

Image restoration through fusion and diffusion

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Abstract – This paper presents a new framework for image restoration using the Partial Derivatives Equations (PDE) approach and image fusion techniques. The degraded image is processed independently using two or more PDE filters than the results are combined through fusion, the final processed image being obtained by a recurrence of the previous steps. The proposed framework allows combinations of any PDE based filters for processing adaptively a given region of the input image with the most appropriate filter. Preliminary results show that this approach is very efficient in designing new image restoration techniques.

Keywords: restoration, PDE, image, fusion

I. INTRODUCTION

Digital image processing had, has and will continue to have an extremely important role in applications from various domains. Virtually any domain uses image acquisition devices complemented with processing facilities for acquiring, processing, representing and interpreting the visual information. Despite the recent technological advances, the correction of the distortions introduced by the acquisition devices is still a problem: blur due to intra-scan movement or to insufficient resolution, noise due to the transmission from distant imaging devices, noise due to the technical limitations of the acquisition equipments. Image processing includes a sub domain – image restoration - that deals with all these problems and produces the closest possible image to the distortion-free original. The PDE framework allows the implementation of virtually any operator that can be then applied for various image processing tasks. PDE-based restoration techniques are modeling an image using a three valued function for the luminance of a pixel $-U(x,y,t)-$; the restoration process is then modeled by the evolution in time of an EDP whose characteristics are imposing the properties of the restored result. The solution, computed for a given time t , defines the restored/enhanced image and is subject to boundary conditions (Neumann typically) and/or of initial values. By imposing the properties of the EDP one can control the aspect of the processed image: the restoration process can be diminished in intensity or even inverted; the filter can smooth or enhance the edges of the image depending on the

spatial partial derivatives of the evolving image. In most cases the equations are of diffusion type (backward or forward). Whilst smoothing an image is equivalent to the heat propagation process, edge enhancement can be modeled by an inversion of the previous process.

Several authors concentrated recently in proposing complex PDE based restoration techniques by using combinations of PDE based filters in the spatial domain. Such approaches are quite heavy both from a theoretical and a practical point of view: when elaborating new PDEs the mathematical soundness must be addressed and the actual discrete model must be also developed. Moreover, due to the non-stationarity of the input image, such approaches can lead to “toy”, artificial like results.

The approach we are proposing is different and, up to our knowledge, no similar solutions exist in the literature. Instead of proposing a new equation to restore an image, we process the image with two or more different PDE filters and we combine the results through fusion. The final result is obtained by a recurrence of the previous steps and is computed using already proven theoretical and discrete PDE based models. Most of the intelligence of such an approach relies on the fusion rule and on the expertise needed to combine different PDEs. The approach is detailed in Section IV and its efficiency in image restoration tasks is illustrated in Section V.

II. PDE BASED FILTERS FOR IMAGE RESTORATION AND ENHANCEMENT

The domain was practically born with the introduction of the anisotropic diffusion method [10]:

$$\frac{\partial U}{\partial t} = \text{div}(c(x, y, t)\nabla U(x, y, t)) \quad (1)$$

Through the introduction of a diffusivity $c(.)$ as a function of the gradient vectors of the evolving image:

$$c(x, y, t) = g(|\nabla U(x, y, t)|) = \frac{1}{1 + \left(\frac{|\nabla U(x, y, t)|}{K}\right)^2} \quad (2)$$

the method eliminates the drawbacks of the isotropic

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diffusion equation (edge displacement, pattern elimination), assimilable to a Gaussian convolution :

$$\frac{\partial U}{\partial t} = \text{div}(\nabla U(x, y, t)) = \Delta U = U_{xx}(x, y, t) + U_{yy}(x, y, t) \quad (3)$$

The properties of the equation can be more easily analyzed if it is put in terms of directional derivatives, considered on the directions of the textures ($\vec{\xi}$) and on the orthogonal ones ($\vec{\eta}$) [13]:

$$\begin{aligned} \frac{\partial U}{\partial t} &= c_{\xi} U_{\xi\xi} + c_{\eta} U_{\eta\eta} \\ c_{\xi} &= g(|\nabla U|) \\ c_{\eta} &= g|\nabla U| + |\nabla U|g'(|\nabla U|) \end{aligned} \quad (4)$$

Contrary to (3), which induces a smoothing process in each pixel, the equation can have zero or negative coefficients in the $\vec{\eta}$ directions, freezing (or inverting) thus the smoothing process. Accordingly, the equation allows the coexistence of apparently complementary processes in each pixel: smoothing along the texture directions and enhancement in the gradient directions. Despite the impressive experimental results, the method has been criticized by several authors [12], [4] that addressed practical problems (noise can be enhanced too) or mathematical formulation problems (due to the edge enhancing effect the equation does not have a unique solution). However several researchers used this model for proposing evolved image restoration PDE-based methods; we only mention here the most important: the ‘‘mean curvature motion’’ filter developed in an axiomatic framework [2], total variation based diffusion filters, tensor driven diffusion filters for edge enhancement [16], [13]. The common characteristic of all these filters resides in the fact that they are using as diffusion directions the directions given by the gradient vectors and the orthogonal ones, computed usually on a Gaussian pre-smoothed image.

Using the same PDE formalism other authors proposed diffusion methods for blur elimination, [9],[17],[5]. The fundamental equation was proposed first by Osher and Rudin:

$$\frac{\partial U}{\partial t} = -\text{sign}(U_{\eta\eta})|\nabla U| \quad (5)$$

and it corresponds to a deliberate inversion of the smoothing process. The stability of the method is the continuous domain cannot be assured and the authors devoted a great part of their work in proposing suitable discrete approximations using the framework of hyperbolic PDEs and viscosity solutions. In practical approaches the equation uses modified initial values, computed by Gaussian pre-smoothing; if the

standard deviation of the kernel is supposed to be σ the equation can be then written:

$$\frac{\partial U}{\partial t} = -\text{sign}((G_{\sigma} * U)_{\eta\eta})|\nabla U| \quad (6)$$

Other authors judged that the combination of the effects of more than one PDE could prove useful. We only mention here the filter introduced in [7], directly in directional terms:

$$\begin{aligned} \frac{\partial U}{\partial t} &= \alpha_{\tau}(U - U_0) + \alpha_{\tau}[h_{\tau}(|G_{\sigma} * \nabla U|)U_{\eta\eta} + U_{\xi\xi}] \\ &\quad - \alpha_c[1 - h_{\tau}(|G_{\sigma} * \nabla U|)]\text{sign}(G_{\sigma} * D)|\nabla U| \end{aligned} \quad (7)$$

The filter has a selective behavior (unidirectional smoothing, isotropic smoothing, enhancement) depending on a fuzzy threshold function $h(\cdot)$. The coefficients α_i are weighting the contribution of each filter in an intuitive way.

More recently a series of researchers proposed methods different in spirit. By addressing the noise sensitivity of gradient-based orientation estimators, they proposed diffusion models integrating smoothed directional information, computed through a Principal Component Analysis (PCA). In [16],[17] the following tensor driven diffusion model is proposed:

$$\begin{cases} \frac{\partial U}{\partial t} = \text{div}(D\nabla U) \\ D = (\vec{v}_1 \mid \vec{v}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \vec{v}_1 \\ \vec{v}_2 \end{pmatrix} \end{cases} \quad (8)$$

Eq. (8) has diffusion directions given by the eigenvectors of the covariance matrix of the gradient field, prior pre-smoothed using a Gaussian filter:

$$J_{\rho}(\nabla U_{\sigma}) = \begin{pmatrix} G_{\rho} * \left(\frac{\partial U_{\sigma}}{\partial x}\right)^2 & G_{\rho} * \frac{\partial U_{\sigma}}{\partial x} \frac{\partial U_{\sigma}}{\partial y} \\ G_{\rho} * \frac{\partial U_{\sigma}}{\partial x} \frac{\partial U_{\sigma}}{\partial y} & G_{\rho} * \left(\frac{\partial U_{\sigma}}{\partial y}\right)^2 \end{pmatrix} \quad (9)$$

The anisotropic behavior of the filter is imposed by the modified choice of the two eigenvalues in (8) λ_i . Whilst the original filter performs extremely well in processing unidirectional textures, a modified version that can process efficiently multidirectional textures has been proposed in [15]. The same paper [15] proposes a linear version of the filter, much faster than the non-linear one. These methods had been then generalized and adapted for processing seismic images in [8].

Finally let us consider the model introduced in [13], [14]:

$$\frac{\partial U}{\partial t} = c_{\xi} U_{\xi\xi} + c_{\eta} U_{\eta\eta} \quad (10)$$

with:

$$\begin{cases} c_{\xi} = \frac{\partial}{\partial \xi} g^{\xi}(|U_{\xi}|U_{\xi}) \\ c_{\eta} = \frac{\partial}{\partial \eta} g^{\eta}(|U_{\eta}|U_{\eta}) \end{cases} \quad (11)$$

The PDE combines local information (directional derivatives) with semi-local information (diffusion directions). This filter allows the existence of enhancement processes both in the texture directions and in the orthogonal ones. The filter proved to be better in terms of performances; its superiority is demonstrated in [13] for the restoration of images composed of oriented patterns; an analysis of variance (ANOVA) was performed in order to investigate and prove the relevancy of the experimental results. The method has been generalized for the 3D case in [11].

III. IMAGE FUSION TECHNIQUES

Image fusion was born for combining information contained in a single image (captured eventually with different sensors) or a collection of images; this type of processing uses thus two or more input images and envisages fusing the complementary information for a better visualization through a better quality image. [6]. Information contained in the input images can be combined using pixel-level, feature-level or decision-level based fusion rules. Pixel-level fusion is the simplest among the three methods and refers to those methods that are working on pixel-by-pixel basis, without trying to integrate information that is semantically meaningful at an upper layer (edges, junctions, lines, homogenous regions). Feature-level fusion usually involves some kind of preprocessing for extracting an image attribute (seldom by segmentation); fusion is then carried out on a feature-by-feature basis. The last type, decision level fusion employs a hierarchical image description in terms of relational graphs and uses, seldom, pattern recognition techniques [6].

Information fusion can be carried out in the spatial (by a simple weighting operation) or in the transform domain. The most used and studied methods are those relying on a pyramidal decomposition of the input images. For a single output image the process is illustrated on the following figure [3]:

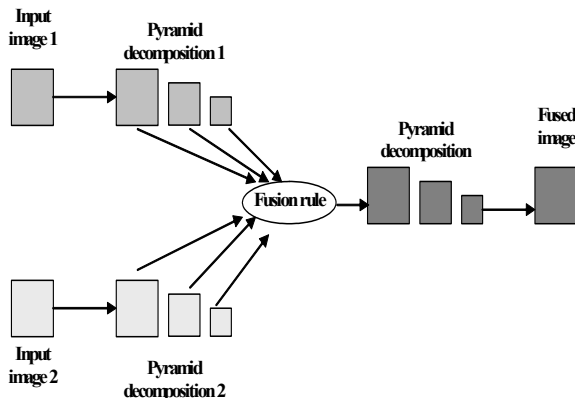


Fig. 1. Image fusion through pyramid decomposition
Each input image is first decomposed in Gaussian and Laplacian pyramids. The input image is equivalent to the first level of Gaussian decomposition $G_0(x, y) \equiv U(x, y)$ and the subsequent levels are obtained by low-pass filtering using a smoothing

kernel (w) followed by subsampling:

$$G_k = [w * G_{k-1}]_{\downarrow 2} \quad (12)$$

(\downarrow_2 denotes sub-sampling operator with 2)

By iterating (12) each level G_k corresponds to a low pass filtered version of the previous level.

To each Gaussian level a Laplacian one can be associated:

$$L_k = G_k - 4w * [G_{k+1}]_{\uparrow 2} \quad (13)$$

(\uparrow_2 denotes the upsampling operator with 2)

A Laplacian level represents a high-pass filtered version of the previous level.

The fusion rule operates on each level of decomposition; if for example multi-focus image distortions are considered a simple „choose max” operating on the Laplacian levels and issuing the fused levels \tilde{L}_k can be used to deal with it. The fused image is then reconstructed using iteratively:

$$\hat{G}_k = \tilde{L}_k + 4w * [G_{k+1}]_{\uparrow 2} \quad (14)$$

The final reconstructed Gaussian level \hat{G}_0 gives then the reconstructed image.

Other pyramid decomposition based fusion methods do exist; we only mention here the ratio of low pass pyramid and the FSD pyramid [6].

IV. PROPOSED APPROACH

Within the PDE framework a complex restoration process is defined generally through use of linear combinations of basic PDEs that will combine accordingly the characteristics of the corresponding filters. Let's take for example equation (7); the PDE embeds in itself three different filters: isotropic diffusion for processing homogeneous regions (3), choc filters for enhancing edges (6), and mean curvature motion filters for smoothing along edges. The behavior of the equation is decided locally with respect of the relationship of the gradient norm to a given threshold. An example of the application of such a filter is shown in Fig.2.

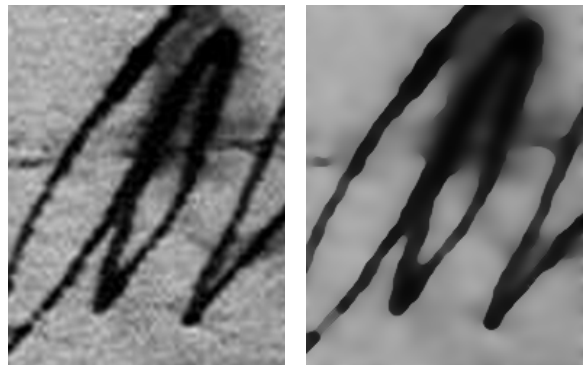


Fig. 2. Complex restoration process through use of eq. (7) From left to right: original image, restored result

One of the main disadvantages of such an approach is that it is quite restrictive in the choice of the constituent filters. Being put directly in terms of directional interpretation, all the PDEs that it embeds must have specific directional interpretations in order

to justify theoretically the properties of the filter. For example, the middle term in eq. (6) includes a mean curvature motion PDE ($\alpha_r U_{\xi\xi}$) for the specific purpose of transforming its unidirectional smoothing action in an isotropic one $\alpha_r (U_{\eta\eta} + U_{\xi\xi})$, whenever the gradient norms falling below a given threshold. Despite being extremely effective in denoising, the same term leads to an over-smoothing of the input image since it under its action any non convex curve will be transformed into a convex one, collapsing finally into a point that will disappear for higher scales t . Being to local, the method can also lead to false results.

The approach we are proposing is different in spirit. Instead of combining local PDE behaviors we take a higher level of abstraction and we are interested in combining results produced by different PDEs through use of image fusion techniques. Without loss of generality, the framework we are proposing is shown graphically in Fig.3 for a restoration process based on the use of two different PDEs.

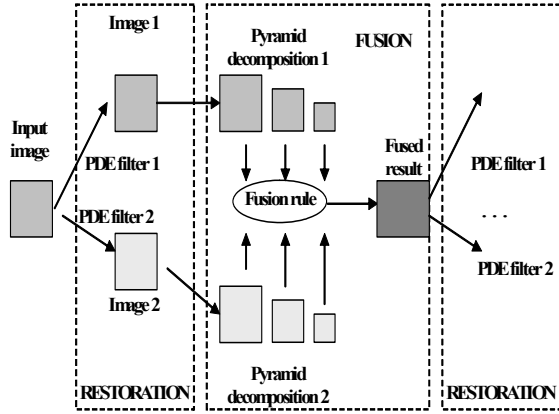


Fig.3. Image restoration through fusion and diffusion

Contrary to existing approaches, we process the entire input image with the same filter and we combine the results using image fusion techniques. The fused image can be further processed using the same mechanism to yield iteratively the solution for a given scale t .

This type of approach allows a complete freedom for the choice of the constituent filters. One can choose for example to combine scalar or tensor driven filters with isotropic or choc PDEs.

Besides the expertise needed for the choice of the PDE based filters, a special attention must be devoted to the fusion rule. Ideally the fusion rule must obey the following constraints:

- when one of the results is significantly more pertinent than the other(s) the fusion rule must allow its selection and insertion in the pyramid decomposition levels.
- at spatial locations where the results are similar the fusion rule must average the results in order to provide a smooth result

In the image fusion framework such a behavior can be obtained using weighted averaging fusion. The pertinence of a result is quantified by a salience measure whereas the similarity between the results is quantified through a match measure.

For image restoration tasks using PDEs, one can choose as salience measure the local energy of the processed region quantified by the variance defined within a neighborhood $W(x,y)$ centered on the pixel under study:

$$\bar{U} = \frac{1}{\text{card}W(x,y)} \sum_{x_i, y_j \in W(x,y)} U(x_i, y_j)$$

$$\sigma^2(x,y) = \frac{1}{\text{card}W(x,y)} \sum_{x_i, y_j \in W(x,y)} [U(x_i, y_j) - \bar{U}]^2 \quad (15)$$

This salience measure is able to quantify the edge preserving properties of a PDE based selective smoothing process.

As match measure any correlation measure can be employed and throughout the rest of the paper we will use a normalized correlation. For the algorithm described in Fig.3 suppose that the salience measures are defined in each pixel of coordinates (x,y) as $\sigma_1^2(x,y), \sigma_2^2(x,y)$; the match measure is then:

$$M_{1,2}(x,y) = \frac{2\sigma_1(x,y)\sigma_2(x,y)}{\sigma_1^2(x,y) + \sigma_2^2(x,y)} \quad (16)$$

$M_{1,2}(x,y)$ has values comprised between 0 and 1, 1 signifying a perfect match between the results produced in the pixel of coordinates (x,y) by two distinct filters.

To impose the selection/averaging fusion rule we must assign weights to each pixel value pertaining to each Gaussian and Laplacian levels of the pyramid decomposition. Let D_{k1} and D_{k2} denote the k level of the pyramid decomposition of the same type, Gaussian or Laplacian. The flowchart of the weights assigning algorithm is shown in Fig.4.

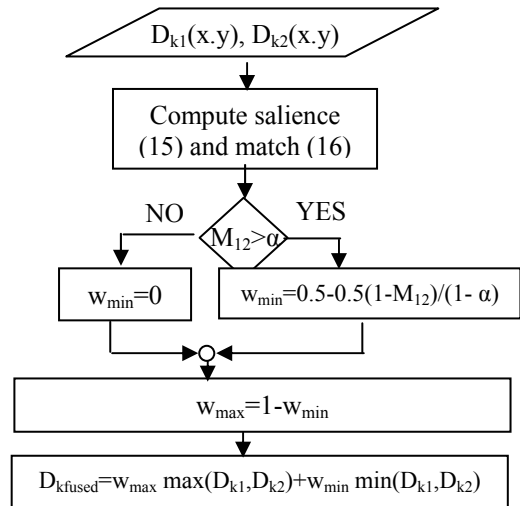


Fig. 4. Weighted averaging of the results the pyramid decomposition domain

The weight assigning algorithm implements the constraints presented previously; if similarity among results is low the rule operates in selection mode and

for high correlations it operates is averaging mode with weights close to 0.5.

For a single level Gaussian and Laplacian decomposition the fused result is computed using the following equation:

$$U_{fused} = \tilde{L}_0 + 4w * [\tilde{G}_1]_{\uparrow 2} \quad (17)$$

with \tilde{L}_0, \tilde{G}_1 denoting the fused Laplacian and Gaussian levels. This result is further used as the initial value of a second restoration step followed eventually by another fusion in the pyramid domain

For decomposition on multiple levels, fusion on the Gaussian domain is only performed on the lowest

level \tilde{G}_{k+1} and the immediate upper level is computed iteratively using:

$$\hat{G}_k = \tilde{L}_k + 4w * [\tilde{G}_{k+1}]_{\uparrow 2} \quad (18)$$

IV. CASE STUDIES. EXPERIMENTAL RESULTS

As me mentioned previously, this framework allows us to combine practically any diffusion filter. We will present only three combinations of PDE filters and we will restrict ourselves in using a single level of decomposition.

Let consider the case of equations (11) and (5). A direct combination in the spatial domain in the spirit of (7) would be difficult to interpret theoretically. The main reason is the fact that (11) uses robust diffusion directions whereas (7) is based on local orientation estimation i.e. the direction η does not have the same meaning on the two equations. However these two filters can be combined easily using the proposed framework. By processing individually the input images with the filter based on (5) edges are enhanced but noise can be amplified too. On the other hand, the use of equation (11) allows restoration of regions combined with an efficient smoothing of the oriented pattern, coupled with junction preservation.

Combination of the intermediate results through selection/weighted average makes sure that only the pertinent information will be re-injected in the intermediate results. For restoring the degraded image from Fig.2 we used 8 steps of restoration consisting each in 30 iterations for explicit approximation schemes for both equations. The result is shown on Fig.5 and is corresponding to 1, 5 and respectively 8 fusion steps. For comparison we show also in Fig.6 the individual results obtained with the two filters corresponding to the same number of iterations (i.e. 240) and the same choice of the parameters.

The choc filter produces some artifacts on the background of the image (constant regions separated by small amplitude jumps) whereas the smoothing term is able to remove noise in this areas. For achieving this goal the parameters of (11) were tuned deliberately to impose mainly a smoothing action leading also to a slight over smoothing. Using the fusion/diffusion approach, information given on edges

by the choc filter is re-injected in the intermediate results leading to a precise edge restoration and enhancement. On the region like part of the image, the match measure computed for the results of the two filters is above the threshold and the effects of the choc filter are removed through averaging.

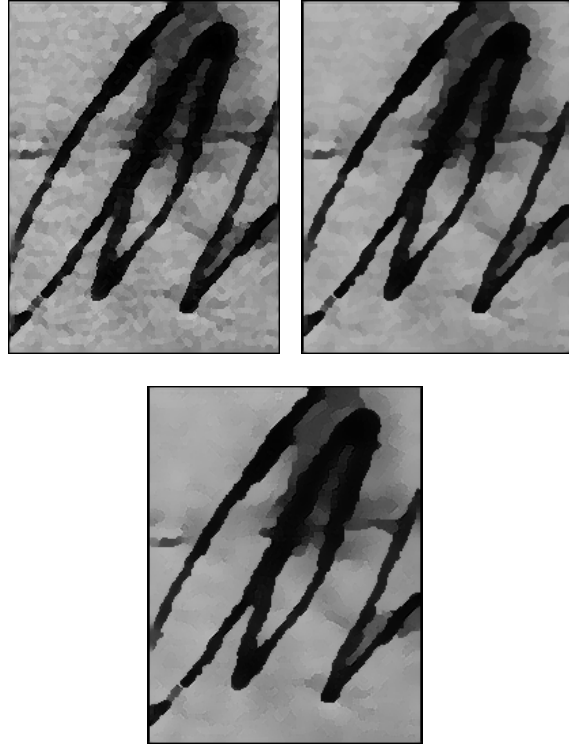


Fig. 5 Restoration of the image in Fig.2 with the proposed approach. From top to bottom and left to right: recurrent steps of fusion/diffusion (see text for details)

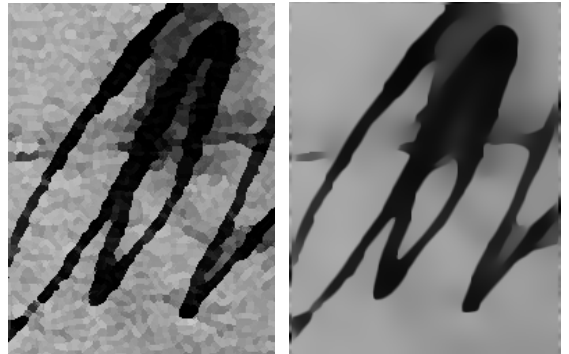


Fig.6 Restoration of the image from Fig.2 using dedicated filters. From left to right: result obtained using (5), result using (11)

A second example for the proposed framework consists in the elimination of blocking effect due to JPEG compression. We want to endow the filter with smoothing capabilities together with the capacity of preserving as much details as possible. For this task we selected the classical Perona Malik filter (1) and the same choc filter (5). Basically the same effect is observable on the result obtained when processing the original image in Fig.7; the region like part is efficiently restored using the smoothing properties of the Perona-Malik filter whereas junctions and details of the image are better retained by the combined

effect of the choc filter and the edge enhancing action of the anisotropic diffusion equation.



Fig.7 Removal of blocking artifacts using the proposed fusion/diffusion approach. From top to bottom and left to right: original image, Perona Malik filtered image, result using the proposed approach

The last application deals with denoising of a fingerprint image. The PDE-based filter from (8) is being reported in the literature as being extremely efficient for this task. As shown in the result in Fig.8.c) this filter smoothes better the oriented patterns than a classical Perona Malik filter (Fig.8.b).

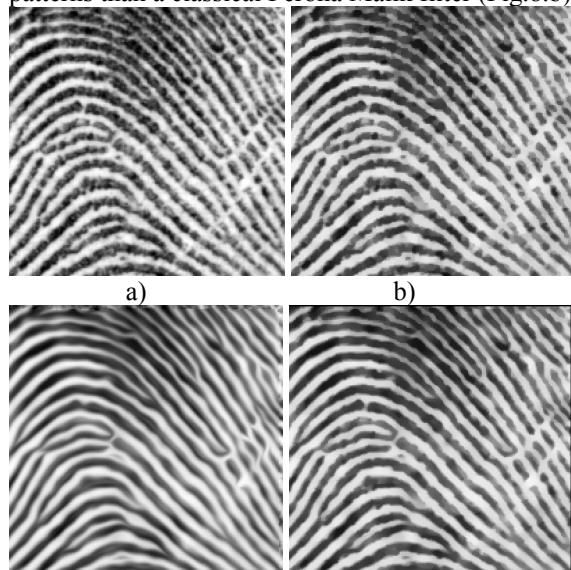


Fig.8 Denoising using the proposed approach. a) Original image, b) Perona Malik result (1); c) Result using (8); d) Result obtained fusing the results of (8) and (1) - 3 fusion steps and 25 restoration time steps for each PDE filter were used

This type of approach is mostly dedicated to image enhancement task and one can notice the artificial aspect of the output image. The use of the Perona Malik equation (1) as a “reaction” term has the expected result: edges are enhanced and no artifacts are created when fusing the results of the two filters.

VI. CONCLUSIONS AND FUTURE WORK

We proposed a new framework for image restoration using the PDE formalism and image fusion techniques that allows the combination of the effects of any PDE based filter. The results are showing that this framework can be effectively used to design new image restoration operators. Future work will be devoted to the study of the influence of the fusion step on the quality of the results (number of decomposition levels, use of the wavelet transform) and to the elaboration of more elaborated fusion rules based on geometrical constraints.

ACKNOWLEDGEMENTS

The research activity presented in this paper was funded by the ”Tehnici de difuzie si de fuziune pentru restaurarea si imbunatatirea imaginilor” PNCDI-II no. 908/2007 research grant.

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