread induced by the fading becomes more important than that of the additive noise affecting the signal.

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Table 1: Compendium of the results for all the tested wavelets.

Eb/No	0	2	4	6	8	10	12	14	16	18	20
Haar	0.1255	0.0879	0.0580	0.0366	0.0218	0.0127	0.0072	0.0041	0.0025	0.0016	0.0011
Daub4	0.1466	0.1086	0.0776	0.0536	0.0361	0.0241	0.0160	0.0109	0.0075	0.0053	0.0040
Daub8	0.1468	0.1088	0.0774	0.0535	0.0363	0.0244	0.0166	0.0115	0.0082	0.0063	0.0051
Daub12	0.1466	0.1086	0.0775	0.0537	0.0365	0.0248	0.0170	0.0122	0.0093	0.0073	0.0061
Daub16	0.1465	0.1086	0.0775	0.0536	0.0368	0.0251	0.0178	0.0131	0.0100	0.0082	0.0071
Daub20	0.1464	0.1085	0.0773	0.0539	0.0370	0.0256	0.0183	0.0137	0.0109	0.0091	0.0081
Syymlet4	0.1469	0.1088	0.0775	0.0536	0.0362	0.0242	0.0162	0.0110	0.0077	0.0056	0.0043
Symmlet6	0.1468	0.1088	0.0777	0.0538	0.0362	0.0243	0.0164	0.0113	0.0081	0.0061	0.0048
Symmlet8	0.1467	0.1088	0.0776	0.0536	0.0365	0.0245	0.0166	0.0116	0.0083	0.0064	0.0053
Symmlet10	0.1467	0.1088	0.0777	0.0536	0.0365	0.0245	0.0168	0.0118	0.0087	0.0067	0.0055
Coiflet1	0.1467	0.1089	0.0776	0.0537	0.0363	0.0242	0.0163	0.0110	0.0078	0.0057	0.0044
Coiflet2	0.1467	0.1089	0.0775	0.0537	0.0362	0.0242	0.0162	0.0110	0.0078	0.0057	0.0044
Coiflet3	0.1468	0.1087	0.0776	0.0538	0.0362	0.0244	0.0165	0.0113	0.0080	0.0060	0.0047
Coiflet4	0.1467	0.1089	0.0776	0.0537	0.0364	0.0245	0.0167	0.0115	0.0083	0.0064	0.0051
Coiflet5	0.1465	0.1086	0.0777	0.0536	0.0364	0.0245	0.0167	0.0117	0.0085	0.0066	0.0054

V. CONCLUSIONS AND FUTURE WORK

The influence of the wavelet mother choice on the BER performance of a WOFDM system is studied in this paper. The simulation scenario focuses on a transmission in a flat, time variant Rayleigh fading channel. The best results are always provided by the Haar wavelet, due to its compactness in time. This property of the Haar wavelet makes it more resilient to the time-variant character of the channel, simulated by a multiplicative Rayleigh sequence. For the other wavelets, the number of vanishing moments (somehow related to the time duration of these wavelets) shows no significant influence. However, Daubechies-4 constantly provides the second BER performance at high SNR values, fact which enforces the conclusions above.

Further work on this area will envision two aspects: the influence of the number of DWT iterations and the WOFDM performance on a frequency selective Rayleigh channel. Indeed, the multicarrier techniques can show their efficiency especially in channels which introduce strong Inter Symbol Interference because of their frequency selectivity. For the number of iterations involved in the DWT computation, we will try to show that, in the flat fading radio channel, the highest BER is observed at the transmission scales where longer duration carriers are used.

ACKNOWLEDGEMENT

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