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Performance Increase in Multiuser Detection Systems Using Convolutional Encoding

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Abstract – The paper aims to present the usage of turbo codes in multiuser detection systems. The result for bit error rate is shown here and they are obtained from Matlab simulation. Results for multiuser conventional detector, multiuser optimum detector as well as combination of turbo and multiuser detection are illustrated.

Keywords: BER, conventional/ optimum detector, (non) orthogonal codes

I. INTRODUCTION

The idea of using multiuser detection algorithms in mobile communication systems is of increasing interest, since the number of users in such a system increases continuously, and, therefore, the performance of the overall system decrease with the number of users. Multiuser detection algorithms are efficient at base station level, where all the signals are demodulated. Using the cross-correlation information between those users, the signals can be combined in such a way as the overall system performance, measured in terms of signal quality or Bit Error Rate (BER) increase. There are still issues in implementing such algorithms at mobile user level, but such problems are under study.

However, with the classical multiuser detection algorithms, there are important issues when the users are not perfectly orthogonal and when they have un-equal amplitude. In those cases the BER have significant increases.

The idea presented in this paper was to use multiuser detection algorithms in conjunction with turbo-encoding technique in order to reduce the effects of non-orthogonality and of different amplitude effects on the system performances.

Several cases have been studied. In this paper will be presented the results obtained by using conventional and optimum detectors and turbo-encoding with MAP / SOVA decoding algorithms, for a system with two and four users, in perfect and un-perfect cross-correlation and amplitude balance conditions.

Extended Monte Carlo simulations have been performed in order to determine the results presented. Several important conclusions have been highlighted based on those results.

However, simulations are still under development, for other multiuser algorithms, for different numbers of users and in different channel conditions.

II. CONVENTIONAL DETECTOR

The basic CDMA N user system model assumes that all users transmits binary data and antipodaly synchronous signature waveforms [1,2,6]. The channel is affected only by additive white Gaussian noise (AWGN), The received signal is therefore,

$$y(t)=\sum_{k=1}^{N}A_k b_k s_k(t)+\sigma n(t) \qquad t\in[0,T] \tag{1}$$

where T is the bit period, b_k {-1,1}is the information bit transmitted by user k during time interval T, A_k is the amplitude of data received from user k , $n(t)$ is the AWGN with unit power spectral density (which models the thermal noise and all other noise sources unrelated to the transmitted signals) and σ is the standard deviation of the noise. The code sequences are normalized such that

$$\int_0^T s_k^2(t)dt=1,\ (\forall)k \tag{2}$$

The cross-correlation between the i^{th} user sequence and the j^{th} one is defined by

$$\rho_{ij}=\int_0^T s_i(t)s_j(t)dt<1 \tag{3}$$

and the cross-correlation matrix is given by

$$\mathbf{R}=\begin{pmatrix}\rho_{11} & \rho_{12} & .. & \rho_{1N}\\ \rho_{21} & \rho_{22} & .. & \rho_{2N}\\ .. & .. & .. & ..\\ \rho_{N1} & \rho_{N2} & .. & \rho_{NN}\end{pmatrix} \tag{4}$$

The simplest strategy to demodulate CDMA signals is the use of a bank of matched filters that operates simultaneously, each of them matched to one user signature signal $h_k(t)=s_k^*(T-t),\ k=1,\ N$. It represents the simplest linear detection approach, and

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maximized the signal to noise ratio at each matched filter output, assuming that the channel noise and interference may be assimilated with AWGN. The block diagram of the conventional detector is shown in figure 1.

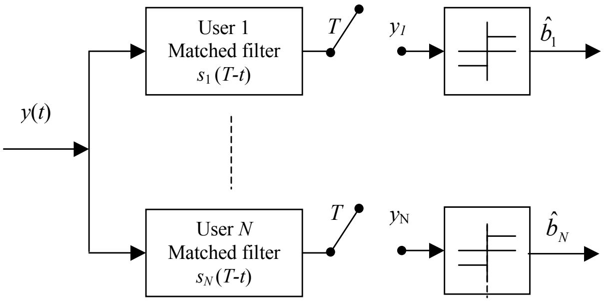

Fig. 1. Conventional Detector

For the received signal given in (1), the k^{th} user matched filter output, sampled at the end of the bit period is

$$y_k = \int_0^T y(t) s_k(t) dt = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k, \; k = 1, N \tag{5}$$

where

$$n_k = \sigma \int_0^T n(t) s_k(t) dt, \quad k = \overline{1, N} \tag{6}$$

is a gaussian random variable with zero mean and variance σ^2. Using a matrix representation, (5) becomes,

$$\mathbf{Y} = \mathbf{RAb} + \mathbf{N} \tag{7}$$

where $\mathbf{Y} = [y_1, y_2, \ldots, y_N]^T$ is a column vector that includes the outputs of the matched filters, **R** is given in (4), $\mathbf{A} = diag\{A_1, A_2, \ldots, A_N\}$ is the matrix of the amplitudes of the received bits, $\mathbf{b} = [b_1, b_2, \ldots, b_N]^T$ is a column vector that contains the bits received from all users, and $\mathbf{N} = [n_1, n_2, \ldots, n_N]^T$ is the sampled noise vector., such that

$$E[\mathbf{N} \cdot \mathbf{N}^T] = \sigma^2 \mathbf{R} \tag{8}$$

The estimated bit, after the threshold comparison, is

$$\hat{b}_k = \mathrm{sgn}(y_k) = \mathrm{sgn}(A_k b_k + n_k) \tag{9}$$

III. OPTIMUM DETECTOR

The conventional receiver requires no knowledge beyond the signature waveforms patterns and the timing for all users in order to demodulate the received signal coherently. The results obtained by the conventional detectors are near optimum for a large number of equal power users, in accordance to central limit theorem. However, if those conditions are not fulfilled, the results are no longer accurate.

The optimal detection strategy, that ensures a minimum error probability for each user, has to take jointly decisions on each user data based on *Maximum Aposteriory Likelihood* (MAP) Criterion.

The received signal is

$$y(t) = A_1 b_1 s_1(t) + A_2 b_2 s_2(t) + \sigma n(t), \quad t \in [0.T] \tag{10}$$

The decison are made using aposteriori probablities. After algebraic manipulations, the estimated bits are

$$\begin{cases} \hat{b}_1 = \mathrm{sgn}\left(A_1 y_1 + \frac{1}{2}|A_2 y_2 - A_1 A_2 \rho| - \frac{1}{2}|A_2 y_2 + A_1 A_2 \rho| \right) \\ \hat{b}_2 = \mathrm{sgn}\left(A_2 y_2 + \frac{1}{2}|A_1 y_1 - A_1 A_2 \rho| - \frac{1}{2}|A_1 y_1 + A_1 A_2 \rho| \right) \end{cases} \tag{11}$$

where y_1 and y_2 are the correlator outputs. It can be easily observed that the results are affected by both the cross – correlation coefficients and the amplitude of the users. A system that implements the above relations is shown in figure 2.

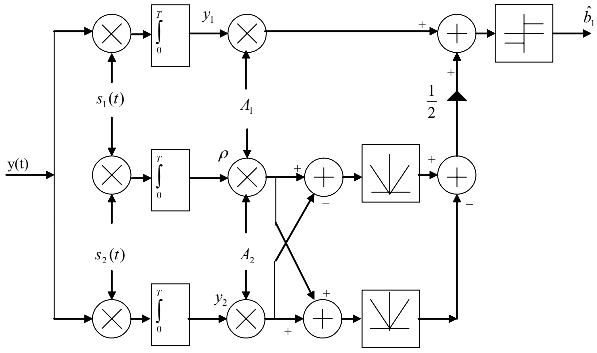

Fig. 2. The optimal muliuser detecor for user 1 in a 2-users system

It can be easily observed that the system implementation is a very complex one, even in the case of a system with two users. However, since the results are better then the ones obtained by the conventional detector, implementations have been made for more then two users based on DSP chips or FPGA.

IV. SIMULATION RESULTS

A. Two user case with orthogonal codes and equal signal power

In the results presented above, the following notations will be used The notations for the simulations are:

- *Mi* for the *i* user to conventional/optimum detector
- *T* for turbo coder/decoder
- *TMi* for the *i* user at turbo-conventional/optimum detector system

A1. ***The conventional detector system with turbo encoding*** with the following characteristics: 3/4 degree polynomial, Log-MAP/SOVA decoding algorithm, conventional detector with 2 users having equal powers and using orthogonal spreading codes.

The vector for received amplitudes is A=[3 3] and the normalized orthogonal spreading codes are:

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$$s1 = [1\ \ 1\ \ 1\ {-1}\ \ 1\ \ 1\ \ 1\ {-1}]/\sqrt{8} \qquad (12)$$

$$s2 = [1\ \ 1\ \ 1\ {-1}\ {-1}\ {-1}\ {-1}\ \ 1]/\sqrt{8} \qquad (13)$$

The number of bits sent by the user is 500. Depending on the noise power and power for the signals sent by the users, different BERs can be achieved as seen in Figures 3 and 4:

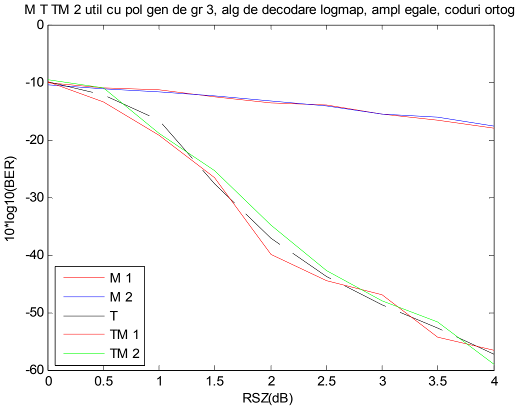

Fig. 3 BER versus SNR for conventional detector, generator polynomial of degree 3 and LogMAP decoding algorithm

An important improvement can be observed when using turbo encoding / decoding with conventional detector that has a variation of BER from -9.53 dB to -17.64 dB for a SNR of 0 dB to 4 dB while the whole system varies from -9.53 dB to -59.09 dB; for example, for a SNR of 2dB we obtain an improvement in BER of 29.96dB, while at 4dB the improvement is of 39.56 dB.

It also can be observed that the turbo-multiuser system has a much faster decrease of SNR when BER increases, even for low SNR's. Moreover, the turbo system compensates the effect of the second user. Since both users operates in the same conditions, the results for turbo-encoded multiuser data have been presented only for one user.

In figure 4 the results obtained using MAP and SOVA algorithms for the same two users systems with ideal cross-correlation and amplitude balance conditions.

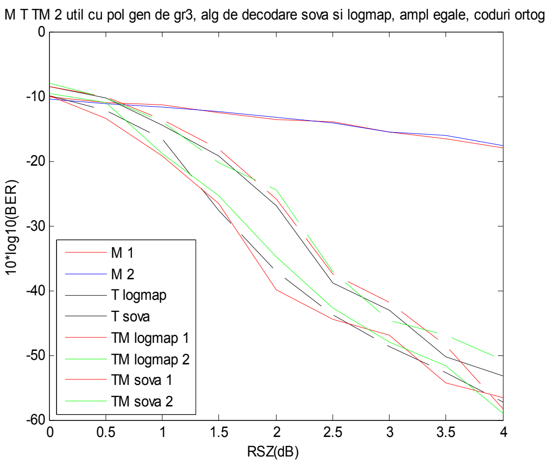

Fig. 4 BER versus SNR for conventional detector, LogMAP and SOVA decoding algorithm

As expected, the results obtained by using the log-MAP decoding algorithm are slightly better then the ones obtained by using the SOVA one. However, the complexity of the algorithm increases exponentially.

Some interesting results have been obtained when comparing 3-degree and 4-degree generator polynomials. All the results presented are obtained using the logMAP algorithm.

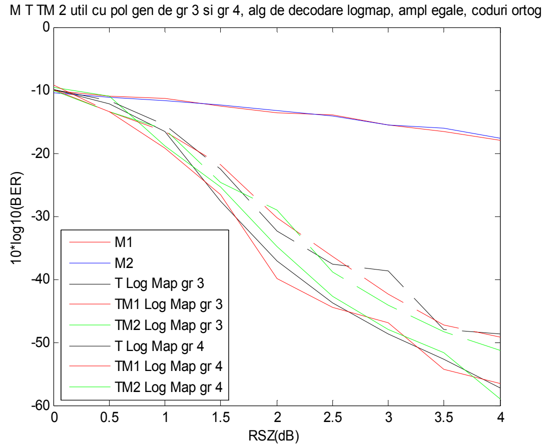

Fig. 5 BER versus SNR for conventional detector, generator polynomial of degree 3 and 4, LogMAP decoding algorithm

It can be observe that the results obtained by using a 4 degree encoding polynomial are slightly better then the ones obtained with the ones obtained with the polynomial of degree 3 (for example, at SNR=2dB we have an improvement of 5.71dB). However, the computational effort and the simulation increase very much, and the improvement does not justify the effort.

A2. ***The optimum detector system with turbo encoding***; the simulation parameters are:

- -the vector for received amplitudes is A=[2 2]
- -the normalized orthogonal spreading codes are (12), (13)
- The number of bits sent by the user is 500.

The simulation results are given in figures 6 and 7.for Log MAP / SOVA detection algorithms, as well as for the simple turbo detector (without the secondary user).

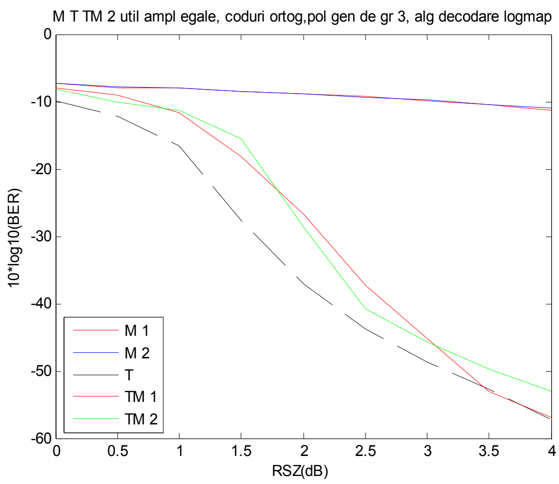

Fig. 6 BER versus SNR for optimum detector, generator polynomial of degree 3 LogMAP decoding algorithm

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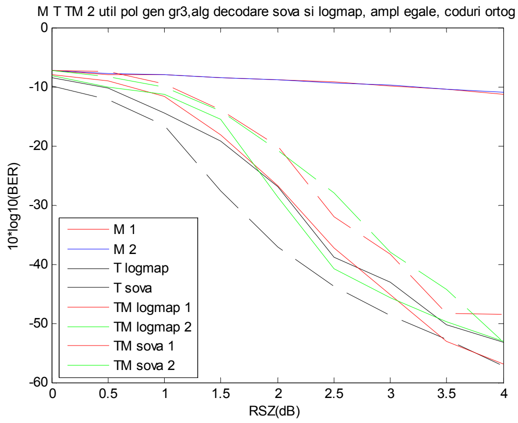

Fig. 7 BER versus SNR for optimum detector, generator polynomial of degree 3 LogMAP/SOVA decoding algorithms

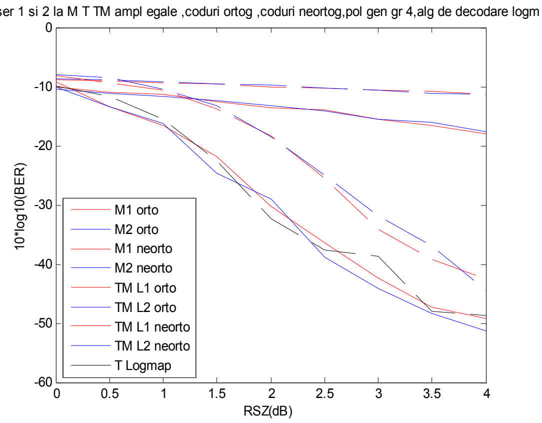

Fig. 8 BER versus SNR for conventional detector, using generator polynomial of degree 3 and LogMAP decoding algorithm

The simple turbo-system has better performances then the optimum multiuser one, but this is not of much relevance, since the results are not evaluated in a multiuser environment.

In this case the logMAP and SOVA algorithms have closer results then in the conventional multiuser system case. For example, at SNR=3dB, the difference between SOVA and LogMAP algoritms is of 7.78dB

B. Two user case with non-orthogonal codes and equal signal power

In a real case, the users not perfectly orthogonal at the receiver input. Even if their orthogonality is insured at the transmitter, because of the nonlinearities of the channel or other disturbance effects they become more or less correlated.

In the following we will analyze the performances of conventional and optimum detectors with turbo decoding when the two users have equal power but non-orthogonal codes.

B1. *The conventional detector system with turbo encoding*

We will consider a two user system, that uses the spreading codes

$$s1 = [1\ \text{-}1\ \text{-}1\ \ 1\ \ 1\ \text{-}1\ \ 1\ \text{-}1]/\sqrt{8} \qquad (14)$$

$$s2 = [1\ \text{-}1\ \ 1\ \text{-}1\ \text{-}1\ \ 1\ \text{-}1\ \ 1]/\sqrt{8} \qquad (15)$$

The number of bits sent by the user is 500 and the cross-correlation matrix is:

$$R = \begin{pmatrix} 1 & -0.5 \\ -0.5 & 1 \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \qquad (16)$$

The main difference with respect to previous case is the fact that the cross-correlation between the 2 users is -0.5.

The results obtained in terms of BER as a function of SNR are shown in figure 8.

It can be seen that the BER performances are severely degraded for strongly correlated user with respect to the uncorrelated ones (the difference is of 8.72dB for SNR = 3.5 dB, and increases as BER increase). At SNR = 0 dB Beer's value is -8 dB, the same as in the orthogonal signals case. As the cross-correlation coefficient between the two users increase, the performances become worse.

B2. *The optimum detector system with turbo encoding*

In the case of a optimum multiuser detector with turbo-decoding, when the users are not orthogonal as in previous case, the results are shown in figure 9.

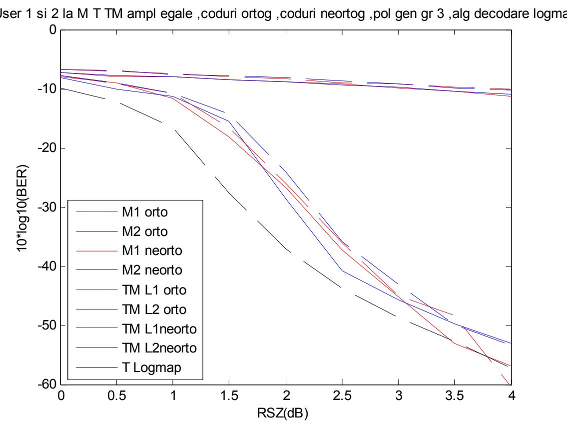

Fig. 9 BER versus SNR for optimum detector, using generator polynomial of degree 3 and LogMAP decoding algorithm

As expected, the results are severely degraded by the cross-correlation coefficient between the two users. From the figure it can be seen that the BER increases from -6.81 dB for SNR = 0 dB up to -10.88 dB at SNR = 4 dB. In case of turbo-optimum detector BER is-7.78 dB for SNR = 0 dB reaching to -53.47 dB at SNR = 4 dB;

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C. Two user case with orthogonal codes and non-equal signal power

Next, we will analyze the effects of un-equal amplitude of the users using ideal correlation codes. This means that one user uses more power then the second one, so its performances will be better in detriment of the less powerful one. As before, we will analyze the conventional and optimum multiuser detector in terms of BER versus SNR

C1. ***The conventional detector system with turbo encoding***
In our simulations, we considered that the received amplitude vector is A=[1.5 4] and the normalized spreading codes are:

$$s1 = [1\ \ 1\ \ 1\ -1\ \ 1\ \ 1\ \ 1\ -1]/\sqrt{8} \qquad (16)$$

$$s2 = [1\ \ 1\ \ 1\ -1\ -1\ -1\ -1\ \ 1]/\sqrt{8} \qquad (17)$$

The number of bits sent by the user is 500. The results are shown in figure 10.

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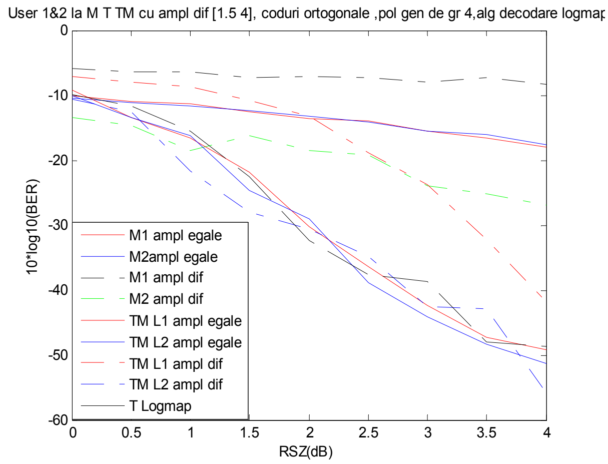

Fig. 10. BER versus SNR for conventional detector, for two users with different signal amplitudes, using generator polynomial of degree 3 and LogMAP decoding algorithm

It can be seen that the more powerful user has very good BER results, close to the ones obtained in the ideal case, when turbo encoding is used, but the performances of the second one are worse. The turbo encoding corrects some of the errors for the second used too, but it still does not achieve the same results as in the ideal case.
As an example, it can be seen that if the signal amplitudes are different in power with an amount of 3.5 dB (amplitude 1.5) we can observe an increase of BER of 17.52 dB at SNR= 2.5 dB and of 6.83dB when SNR is 4 dB BER. I

B1. ***The optimum detector system with turbo encoding***
Using the same simulation conditions for the optimum detector strategy, the results are shown in figure 11. The only difference is that the amplitude vector is [1;2] meaning that, if the 1st user has a power of 0 dB the second one has a power with 6 dB larger.

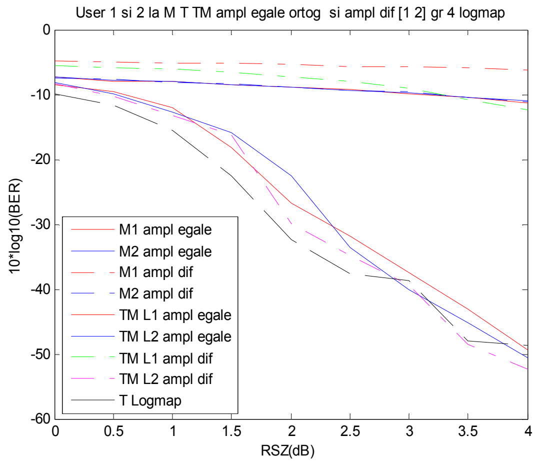

Fig. 11. BER versus SNR for optimum detector, for two users with different signal amplitudes, using generator polynomial of degree 3 and LogMAP decoding algorithm

As expected, the more powerful user has BER results close to the optimal case, with and without turbo encoding. The second user has very poor results, but the turbo encoding compensates part of this loss. However, the results are far from the optimum case.
As an example, it can be observed that for 0 dB power the system acts as a simple optimum detector starting from BER = -5.46 at SNR = 0 dB and achieving BER = -12.29 at SNR = 4 dB with a slight decrease.

D. Four users case with orthogonal codes and equal signal power.

In the following we will investigate the results obtained with 4 users in ideal conditions.
In the simulations we will consider that the received amplitudes vector is [3 3 3 3] and the spreading codes are:

$$s1 = [1\ \ 1\ \ 1\ -1\ \ 1\ \ 1\ \ 1\ -1]/\sqrt{8} \qquad (18)$$

$$s2 = [1\ \ 1\ \ 1\ -1\ -1\ -1\ -1\ \ 1]/\sqrt{8} \qquad (19)$$

$$s3 = [1\ \ 1\ -1\ \ 1\ \ 1\ \ 1\ -1\ \ 1]/\sqrt{8} \qquad (20)$$

$$s4 = [1\ \ 1\ -1\ \ 1\ -1\ -1\ \ 1\ -1]/\sqrt{8} \qquad (21)$$

The number of bits used for Monte Carlo simulation is 500. The results are shown in figure 12 for the conventional detector.
The results are similar with the ones obtained in the two user case, with the remark that the existence of more users degrade the overall performances of the system.
As a numerical example, it can be seen that the conventional detector has a small variation of BER for SNR between 0 and 4 dB meaning -9.89 dB to -17.63 dB, for all 4 users. Compared to the turbo-conventional detector system BER decreases with 23.16 dB at SNR = 2 dB and for SNR = 4 dB BER decreases with 40.37 dB. So the turbo coding/decoding technique compensates the effect of the other users.

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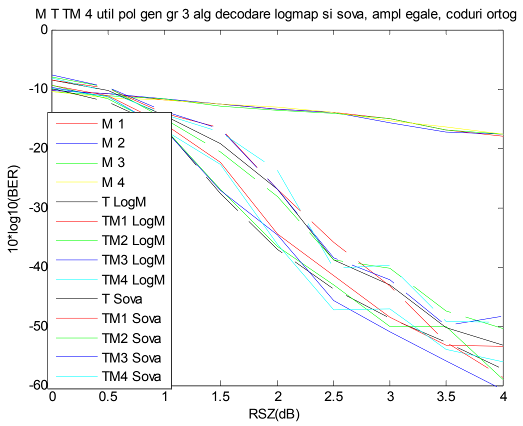

Fig. 12. BER versus SNR for conventional detector, for 4 users with equal amplitudes, using LogMAP and SOVA decoding algorithms

E. **Four users case with non-orthogonal codes and equal signal power.**

The non-orthogonal codes are:

$$s1 = [1\ -1\ -1\ \ 1\ \ 1\ -1\ \ 1\ -1]\,/\sqrt{8} \tag{22}$$

$$s2 = [1\ -1\ \ 1\ -1\ -1\ \ 1\ -1\ \ 1]\,/\sqrt{8} \tag{23}$$

$$s3 = [1\ -1\ \ 1\ \ 1\ \ 1\ 1\ -1\ -1]\,/\sqrt{8} \tag{24}$$

$$s4 = [1\ -1\ \ 1\ -1\ \ 1\ -1\ \ 1\ \ 1]\,/\sqrt{8} \tag{25}$$

and the cross-correlation matrix is:

$$R=\begin{pmatrix} 1 & -0.5 & 0.25 & 0.25 \\ -0.5 & 1 & 0.25 & 0.25 \\ 0.25 & 0.25 & 1 & 0 \\ 0.25 & 0.25 & 0 & 1 \end{pmatrix}=\begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix} \tag{26}$$

It can be seen that users 1 and 2 are strongly correlated (cross-correlation coefficient of 0.5), while users 3 and 4 are not. The results are shown in figure 13.

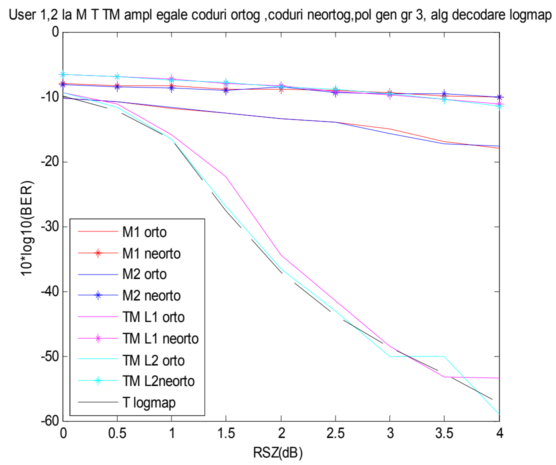

Fig. 12. BER versus SNR for conventional detector, for 4 users with non-orthogonal codes

It cand be seen that the turbo-encoding cannot compensate the effect of the strong cross-correlation between users 1 and 2, while users 3 and 4 behave close to the ideal case. For example for SNR = 3 dB we have an increase of 39.20dB in terms of BER, while and at SNR = 4 dB the increase is of 45.72dB;

V. CONCLUSIONS

One important conclusion that can be highlighted from the results above is that use of turbo encoding in connection with multiuser detection brings important improvement in terms of BER in ideal conditions. The LogMAP algorithm is more powerful then the SOVA one, but the computational effort is significantly higher. The turbo encoding compensates the effect of the other users in the system

If the users are not perfectly orthogonal, the performances of the system, with both optimum and conventional detectors are severely degrade. As the users are closely correlated, the performances of the overall system are worse. The turbo encoding compensates some of the errors, but the overall system performances are still worse then the ideal case.

When the users have orthogonal codes but their amplitude are not the same, the stronger user has better performance, in detriment of the less powerful one. Still the turbo encoding proves to be beneficial for both users, its effect on the less powerful user being more important than on the more powerful one.

The results can be extended in the case of four users, with similar conclusions. However, as the number of users increase, the performances of the system are degrading. The non-orthogonality of the users is a big draw-back of the system, and it can hardly be compensate by using convolutional encoding. The difference in amplitude is also a detriment, but it can more easily be encountered by coding.

Simulations are still under development for systems that are using other multiuser algorithms, longer spreading codes or a larger number of users.

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